Section 1.1: Review of Equations

Solve the equation for the variable.

When solving an equation for a variable, we do so by isolating the variable. That is, we move everything that is on the same side of the equal sign as the variable to the other side of the equation so that the variable is by itself. The following examples illustrate the various types of situations that might occur and what steps are needed to solve for the variable in each case.

Example 1.

$x+7=-5$	The 7 is added to the x
$\underline{\;-7\quad -7\;}$	Subtract 7 from both sides so that only x is on the left side
$x+0=-12$	
$x=-12$	Our solution!

Example 2.

$x-5=4$	The 5 is negative, or subtracted from x
$\underline{\;+5\quad +5\;}$	Add 5 to both sides so that only x is on the left side
$x+0=9$	
$x=9$	Our solution!

Example 3.

$-5x=30$	Variable is multiplied by -5
$\dfrac{-5x}{-5}=\dfrac{30}{-5}$	Divide both sides by -5, so that only x is on the left side
$1x=-6$	
$x=-6$	Our Solution!

Example 4.

$\dfrac{x}{5}=-3$	Variable is divided by 5
$(5)\dfrac{x}{5}=-3(5)$	Multiply both sides by 5, so that only x is on the left side
$1x=-15$	
$x=-15$	Our Solution!

Example 5.

$4-2x=10$	Start by focusing on the positive 4
$\underline{-4 \qquad -4}$	Subtract 4 from both sides
$\frac{-2x}{-2} = \frac{6}{-2}$	Negative (subtraction) stays on the $2x$. Divide by -2, the coefficient of $-2x$
$x=-3$	Our Solution!

Example 6.

$4(2x-6)=16$	Distribute 4 through parentheses
$8x-24=16$	Focus on the subtraction first
$\underline{+24 \quad +24}$	Add 24 to both sides
$\frac{8x}{8}=\frac{40}{8}$	Notice the variable is multiplied by 8. Divide both sides by 8, the coefficient of $8x$
$x=5$	Our Solution!

Example 7.

$$4x-6=2x+10$$

Notice here the x is on both the left and right sides of the equation. This can make it difficult to decide which side to work with. We resolve this by moving one of the terms with x to the other side of the equation, much like we moved a constant term. It doesn't matter which term gets moved, $4x$ or $2x$.

$4x-6=2x+10$	Notice the variable on both sides
$\underline{-2x \qquad -2x}$	Subtract $2x$ from both sides
$2x-6=10$	Focus on the subtraction first
$\underline{+6 \quad +6}$	Add 6 to both sides
$\frac{2x}{2}=\frac{16}{2}$	Notice the variable is multiplied by 2. Divide both sides by 2, the coefficient of $2x$
$x=8$	Our Solution!

Example 8.

$4(2x-6)+9=3(x-7)+8x$	Distribute 4 and 3 through parentheses
$8x-24+9=3x-21+8x$	Combine like terms $-24+9$ and $3x+8x$
$8x-15=11x-21$	Notice the variable is on both sides
$\underline{-8x \qquad -8x}$	Subtract $8x$ from both sides
$-15=3x-21$	Focus on subtraction of 21
$\underline{+21 \qquad +21}$	Add 21 to both sides
$6=3x$	Notice the variable is multiplied by 3
$\frac{6}{3}=\frac{3x}{3}$	Divide both sides by 3, the coefficient of $3x$
$2=x$	Our Solution!

Example 9.

