# Section 1.5: Point-Slope Form

**Objective: Give the equation of a line with a known slope and point.**

The slope-intercept form has the advantage of being simple to remember and use, however, it has one major disadvantage: we must know the $y-$intercept in order to use it! Generally we do not know the $y-$intercept; we only know one or more points (that are not the $y-$intercept). In these cases we can't (easily) use the slope-intercept equation, so we will use a different more flexible formula. If we let the slope of a line be $m$, and a specific point on the line be $(x_1, y_1)$, and any other point on the line be $(x, y)$, then we can use the point-slope formula to make a second equation.

**Example 1.**

| | |
|---|---|
| $m, (x_1, y_1), (x, y)$ | Recall slope formula |
| $\dfrac{y_2 - y_1}{x_2 - x_1} = m$ | Plug in values |
| $\dfrac{y - y_1}{x - x_1} = m$ | Multiply both sides by $(x - x_1)$ |
| $y - y_1 = m(x - x_1)$ | Our Solution |

If we know the slope, $m$ of a line and any point on the line $(x_1, y_1)$ we can easily plug these values into the equation above which will be called the point-slope formula.

$$\textbf{Point – Slope Formula: } \boldsymbol{y - y_1 = m(x - x_1)}$$

**Example 2.**

Write the equation of the line through the point $(3, -4)$ with a slope of $\frac{3}{5}$ in point-slope form.

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | Plug values into point-slope formula |
| $y - (-4) = \dfrac{3}{5}(x - 3)$ | Simplify signs |
| $y + 4 = \dfrac{3}{5}(x - 3)$ | Our Solution |

Often, we will prefer final answers be written in slope-intercept form. If the directions ask for the answer in slope-intercept form we will simply distribute the slope, then solve for $y$.

**Example 3.**

Write the equation of the line through the point $(-6, 2)$ with a slope of $-\dfrac{2}{3}$ in slope-intercept form.

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | Plug values into point-formula |
| $y - 2 = -\dfrac{2}{3}\left(x - (-6)\right)$ | Simplify signs |
| $y - 2 = -\dfrac{2}{3}(x + 6)$ | Distribute slope |
| $\begin{array}{l} y - 2 = -\dfrac{2}{3}x - 4 \\ \underline{\phantom{y-2}+2 \qquad\quad +2} \\ y = -\dfrac{2}{3}x - 2 \end{array}$ | Our Solution |

An important thing to observe about the point-slope formula is that the operation between the $x$'s and $y$'s is subtraction. This means when you simplify the signs you will have the opposite of the numbers in the point. We need to be very careful with signs as we use the point-slope formula.

In order to find the equation of a line we will always need to know the slope. If we don't know the slope to begin with we will have to do some work to find it first before we can get an equation.

**Example 4.**

Find the equation of the line through the points $(-2, 5)$ and $(4, -3)$ in point-slope form.

| | |
|---|---|
| $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ | First we must find the slope |
| $m = \dfrac{-3 - 5}{4 - (-2)} = \dfrac{-8}{6} = -\dfrac{4}{3}$ | Plug values in slope formula and evaluate |
| $y - y_1 = m(x - x_1)$ | With slope and either point, use point-slope formula |
| $y - 5 = -\dfrac{4}{3}(x - (-2))$ | Simplify signs |
| $y - 5 = -\dfrac{4}{3}(x + 2)$ | Our Solution |

**Example 5.**

Find the equation of the line through the points $(-3, 4)$ and $(-1, -2)$ in slope-intercept form.

| | |
|---|---|
| $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ | First we must find the slope |
| $m = \dfrac{-2-4}{-1-(-3)} = \dfrac{-6}{2} = -3$ | Plug values in slope formula and evaluate |
| $y - y_1 = m(x - x_1)$ | With slope and either point, use point-slope formula |
| $y - 4 = -3(x - (-3))$ | Simplify signs |
| $y - 4 = -3(x + 3)$ | Distribute slope |
| $\begin{aligned} y - 4 &= -3x - 9 \\ +4 \quad\quad & \quad\quad +4 \\ \hline y &= -3x - 5 \end{aligned}$ | Solve for $y$ <br> Add 4 to both sides <br> Our Solution |

**Example 6.**

Find the equation of the line through the points $(6, -2)$ and $(-4, 1)$ in slope-intercept form.

| | |
|---|---|
| $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ | First we must find the slope |
| $m = \dfrac{1-(-2)}{-4-6} = \dfrac{3}{-10} = -\dfrac{3}{10}$ | Plug values in slope formula and evaluate |
| $y - y_1 = m(x - x_1)$ | Use slope and either point, use point-slope formula |
| $y - (-2) = -\dfrac{3}{10}(x - 6)$ | Simplify signs |
| $y + 2 = -\dfrac{3}{10}(x - 6)$ | Distribute slope |
| $\begin{aligned} y + 2 &= -\dfrac{3}{10}x + \dfrac{9}{5} \\ -2 \quad\quad & \quad\quad -\dfrac{10}{5} \\ \hline y &= -\dfrac{3}{10}x - \dfrac{1}{5} \end{aligned}$ | Solve for $y$. Subtract 2 form both sides <br><br> Using $\dfrac{10}{5}$ on right so we have a common denominator <br><br> Our Solution |

Note: When the slope is undefined or both points have the same x-coordinate, there is no value for $m$ because the line is vertical. That is, all of the x-coordinates of the points on that line have the same value, which is the x-coordinate of the given point or points. Therefore, the equation of the line is simply the value of the x-coordinate of the given point or points.

