“Don’t wait until you reach your

goal to be proud of yourself.

Be proud of every step you take.”

KAREN SALMANSOHN

CHAPTER 4

Linear Equations in Two Variables

Section 4.1 Objectives

- Write the ordered pair for a given point on a graph.
- Plot the point for a given ordered pair.
- Determine if an ordered pair is a solution of a given equation.
- Determine the unknown coordinate in an ordered pair solution of a given equation.
- Determine the *x*-intercept and *y*-intercept of the graph of a given linear equation.

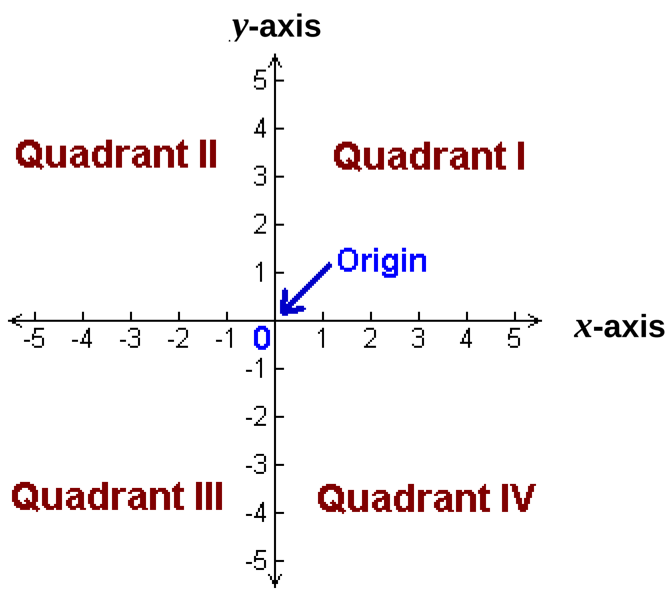
SECTION 4.1 Points and Lines

INTRODUCTION

This section deals with points and lines. Specifically, you will learn two ways of representing points and lines. One way is algebraically and the other way is graphically. You will learn how algebra is used to represent a point as an ordered pair and to represent a line as an equation. You will also see that points and lines can be graphed by drawing them on a grid called the coordinate plane. We begin with a description of the coordinate plane.

COORDINATE PLANE

A coordinate plane is formed by two number lines, a horizontal number line and a vertical number line, that intersect at a point called the ***origin***. The number lines are called ***axes***. The horizontal number line is the ***x-axis*** and the vertical number line is the ***y-axis***. These two number lines divide the plane (a two-dimensional surface) into four ***quadrants***. They are labeled below.



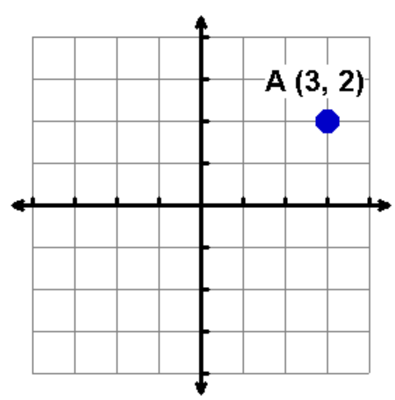
POINTS ON THE COORDINATE PLANE

Now we will study points that lie on the coordinate plane.

The algebraic representation of a point gives the position of the point on the coordinate plane. The algebraic representation of the point to the right is the ordered pair (3, 2).

The graphical representation of a point visually shows the point on the coordinate plane. The graphical representation of the point to the right is simply the dot you see in Quadrant I.

The letter A that you see before (3, 2) is the name of the point. A point is always named with a single capital letter.



ORDERED PAIRS FOR POINTS

The position of a point on the coordinate plane is given using an ***ordered pair*** like (3, 2). An ordered pair contains an ***x-coordinate*** and a ***y-coordinate*** that tell you exactly where the point lies on the coordinate plane.

The term "ordered" pair means that the order of the two coordinates is important. The *x*- coordinate is always written first, and the *y*-coordinate is always written second. You can remember this because this is the same order that the letters *x* and *y* appear in the English alphabet. Notice also that the *x* and *y* coordinates are separated by a comma and parentheses are put around them.

Consider the following ordered pairs:

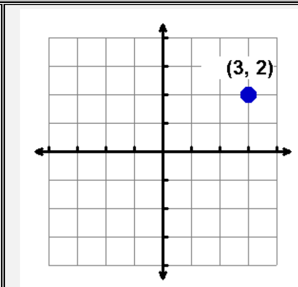
A $(5, 2)$	*x*-coordinate = 5	*y*-coordinate = 2
B $(-4, 3)$	*x*-coordinate = –4	*y*-coordinate = 3

ORDERED PAIRS

An ordered pair is a pair of numbers used to give the position of a point on the coordinate plane.

Ordered Pair: (x, y)

The first number in the ordered pair is the *x*-coordinate.
The second number in the ordered pair is the *y*-coordinate.



Now you will learn how to determine the ordered pair for a point that is plotted on the coordinate plane. In other words, just the point (dot) will be shown on the coordinate plane and you will determine the *x*-coordinate and *y*-coordinate that indicate the position of the point.

DETERMINING THE ORDERED PAIR OF A POINT

1. Start at the origin and first move left or right (in the *x* direction) until you are lined up with the point. This gives the *x*-coordinate.

$$x \begin{cases} \text{positive if right} \\ \text{negative if left} \end{cases}$$

2. Then, from that location, move up or down (in the *y* direction) until you are at the point. The number of units moved up or down is the *y*-coordinate.

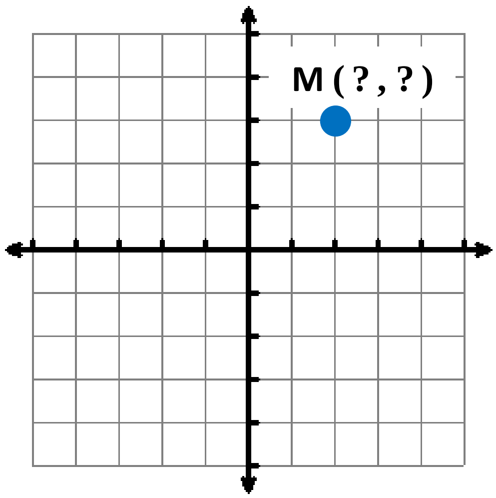
$$y \begin{cases} \text{positive if up} \\ \text{negative if down} \end{cases}$$

3. Finally, write the two coordinates as an ordered pair (x, y).

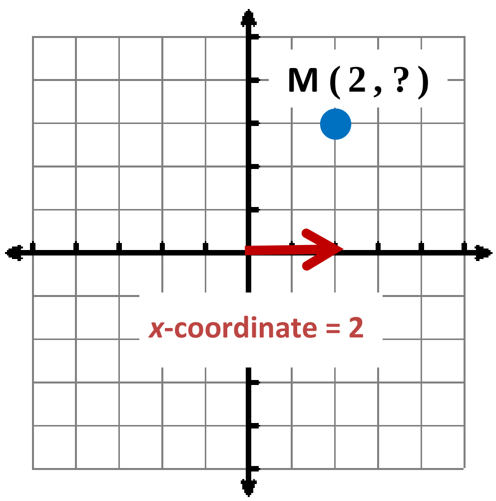
<u>Note</u>: The ordered pair of the origin is (0, 0).