$\frac{3}{4}x-\frac{7}{2}=\frac{5}{6}$	Focus on subtraction
$+\frac{7}{2} \quad +\frac{7}{2}$	Add $\frac{7}{2}$ to both sides

We will need to get a common denominator to add $\frac{5}{6}+\frac{7}{2}$. We have a common denominator of 6. So, we rewrite the fraction $\frac{7}{2}$ in terms of the common denominator by multiplying both the numerator and the denominator by 3, $\frac{7}{2}\left(\frac{3}{3}\right)=\frac{21}{6}$. We can now add the fractions:

$\frac{3}{4}x-\frac{21}{6}=\frac{5}{6}$	Same problem, with common denominator 6
$\underline{+\frac{21}{6} \quad +\frac{21}{6}}$	Add $\frac{21}{6}$ to both sides
$\frac{3}{4}x=\frac{26}{6}$	Reduce $\frac{26}{6}$ to $\frac{13}{3}$
$\frac{3}{4}x=\frac{13}{3}$	Focus on multiplication by $\frac{3}{4}$

We can get rid of $\frac{3}{4}$ by dividing both sides by $\frac{3}{4}$. Dividing by a fraction is the same as multiplying by the reciprocal, so we will multiply both sides by $\frac{4}{3}$.

$$\left(\frac{4}{3}\right)\frac{3}{4}x=\frac{13}{3}\left(\frac{4}{3}\right) \quad \text{Multiply by reciprocal}$$

$$x=\frac{52}{9} \quad \text{Our solution!}$$

While this process does help us arrive at the correct solution, the fractions can make the process quite difficult. This is why we use an alternate method for dealing with fractions – clearing fractions. We can easily clear the fractions by finding the LCD and multiplying each term by the LCD. This is shown in the next example, which is the same problem as our first example; but, this time we will solve by clearing the fractions.

Example 10.

$$\frac{3}{4}x-\frac{7}{2}=\frac{5}{6} \quad \text{LCD}=12\text{, multiply each term by } 12$$

$$\frac{(12)3}{4}x-\frac{(12)7}{2}=\frac{(12)5}{6} \quad \text{Reduce the fractions}$$

$$(3)3x-(6)7=(2)5 \quad \text{Multiply out each term}$$

$$9x-42=10 \quad \text{Focus on subtraction by } 42$$

$$\underline{\quad +42 \quad +42} \quad \text{Add } 42 \text{ to both sides}$$

$$9x=52 \quad \text{Notice the variable is multiplied by } 9$$

$$\frac{9x}{9}=\frac{52}{9} \quad \text{Divide both sides by } 9\text{, the coefficient of } 9x$$

$$x=\frac{52}{9} \quad \text{Our Solution!}$$

World View Note: The study of algebra originally was called the "Cossic Art" from the Latin, the study of "things" (which we now call variables).

1.1 Practice

Solve each equation.

1) $v+9=16$
2) $14=b+3$
3) $x-11=-16$
4) $-14=x-18$
5) $340=-17x$
6) $4r=-28$
7) $-9=\frac{n}{12}$
8) $\frac{k}{13}=-16$
9) $24=2n-8$
10) $-5m+2=27$
11) $\frac{b}{3}+7=10$
12) $4+\frac{a}{3}=1$
13) $-21x+12=-6-3x$
14) $-1-7m=-8m+7$
15) $-7(x-2)=-4-6(x-1)$
16) $-6(x-8)-4(x-2)=-4$
17) $-2(8n-4)=8(1-n)$
18) $-4(1+a)=2a-8(5+3a)$
19) $\frac{3}{2}n-\frac{8}{3}=-\frac{29}{12}$
20) $\frac{3}{2}-\frac{7}{4}v=-\frac{9}{8}$
21) $\frac{45}{16}+\frac{3}{2}n=\frac{7}{4}n-\frac{19}{16}$
22) $\frac{2}{3}m+\frac{9}{4}=\frac{10}{3}-\frac{53}{18}m$

1.1 Answers

1) 7
2) 11
3) -5
4) 4
5) -20
6) -7
7) -108
8) -208
9) 16
10) -5
11) 9
12) -9
13) 1
14) 8
15) 12
16) 6
17) 0
18) -2
19) $\frac{1}{6}$
20) $\frac{3}{2}$
21) 16
22) $\frac{39}{130} = \frac{3}{10}$