# 1.5 Practice

**Write the point-slope form of the equation of the line through the given point with the given slope.**

1) through $(2, 2)$, slope $= \frac{1}{2}$

2) through $(2, 1)$, slope $= -\frac{1}{2}$

3) through $(-1, -5)$, slope $= 9$

4) through $(2, -2)$, slope $= -2$

5) through $(-4, 1)$, slope $= \frac{3}{4}$

6) through $(4, -3)$, slope $= -2$

7) through $(0, -2)$, slope $= -3$

8) through $(-1, 1)$, slope $= 4$

9) through $(0, -5)$, slope $= -\frac{1}{4}$

10) through $(0, 2)$, slope $= -\frac{5}{4}$

11) through $(-5, -3)$, slope $= \frac{1}{5}$

12) through $(-1, -4)$, slope $= -\frac{2}{3}$

13) through $(-1, 4)$, slope $= -\frac{5}{4}$

14) through $(1, -4)$, slope $= -\frac{3}{2}$

**Write the slope-intercept form of the equation of the line through the given point with the given slope.**

15) through: $(-1, -5)$, slope $= 2$

16) through: $(2, -2)$, slope $= -2$

17) through: $(5, -1)$, slope $= -\frac{3}{5}$

18) through: $(-2, -2)$, slope $= -\frac{2}{3}$

19) through: $(-4, 1)$, slope $= \frac{1}{2}$

20) through: $(4, -3)$, slope $= -\frac{7}{4}$

21) through: $(4, -2)$, slope $= -\frac{3}{2}$

22) through: $(-2, 0)$, slope $= -\frac{5}{2}$

23) through: $(-5, -3)$, slope $= -\frac{2}{5}$

24) through: $(3, 3)$, slope $= \frac{7}{3}$

25) through: $(2, -2)$, slope $= 1$

26) through: $(-4, -3)$, slope $= 0$

27) through: $(-2, -5)$, slope $= 2$

28) through: $(-4, 2)$, slope $= -\frac{1}{2}$

29) through: $(5, 3)$, slope $= \frac{6}{5}$

**Write the point-slope form of the equation of the line through the given points.**

30) through: $(-4, 3)$ and $(-3, 1)$

31) through: $(1, 3)$ and $(-3, 3)$

32) through: $(5, 1)$ and $(-3, 0)$

33) through: $(-4, 5)$ and $(4, 4)$

34) through: $(-4, -2)$ and $(0, 4)$

35) through: $(-4, 1)$ and $(4, 4)$

36) through: $(3, 5)$ and $(-5, 3)$

37) through: $(-1, -4)$ and $(-5, 0)$

38) through: $(3, -3)$ and $(-4, 5)$

39) through: $(-1, -5)$ and $(-5, -4)$

**Write the slope-intercept form of the equation of the line through the given points.**

40) through: $(-5, 1)$ and $(-1, -2)$

41) through: $(-5, -1)$ and $(5, -2)$

42) through: $(-5, 5)$ and $(2, -3)$

43) through: $(1, -1)$ and $(-5, -4)$

44) through: $(4, 1)$ and $(1, 4)$

45) through: $(0, 1)$ and $(-3, 0)$

46) through: $(0, 2)$ and $(5, -3)$

47) through: $(0, 2)$ and $(2, 4)$

48) through: $(0, 3)$ and $(-1, -1)$

49) through: $(-2, 0)$ and $(5, 3)$

# 1.5 Answers

1) $y - 2 = \frac{1}{2}(x - 2)$

2) $y - 1 = -\frac{1}{2}(x - 2)$

3) $y + 5 = 9(x + 1)$

4) $y + 2 = -2(x - 2)$

5) $y - 1 = \frac{3}{4}(x + 4)$

6) $y + 3 = -2(x - 4)$

7) $y + 2 = -3x$

8) $y - 1 = 4(x + 1)$

9) $y + 5 = -\frac{1}{4}x$

10) $y - 2 = -\frac{5}{4}x$

11) $y + 3 = \frac{1}{5}(x + 5)$

12) $y + 4 = -\frac{2}{3}(x + 1)$

13) $y - 4 = -\frac{5}{4}(x + 1)$

14) $y + 4 = -\frac{3}{2}(x - 1)$

15) $y = 2x - 3$

16) $y = -2x + 2$

17) $y = -\frac{3}{5}x + 2$

18) $y = -\frac{2}{3}x - \frac{10}{3}$

19) $y = \frac{1}{2}x + 3$

20) $y = -\frac{7}{4}x + 4$

21) $y = -\frac{3}{2}x + 4$

22) $y = -\frac{5}{2}x - 5$

23) $y = -\frac{2}{5}x - 5$

24) $y = \frac{7}{3}x - 4$

25) $y = x - 4$

26) $y = -3$

27) $y = 2x - 1$

28) $y = -\frac{1}{2}x$

29) $y = \frac{6}{5}x - 3$

30) $y - 3 = -2(x + 4)$

31) $y = 3$

32) $y - 1 = \frac{1}{8}(x - 5)$

33) $y - 5 = -\frac{1}{8}(x + 4)$

34) $y + 2 = \frac{3}{2}(x + 4)$

35) $y - 1 = \frac{3}{8}(x + 4)$

36) $y - 5 = \frac{1}{4}(x - 3)$

37) $y + 4 = -(x + 1)$

38) $y + 3 = -\frac{8}{7}(x - 3)$

39) $y + 5 = -\frac{1}{4}(x + 1)$

40) $y = -\frac{3}{4}x - \frac{11}{4}$

41) $y = -\frac{1}{10}x - \frac{3}{2}$

42) $y = -\frac{8}{7}x - \frac{5}{7}$

43) $y = \frac{1}{2}x - \frac{3}{2}$

44) $y = -x + 5$

45) $y = \frac{1}{3}x + 1$

46) $y = -x + 2$

47) $y = x + 2$

48) $y = 4x + 3$

49) $y = \frac{3}{7}x + \frac{6}{7}$