EXAMPLES: Write the ordered pair for each point.

1. Write the ordered pair (x, y) for point M.



We need to determine the *x*-coordinate and the *y*-coordinate of point M.

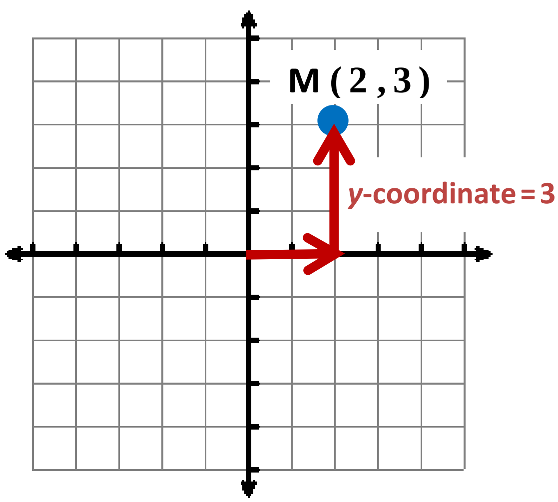


Start at the origin.

The *x*-coordinate represents a movement right or left.

In this problem, we need to move **right** until we are lined up with point M.

Since we moved 2 units to the **right**, the *x*-coordinate is 2.



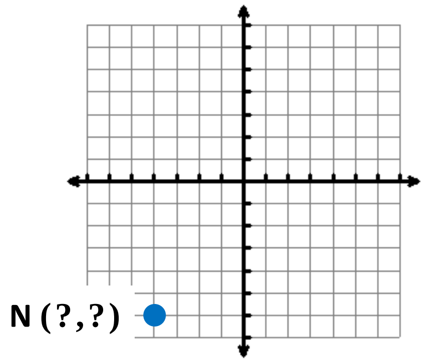
The *y*-coordinate represents a movement up or down.

From our current location, we need to move 3 units **up** to get to point M.

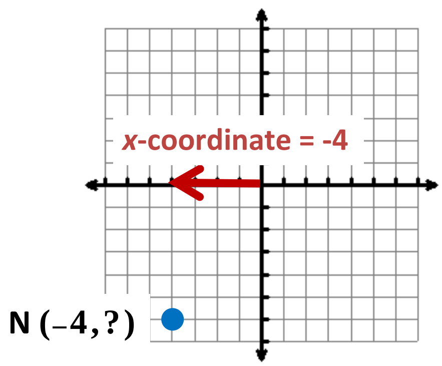
Since we moved 3 units **up**, the *y*-coordinate is 3.

So, point M is represented by the ordered pair $(2,3)$.

2. Write the ordered pair (x, y) for point N.



We need to determine the *x*-coordinate and the *y*-coordinate of point N.

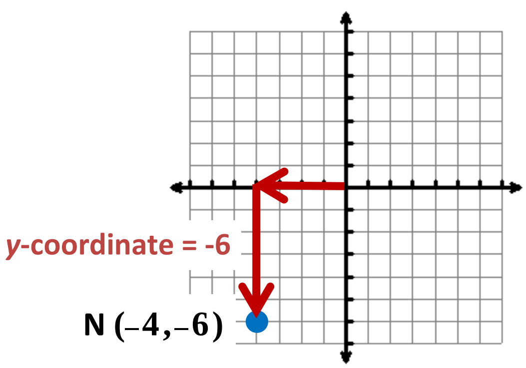


Start at the origin.

The *x*-coordinate represents a movement right or left.

In this problem, we need to move **left** until we are lined up with point N.

Since we moved 4 units to the **left**, the *x*-coordinate is –4.



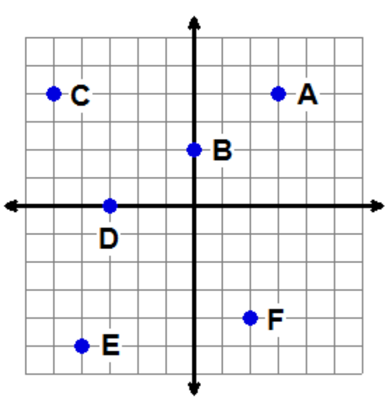
The *y*-coordinate represents a movement up or down.

From our current location, we need to move 6 units **down** to get to point N.

Since we moved 6 units **down**, the *y*-coordinate is –6.

So, point N is represented by the ordered pair $(-4, -6)$.

PRACTICE: Write the ordered pair (x, y) for each point.



ANSWERS:

A (3, 4)

B (0, 2)

C (–5, 4)

D (–3, 0)

E (–4, –5)

F (2, –4)

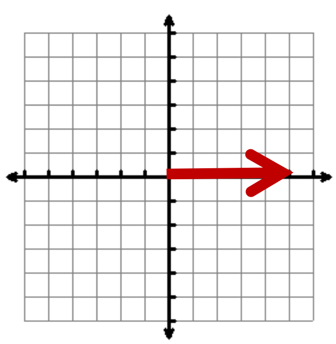
PLOTTING POINTS

The graph of a point is simply a dot on the coordinate plane. To graph, or plot, a point means to draw a dot at the coordinates that correspond to its ordered pair. The steps for doing this are listed below.

PLOTTING A POINT
1. Begin at the origin. 2. Move left or right the number of units corresponding to the *x*-coordinate. Positive *x* → Move Right Negative *x* → Move Left *<u>Note</u>: Do not put a dot yet, just hold the position with your pencil.* 3. From the current position, move up or down the number of units corresponding to the *y*-coordinate. Positive *y* → Move Up Negative *y* → Move Down 4. Plot the dot at the end position!
(*x*, *y*) **Right if Positive** / **Left if Negative** ← *x* *y* → **Up if Positive** / **Down if Negative**

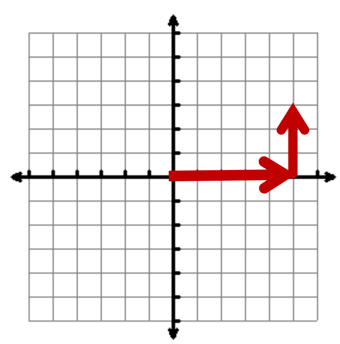
<u>EXAMPLES</u>: Plot each point.

1. Plot the point A (5, 3).



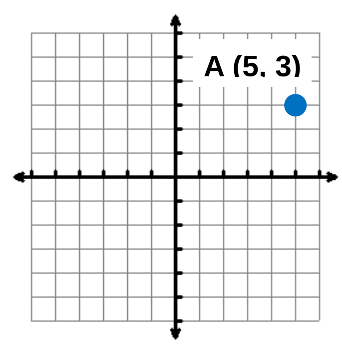
The first coordinate, the *x*-coordinate, indicates how far to move left or right from the origin.

In this problem, the *x*-coordinate is 5. Since it is <u>positive</u>, we move 5 units to the <u>right</u>.



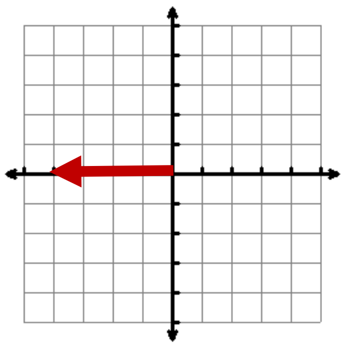
The second coordinate, the *y*-coordinate, indicates how far to move up or down *from our current location.*

In this problem, the *y*-coordinate is 3. Since it is <u>positive</u>, we move 3 units <u>up</u>.



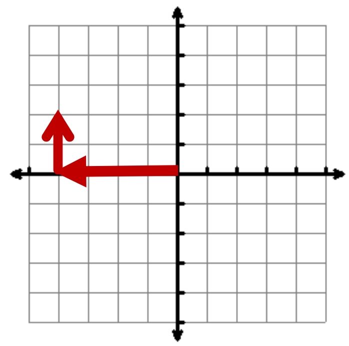
Place a dot at the ending position to represent the point.

2. Plot the point B (–4, 2).



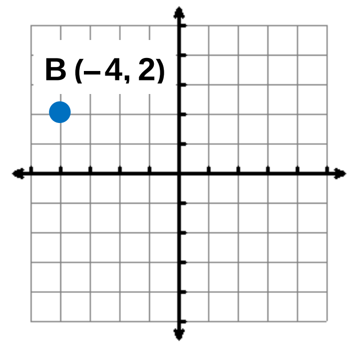
The x-coordinate indicates how far to move left or right from the origin.

The x-coordinate is –4. Since it is <u>negative</u>, we move 4 units to the <u>left</u>.



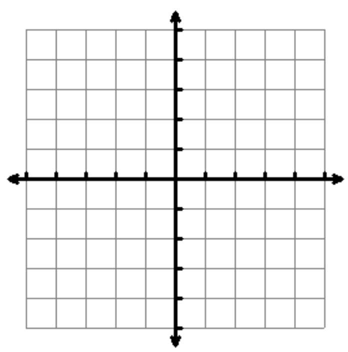
The y-coordinate indicates how far to move up or down *from our current location.*

The y-coordinate is 2. Since it is <u>positive</u>, we move 2 units <u>up</u>.

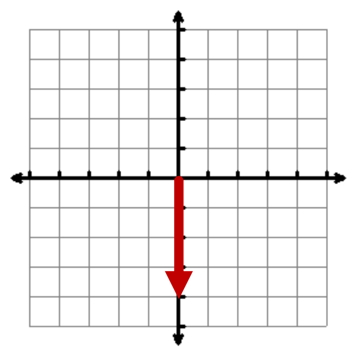


Place a dot at the ending position to represent the point.

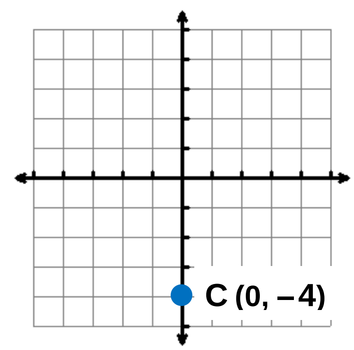
3. Plot the point C (0, –4).



The x-coordinate is 0. So, <u>do not move</u> left or right, just remain at the origin.

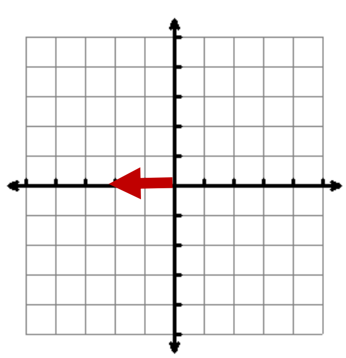


The y-coordinate is –4. Since it is <u>negative</u>, we move 4 units <u>down.</u>

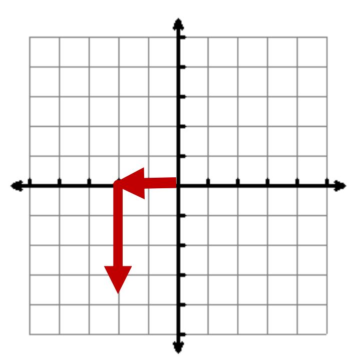


Place a dot at the ending position to represent the point.

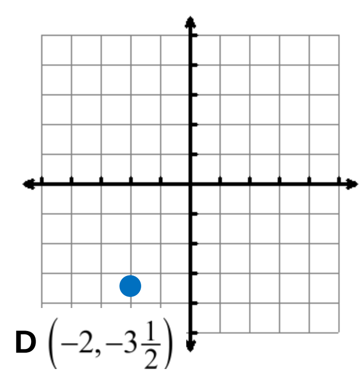
4. Plot the point D $\left(-2, -3\frac{1}{2}\right)$.



The x-coordinate is –2. Since it is <u>negative</u>, we move 2 units to the <u>left</u> of the origin.

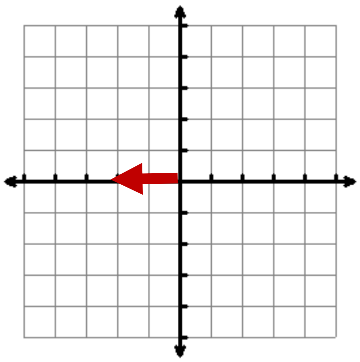


The y-coordinate is $-3\frac{1}{2}$. Since it is <u>negative</u>, we move $3\frac{1}{2}$ units <u>down</u> from the current position.

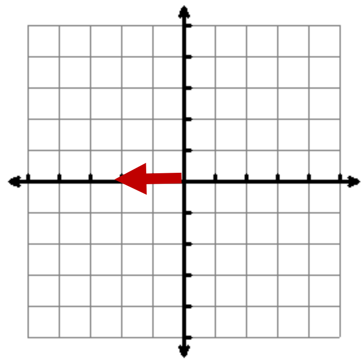


Place a dot at the ending position to represent the point.

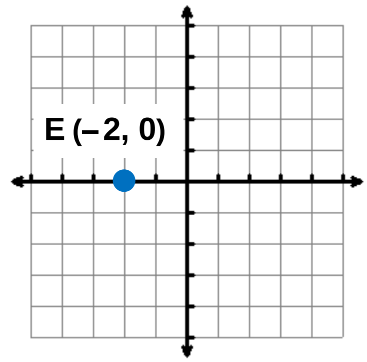
5. Plot the point E $(-2,0)$.



The x-coordinate is -2. Since it is negative, we move 2 units to the left.



The y-coordinate is 0. So, do not move up or down at all.



Place a dot at the ending position to represent the point.

PRACTICE: Plot the following points and label each using the capital letter.

A $(1,5)$

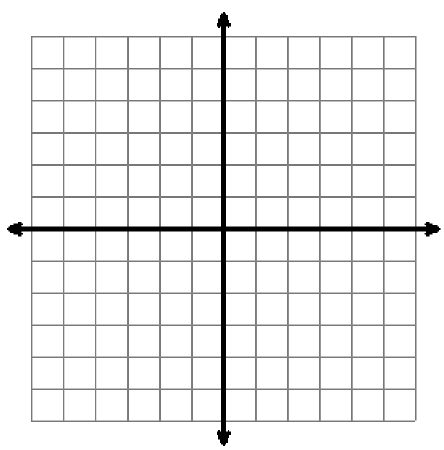
B $(2,-3)$

C $(-3,-1)$

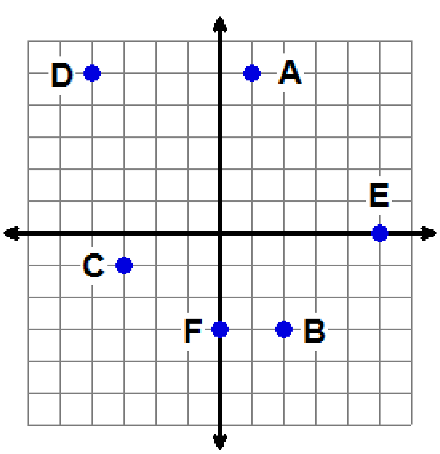
D $(-4,5)$

E $(5,0)$

F $(0,-3)$



ANSWERS:

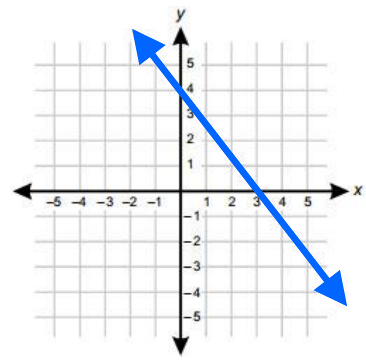


LINES ON THE COORDINATE PLANE

Now we will turn our attention to lines on the coordinate plane. The graphical representation of a line is a drawing that visually shows the line on the coordinate plane, and the algebraic representation of a line is an equation.

GRAPH OF A LINE

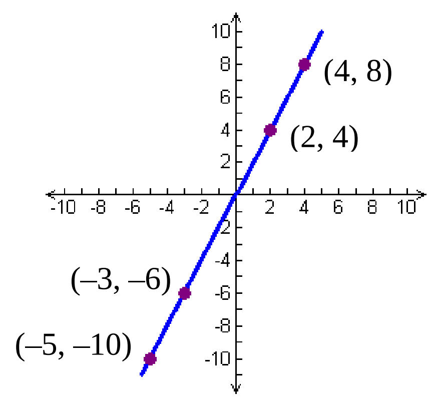
The graph of a line is a drawing of the line on the coordinate plane. A line is perfectly straight. A line may be positioned in any direction: horizontal, vertical, slanted upwards, or slanted downwards. Arrows are placed on a line to show that the line extends infinitely in both directions.



Note: *You should be aware that the graphs of lines shown in this text may not display the arrows due to a limitation of the technology used to draw the lines. Despite this, you should remember that all lines are endless.*

EQUATION OF A LINE

Think of a line as being made up of an infinite number of points. There is a relationship that exists between the x-coordinates and the y-coordinates of all the points on a particular line. That relationship can be expressed as an algebraic equation.



For example, we show four points on the line to the right. When you look at each ordered pair, you can see that the y-coordinate is twice the x-coordinate. This will be true for every point on the line.

This relationship between x and y can be expressed as the algebraic equation $y = 2x$. So, we say that the equation of the line graphed to the right is the ***linear equation*** $y = 2x$.

Equations whose graphs are lines are called ***linear equations in two variables***.

Often the equation representing a line will be given in the form Ax + By = C where A, B, and C are real numbers. This form is called the ***standard form of a linear equation in two variables***. Some examples of linear equations for different lines are: $2x - y = 0$, $x + 5y = 8$, and $6x - 3y = 12$.

LINEAR EQUATION IN TWO VARIABLES
Standard Form of a Linear Equation: $Ax + By = C$ The graph of a linear equation in two variables is a line.

Later in this chapter, you will learn how to determine the algebraic equation of a given line. You will also learn how to graph a line. For now, we focus only on finding solutions of a linear equation.

SOLUTION OF A LINEAR EQUATION IN TWO VARIABLES

A *solution* for the equation of a line is different from the solutions to other equations that you have solved. A solution for the equation of a line is an ordered pair representing a point that lies on the line. This means that the coordinates of the ordered pair must satisfy the equation for that line. To determine if a given ordered pair is a solution of a linear equation, you will use a familiar procedure. You will substitute the given values (the *x* and *y* coordinates) in the equation and determine if the simplified equation is true.

SOLUTION OF A LINEAR EQUATION IN TWO VARIABLES
Solution of a Linear Equation: • An ordered pair representing a point that lies on the line • The coordinates of the ordered pair (*x*, *y*) must satisfy the equation To determine if a given ordered pair is a solution of a given linear equation: 1. Substitute the *x*-coordinate and the *y*-coordinate into the equation. 2. Simplify until there is just one number on both sides of the equation. 3. If the two sides of the equation are equal, then the ordered pair is a solution. Otherwise, the ordered pair is not a solution.

EXAMPLES: Determine if the given ordered pair is a solution of the equation $x+2y=6$.

1. Is (4, 1) a solution of the equation $x+2y=6$?

$x=4$ and $y=1$ — In the ordered pair (4, 1), the *x*-coordinate is 4 and the *y*-coordinate is 1.

$x+2y=6$ — In the equation, replace *x* with 4 and replace *y* with 1.

$$\mathbf{4}+2(\mathbf{1})\stackrel{?}{=}6$$ Simplify the left side of the equation.

$$4+2\stackrel{?}{=}6$$

$$6=6 \checkmark$$ This is a true statement. Therefore, (4, 1) is a solution.

YES — Note: This means that (4, 1) is a point on the line $x+2y=6$.

2. Is $(-4,5)$ a solution of the equation $x+2y=6$?

$x=-4$ and $y=5$ — In the ordered pair (–4, 5), the *x*-coordinate is –4 and the *y*-coordinate is 5.

$x+2y=6$ — In the equation, replace *x* with –4 and replace *y* with 5.

$$\mathbf{-4}+2(\mathbf{5})\stackrel{?}{=}6$$ Simplify the left side of the equation.

$$-4+10\stackrel{?}{=}6$$

$$6=6 \checkmark$$ This is a true statement. Therefore, (–4, 5) is a solution.

YES — Note: This means that (–4, 5) is a point on the line $x+2y=6$.

In the last two examples, we found that both (4, 1) and (–4, 5) are solutions of the equation $x+2y=6$.

In fact, there are an infinite number of solutions for the equation because there are an infinite number of points on the line. However, not every point on the coordinate plane will satisfy the equation of a particular line. Only points that result in a true statement are solutions. Consider one more example below for the same equation that was used in examples 1 and 2.

3. Is (3, 4) a solution of the equation $x + 2y = 6$?

$x=3$ and $y=4$	In the ordered pair (3, 4), the *x*-coordinate is 3 and the *y*-coordinate is 4.
$x+\ 2y\ =\ 6$ $\downarrow \quad \downarrow$	In the equation, replace *x* with 3 and replace *y* with 4.
$\mathbf{3}+2(\mathbf{4}) \stackrel{?}{=} 6$ $3+\ 8 \stackrel{?}{=} 6$	Simplify the left side of the equation.
$11 = 6$ ✕	This is <u>not</u> a true statement. Therefore, (3, 4) is <u>not</u> a solution.
NO	Note: This means that (3, 4) is <u>not</u> a point on the line $x + 2y = 6$.

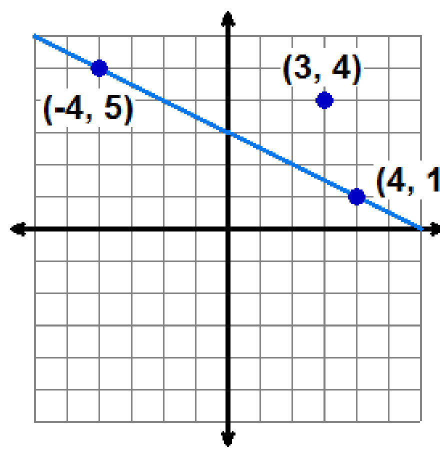
Our algebraic work in the last three examples showed if the given ordered pairs were solutions of the linear equation $x+2y=6$.

- Solutions: (4, 1) and (–4, 5)
- Not a Solution: (3, 4)

The graph to the right confirms our algebraic results. Remember that a solution represents a point that lies on the line.

We graphed the line $x+2y=6$.

You can see that the points (4, 1) and (–4, 5) lie on the line.
You can also see that the point (3, 4) does not lie on the line.



<u>Note</u>: *You are not expected to know how to graph the line.*
You will learn this later in the chapter.

<u>PRACTICE</u>: Determine if the ordered pair is a solution of the equation.

1. Is $(-2,3)$ a solution of the equation $x-4y=-14$?
2. Is $(2,-5)$ a solution of the equation $6x-y=9$?
3. Determine if each ordered pair below is a solution of the equation $3x+y=9$.
   a. $(2,3)$
   b. $(0,3)$
   c. $(5,-6)$
4. Determine if each ordered pair below is a solution of the equation $4x-6y=20$.
   a. $(-1,-4)$
   b. $(5,0)$
   c. $(-2,1)$

<u>ANSWERS</u>:

1. Yes
2. No
3. a. Yes
   b. No
   c. Yes
4. a. Yes
   b. Yes
   c. No

DETERMINING AN UNKNOWN COORDINATE IN A SOLUTION

Now you will be given the equation of a line and an ordered pair with only one of the two coordinates given. You will use algebra to solve for the unknown coordinate so that the ordered pair is a solution of the given equation.

DETERMINING AN UNKNOWN COORDINATE IN A SOLUTION
1. Substitute the given coordinate in the equation. 2. Use algebra to solve for the remaining variable.

EXAMPLES: Determine the unknown coordinate so that the ordered pair is a solution of the equation.

1. Find the unknown coordinate so that (___ , – 2) is a solution of $6x - 3y = 12$.

$x = ?$ and $y = -2$ The ordered pair shows that the *x*-coordinate is unknown and the *y*-coordinate is –2.

$6x - 3y = 12$ In the equation, replace *y* with the given value –2.

$6x - 3(-2) = 12$ Simplify the left side of the equation.

$6x \cancel{+6} = 12$ Now use algebra to solve for *x*.

$\cancel{-6} \quad -6$ Subtract 6 from both sides to get the variable term 6*x* alone on the left side.

$6x = 6$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{6}{6}$ Divide both sides of the equation by 6 to get the variable alone.

$x = 1$ So, the point (1, –2) is a solution of the equation $6x - 3y = 12$.
This means that (1, –2) is a point on the line $6x - 3y = 12$.

2. Find the unknown coordinate so that (0, ___) is a solution of $6x - 3y = 12$.

$x = 0$ and $y = ?$ The ordered pair shows that the *x*-coordinate is 0 and the *y*-coordinate is unknown.

$6x - 3y = 12$ In the equation, replace *x* with the given value 0.

$6(0) - 3y = 12$ Simplify the left side of the equation.

$0 - 3y = 12$ Now use algebra to solve for *y*.

$\frac{\cancel{-3}y}{\cancel{-3}} = \frac{12}{-3}$ Divide both sides by -3 to get the variable alone.

$y = -4$ So, the point (0, –4) is a solution of the equation $6x - 3y = 12$.
This means that (0, –4) is a point on the line $6x - 3y = 12$.

3. Find the unknown coordinate so that (1, ___) is a solution of $2x-y=4$.

$x=1$ and $y=?$	The ordered pair shows that the x-coordinate is 1 and the y-coordinate is unknown.
$2x-y=4$	In the equation, replace x with 1.
$2(\mathbf{1})-y=4$	Simplify the left side of the equation.
$\cancel{2}-y=4$	Now use algebra to solve for y.
$\cancel{-2} \quad \cancel{-2}$	Subtract 2 from both sides.
$-y=2$	
$\frac{\cancel{-1}y}{\cancel{-1}}=\frac{2}{-1}$	Divide both sides by -1.
$y=-2$	So, the point (1, –2) is a solution of the equation $2x-y=4$. This means that (1, –2) is a point on the line $2x-y=4$.

4. Find the unknown coordinate so that (___ , 0) is a solution of $2x-y=4$.

$x=?$ and $y=0$	The ordered pair shows that the x-coordinate is unknown and the y-coordinate is 0.
$2x-y=4$	In the equation, replace y with 0.
$2x-(\mathbf{0})=4$	Simplify the left side of the equation.
$2x=4$	
$\frac{\cancel{2}x}{\cancel{2}}=\frac{4}{2}$	Divide both sides by 2.
$x=2$	So, the point (2, 0) is a solution of the equation $2x-y=4$. This means that (2, 0) is a point on the line $2x-y=4$.

PRACTICE: Determine the unknown coordinate in each ordered pair so that it is a solution of the equation.

1. Determine the unknown coordinate so that $(\_\_,-1)$ is a solution of $3x-2y=8$.
2. Determine the unknown coordinate so that $(0,\_\_)$ is a solution of $-5x+8y=56$.
3. Determine the unknown coordinate in each ordered pair so that it is a solution of the equation $-2x+4y=12$.
   a. (4, ___)
   b. (___ ,1)
   c. (0, ___)
   d. (___ ,0)

ANSWERS:

1. $(\mathbf{2},-1)$	3a. $(4,\mathbf{5})$	3c. $(0,\mathbf{3})$
2. $(0,\mathbf{7})$	3b. $(\mathbf{-4},1)$	3d. $(\mathbf{-6},0)$

ORGANIZING SOLUTIONS OF EQUATIONS USING TABLES

Remember that a line is made up of an infinite number of points. Therefore, the equation of a line has an infinite number of solutions. We cannot list *all* the solutions, but we can find and list some of the solutions. In the last set of examples, we showed the procedure for finding some solutions. You used this procedure when you completed the last set of practice problems.

In practice problem #3, you found four ordered pairs that are solutions of the equation $-2x+4y=12$.

The results are shown below.

Solutions of
$-2x+4y=12$:

(4, 5)

(–4, 1)

(0, 3)

(–6, 0)

Another way to organize ordered pair solutions of an equation is to use a ***table format***. The x-coordinates are shown in the left-hand column and the corresponding y-coordinates are shown in the right-hand column.

The results of practice problem #3 are shown in a table below.

Solutions of
$-2x+4y=12$:

x	y
4	5
–4	1
0	3
–6	0

Since there is an infinite number of points on a line, you can find an ordered pair that is a solution by choosing *any* value of x or y, substituting that value into the equation, and then solving for the corresponding coordinate. This process can be repeated to find as many ordered pair solutions as you want.

TABLES

Table: a format used to organize ordered pair solutions of an equation

- First Column – contains the x-coordinates
- Second Column – contains the corresponding y-coordinates

To Find a Solution of an Equation:

1. Choose any value for x or y.
2. Substitute that value in the equation.
3. Solve for the corresponding coordinate.

Repeat these steps to find additional solutions.

x	y
4	**5**
–4	1
0	**3**
–6	0

In the examples and practice exercises that follow, one of the coordinates in each ordered pair will be chosen for us and we will solve for the unknown coordinate.

EXAMPLE: Complete the table to find several solutions of the equation $x + y = 5$.

Solutions of
$x + y = 5$

x	y
1	4
2	
–1	
	–2

First Row of Table: This row is complete. It shows that (1, 4) is a solution.

Second Row: The x-coordinate is given as 2 and the y-coordinate is unknown. So, in the equation, replace x with 2 and use algebra to solve for y. ➡

$$\begin{aligned} x + y &= 5 \\ \mathbf{2} + y &= 5 \\ -2 \quad & \quad -2 \\ \hline y &= 3 \end{aligned}$$

Third Row: The x-coordinate is given as –1 and the y- coordinate is unknown. In the equation, replace x with –1 and solve for y. ➡

$$\begin{aligned} x + y &= 5 \\ \mathbf{-1} + y &= 5 \\ +1 \quad & \quad +1 \\ \hline y &= 6 \end{aligned}$$

Fourth Row: The y-coordinate is given as –2 and the x-coordinate is unknown. In the equation, replace y with –2 and solve for x. ➡

$$\begin{aligned} x + \quad y &= 5 \\ x + \mathbf{(-2)} &= 5 \\ +2 \quad & \quad +2 \\ \hline x \quad &= 7 \end{aligned}$$

Completed Table

Solutions of
$x + y = 5$

x	y
1	4
2	**3**
–1	**6**
7	–2

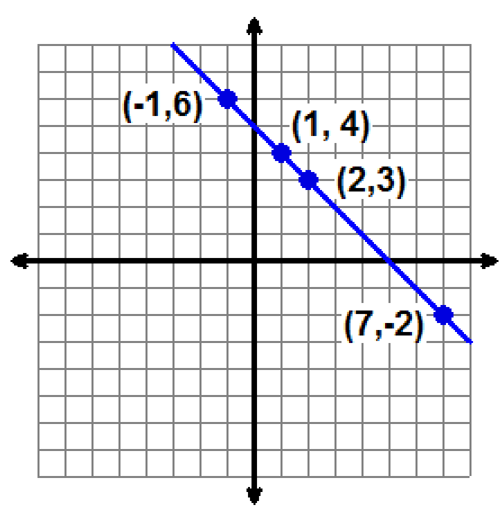
Use the results from above to fill in the unknown values in rows two, three, and four of the table.

Our algebraic work above showed that the following ordered pairs are solutions of the equation $x + y = 5$:

(1, 4)
(2, 3)
(–1, 6)
(7, –2)

This means that all four points lie on the line $x + y = 5$.

We graphed the line to the right so you can see for yourself that the points do lie on the line.



Note: You are not expected to know how to graph the line. You will learn this later in the chapter.

PRACTICE: Complete each table to find solutions of the given equation.

1. Solutions of $x + 2y = 6$

x	y
-4	
	-4
	0

2. Solutions of $-4x + y = 4$

x	y
0	
3	
	0

ANSWERS:

1. Solutions of $x + 2y = 6$

x	y
-4	**5**
14	-4
6	0

2. Solutions of $-4x + y = 4$

x	y
0	**4**
3	**16**
–1	0

INTERCEPTS

There are two solutions of a linear equation that are very important. One is an ordered pair with an x-coordinate of 0, and the other is an ordered pair with a y-coordinate of 0. These two solutions are important because they correspond to the points where the line crosses the axes of the coordinate plane.

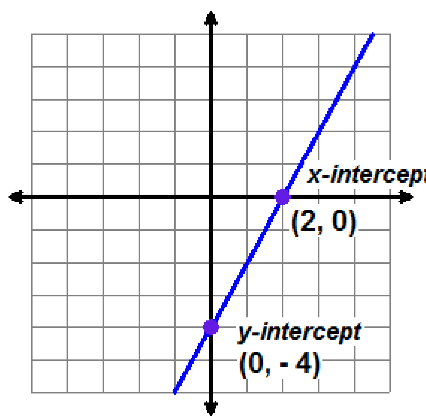
We explain this further by showing an example below. We show the graph of the line $2x - y = 4$, and we show two solutions on the line, (2, 0) and (0, – 4). Notice that each ordered pair has a coordinate of 0.

The line intersects (crosses) the ***x***-axis at (2, 0). So, we say that the ***x-intercept*** of the line is 2.

Notice that the corresponding ***y***-coordinate is 0.

The line intersects (crosses) the ***y***-axis at (0, – 4). So, we say that the ***y-intercept*** of the line is – 4.

Notice that the corresponding ***x***-coordinate is 0.



INTERCEPTS OF A LINE

***x*-intercept**: the x-coordinate of the point where a line crosses the x-axis	***y*-intercept**: the y-coordinate of the point where a line crosses the y-axis
To find the x-intercept: 1. In the equation, replace y with 0. 2. Solve for x.	To find the y-intercept: 1. In the equation, replace x with 0. 2. Solve for y.

EXAMPLES: Determine the x or y intercept as specified.

1. Find the x-intercept of the line $-3x+4y=12$.

$$
\begin{aligned}
-3x + 4y &= 12 && \text{In the equation, replace } y \text{ with 0.}\\
-3x+4(0) &= 12 && \text{Simplify the left side of the equation.}\\
-3x &= 12 && \\
\frac{\cancel{-3}x}{\cancel{-3}} &= \frac{12}{-3} && \text{Divide both sides of the equation by –3.}\\
x &= -4 && \text{When } y=0,\ x=-4.
\end{aligned}
$$

***x*-intercept:** –4 The x-intercept is –4, so the line crosses the x-axis at the point (–4, 0).

2. Find the y-intercept of the line $-3x+4y=12$.

$$
\begin{aligned}
-3x + 4y &= 12 && \text{In the equation, replace } x \text{ with 0.}\\
-3(0)+4y &= 12 && \text{Simplify the left side of the equation.}\\
4y &= 12 && \\
\frac{\cancel{4}y}{\cancel{4}} &= \frac{12}{4} && \text{Divide both sides of the equation by 4.}\\
y &= 3 && \text{When } x=0,\ y=3.
\end{aligned}
$$

***y*-intercept:** 3 The y-intercept is 3, so the line crosses the y-axis at the point (0, 3).

PRACTICE: Determine the x and/or y intercept as specified.

1. Find the x-intercept of the line $-2x+3y=-6$.
2. Find the y-intercept of the line $-2x+3y=-6$.
3. Find the y-intercept of the line $5x-3y=-30$.
4. Find the x-intercept of the line $5x-3y=-30$.
5. Find the x and y intercepts of the line $2x-4y=8$.
6. Find the x and y intercepts of the line $x+3y=9$.

ANSWERS:

1. $x = 3$
2. $y = –2$
3. $y = 10$
4. $x = –6$
5. $x = 4$ and $y = –2$
6. $x = 9$ and $y = 3$

SECTION 4.1 SUMMARY
Points and Lines

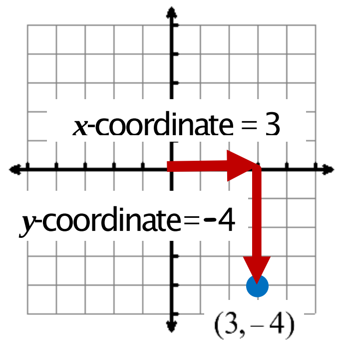
ORDERED PAIRS AND PLOTTING POINTS

An ordered pair (x, y) gives the position of a point on the coordinate plane.

To Write an Ordered Pair:

1. Start at the origin. Move left or right to align with the point. The number of units moved is the x-coordinate: Right: $+x$ Left: $-x$
2. From that spot, move up or down until you reach the point. The number of units moved is the y-coordinate: Up: $+y$ Down: $-y$
3. Write the two coordinates as an ordered pair (x, y).

Example:



To Plot a Point:

1. Start at the origin. Move left or right the number of units equal to the x- coordinate:
   $+x$ move Right
   $-x$ move Left

   Note: Do not put a dot yet.
2. From that position, move up or down the number of units equal to the y-coordinate:
   $+y$ move Up
   $-y$ move Down
3. Put a dot at the end position.

LINEAR EQUATIONS AND THEIR SOLUTIONS

Linear Equation – an equation whose graph is a line; Standard Form: $Ax + By = C$

Solution of a Linear Equation

- An ordered pair representing a point that lies on the line
- The coordinates of the ordered pair (x, y) must satisfy the equation

To Determine if an Ordered Pair is a Solution of an Equation:

1. Substitute the x and y coordinates into the equation.
2. Simplify both sides of the equation.
3. If the two sides are equal, then the ordered pair is a solution.

Example: Is (5, 6) a solution of the equation $4x + y = 26$?

$$4(5) + 6 = 26$$
$$20 + 6 = 26$$
$$26 = 26 \checkmark$$

Yes, the point is a solution.

To Determine an Unknown Coordinate in a Solution:

1. Substitute the given coordinate in the equation.
2. Solve for the unknown coordinate.

Example: Find the unknown coordinate so that (1, __) is a solution of $3x + y = 15$.

$$3(1) + y = 15$$
$$3 + y = 15$$
$$-3 \qquad -3$$
$$y = 12$$

Table: used to organize ordered pair solutions of an equation

1. Choose any value for x or y.
2. Substitute that value in the equation.
3. Solve for the unknown coordinate.

Example: Complete the table to find solutions of $x + y = 5$.

x	y
2	
–1	
	–2

Solve as shown in example to left.

INTERCEPTS

***x*-Intercept:** the point where a line crosses the x-axis

To Find the x-Intercept:

1. Replace y with 0.
2. Solve for x.

Note: The line crosses the x-axis at (3, 0)

Example: Find the x-intercept of $2x + y = 6$.

$$2x + 0 = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

x-Intercept: 3

***y*-intercept:** the point where a line crosses the y-axis

To Find the y-Intercept:

1. Replace x with 0.
2. Solve for y.

Note: The line crosses the y-axis at (0, 6)

Example: Find the y-intercept of $2x + y = 6$.

$$2(0) + y = 6$$
$$0 + y = 6$$
$$y = 6$$

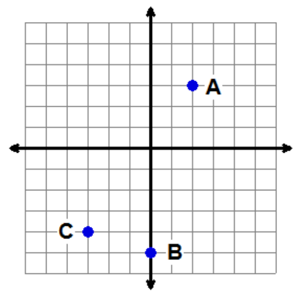
y-Intercept: 6

SECTION 4.1 EXERCISES

Points and Lines

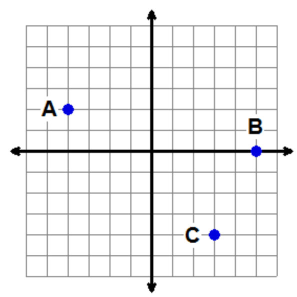
Write the ordered pair (x, y) for each of the points shown on the graph.

1. A __________
2. B __________
3. C __________



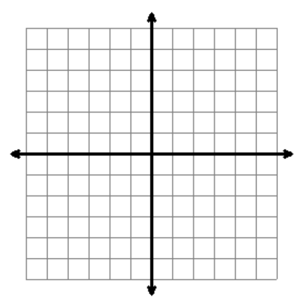
Write the ordered pair (x, y) for each of the points shown on the graph.

4. A __________
5. B __________
6. C __________



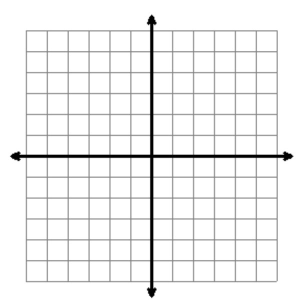
Plot the points on the coordinate plane and label them using the capital letters.

7. A $(-5,0)$
8. B $(1,-3)$
9. C $(-2,5)$



Plot the points on the coordinate plane and label them using the capital letters.

10. A $(3,1)$
11. B $(0,2)$
12. C $(-5,-1)$



Determine if the ordered pair is a solution of the given equation.

13. Is $(3,4)$ a solution of the equation $-2x+5y=9$?
14. Is $(-5,0)$ a solution of the equation $4x-6y=-20$?
15. Is $(2,-5)$ a solution of the equation $-x+2y=-12$?
16. Is $(-6,9)$ a solution of the equation $2x+3y=21$?
17. Is $(0,9)$ a solution of the equation $-3x-4y=7$?
18. Is $(-1,-8)$ a solution of the equation $5x-y=3$?

Determine the unknown coordinate so that the ordered pair is a solution of the given equation.

19. Find the unknown coordinate so that $(\ \_\_\ , 0\ )$ is a solution of $-6x-5y=18$.
20. Find the unknown coordinate so that $(\ -2, \_\_\ )$ is a solution of $3x+2y=-12$.
21. Find the unknown coordinate so that $(\ \_\_\ , 2\ )$ is a solution of $-x+4y=12$.
22. Find the unknown coordinate so that $(\ 0, \_\_\ )$ is a solution of $4x-3y=-6$.
23. Find the unknown coordinate so that $(\ \_\_\ , -2\ )$ is a solution of $2x - y = 10$.
24. Complete the table below to find solutions to the equation $2x-y=10$.

x	y
-1	
	4
0	
	0

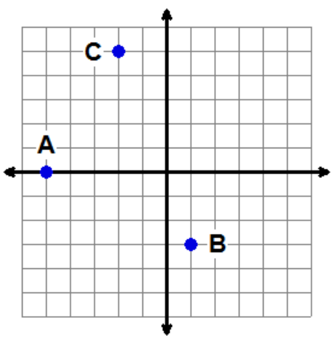
Determine the intercept of the graph of the given equation.

25. Find the y-intercept of the graph of $-4x+5y=20$.
26. Find the x-intercept of the graph of $-x+3y=3$.
27. Find the x-intercept of the graph of $3x-2y=15$.
28. Find the y-intercept of the graph of $-5x-2y=14$.
29. Find the x-intercept of the graph of $6x+y=-6$.
30. Find the y-intercept of the graph of $2x+4y=7$.

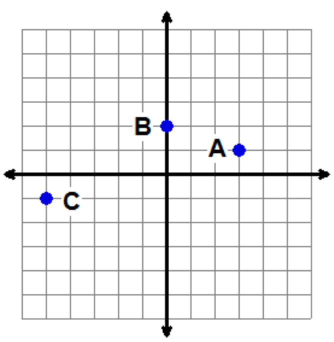
Answers to Section 4.1 Exercises

1. A $(2,3)$
2. B $(0,-5)$
3. C $(-3,-4)$
4. A $(-4,2)$
5. B $(5,0)$
6. C $(3,-4)$

7. A
8. B
9. C



10. A
11. B
12. C



13. No
14. Yes
15. Yes
16. No
17. No
18. Yes

19. $(\mathbf{-3},0)$
20. $(-2,\mathbf{-3})$
21. $(\mathbf{-4},2)$
22. $(0,\mathbf{2})$
23. $(\mathbf{4},-2)$
24.

x	y
–1	**–12**
7	4
0	**–10**
5	0

25. $y = 4$
26. $x = -3$
27. $x = 5$
28. $y = -7$
29. $x = -1$
30. $y = \frac{7}{4}$

Mixed Review

Sections 1.1 – 4.1

1. Solve $\frac{3}{2}x+\frac{6}{5}=x+\frac{4}{3}$

2. Solve $-3(5x+7)+1\le 8(1-x)$, graph the solution, and write the solution in interval notation.

3. Write an algebraic equation for the word problem. Then solve the equation to answer the question.
   Your scores on your last two games of bowling were 130 and 112. What score do you need on your third game to get an average score of 125 for all three games?

4. A formula used in physics is $v=u+at$. If v = 83, u = 29, and a = 18, find t.

5. Solve $-5x+4y=-21$ for y.

6. Solve $\frac{\frac{9}{5}}{3}=\frac{n}{10}$

7. A tutoring center has a ratio of 5 tutors for every 16 students. If the center serves 80 students, how many tutors should the center staff?

8. Convert 222 inches to feet. Use the conversion fact: 12 inches (in) = 1 foot (ft)

9. 120 is what percent of 750?

10. Maryland's sales tax rate is 6%. If you buy a large screen TV for $1800, how much will you pay in sales tax? What will be the total cost of the TV?

Answers to Mixed Review

1. $x=\frac{4}{15}$

2. $x\ge -4$
   $[-4,\infty)$

3. 133

4. $t = 3$

5. $y=\frac{5x-21}{4}$

6. $n = 6$

7. 25 tutors

8. 18.5 ft

9. 16%

10. Tax: $108
    Total Cost: $1908