Section 3.4: Introduction to Polynomials

Objective: Evaluate, add, and subtract polynomials.

Many applications in mathematics have to do with what are called polynomials. Polynomials are made up of terms. **Terms** are a product of numbers or variables. For example, $5x$, $2y^2$, -5, ab^3c and x are all terms. Terms are connected to each other by addition. Expressions are often named based on the number of terms in them. A **monomial** has one term, such as $3x^2$. A **binomial** has two terms, such as a^2-b^2. A **trinomial** has three terms, such as ax^2+bx+c. The term **polynomial** means many terms. Monomials, binomials, trinomials, and expressions with more terms all fall under the umbrella of "polynomials".

If we know what the variable in a polynomial represents, we can replace the variable with the number and evaluate the polynomial as shown in the following example.

Example 1. Evaluate the given expression.

$2x^2-4x+6$ when $x=-4$	Replace variable x with -4
$2(-4)^2-4(-4)+6$	Exponents first
$2(16)-4(-4)+6$	Multiplication (we can do all terms at one)
$32+16+6$	Add
54	Our Solution

It is important to be careful with negative variables and exponents. Remember the exponent only affects the base it is physically attached to. This means that $-3^2=-9$ because the exponent is only attached to the 3. Also, $(-3)^2=9$ because the exponent is attached to the parentheses and affects everything inside. When we replace a variable with parentheses like in the previous example, the substituted value is in parentheses. So the $(-4)^2=16$ in the example. However, consider the next example.

Example 2. Evaluate the given expression.

$-x^2+2x+6$ when $x=3$	Replace variable x with 3
$-(3)^2+2(3)+6$	Exponent only on the 3
$-9+2(3)+6$	Multiply
$-9+6+6$	Add
3	Our Solution

World View Note: Ada Lovelace in 1842 described a Difference Engine that would be used to calculate values of polynomials. Her work became the foundation for what would become the modern computer more than 100 years after her death from cancer. The programming language Ada was named in her honor.

Generally when working with polynomials, we do not know the value of the variable, so we will try and simplify instead. The simplest operation with polynomials is addition. When adding polynomials, we combine like terms. Consider the following example.

Example 3. Simplify.

$(4x^3-2x+8)+(3x^3-9x^2-11)$	Remove the parentheses
$4x^3-2x+8+3x^3-9x^2-11$	Combine like terms $4x^3+3x^3$ and $8-11$
$7x^3-9x^2-2x-3$	Our Solution

Generally, final answers for polynomials are written so the exponent on the variable counts down. Example 3 demonstrates this with the exponent counting down 3, 2, 1, 0 (recall $x^0=1$). Subtracting polynomials is almost as fast. One extra step comes from the minus in front of the parentheses. When we have a negative in front of parentheses, we distribute it through, changing the signs of everything inside.

Example 4. Simplify.

$(5x^2-2x+7)-(3x^2+6x-4)$	Distribute negative through second part
$5x^2-2x+7-3x^2-6x+4$	Combine like terms: $5x^2-3x^2$, $-2x-6x$ and $7+4$
$2x^2-8x+11$	Our Solution

Addition and subtraction can also be combined into the same problem as shown in this final example.

Example 5. Simplify.

$(2x^2-4x+3)+(5x^2-6x+1)-(x^2-9x+8)$	Distribute negative through
$2x^2-4x+3+5x^2-6x+1-x^2+9x-8$	Combine like terms
$6x^2-x-4$	Our Solution

3.4 Practice

Evaluate each expression in exercises 1-10.

1) $-a^3 - a^2 + 6a - 21$ when $a = -4$
2) $n^2 + 3n - 11$ when $n = -6$
3) $n^3 - 7n^2 + 15n - 20$ when $n = 2$
4) $n^3 - 9n^2 + 23n - 21$ when $n = 5$
5) $-5n^4 - 11n^3 - 9n^2 - n - 5$ when $n = -1$
6) $x^4 - 5x^3 - x + 13$ when $x = 5$
7) $x^2 + 9x + 23$ when $x = -3$
8) $-6x^3 + 41x^2 - 32x + 11$ when $x = 6$
9) $x^4 - 6x^3 + x^2 - 24$ when $x = 6$
10) $m^4 + 8m^3 + 14m^2 + 13m + 5$ when $m = -6$

Simplify each expression in exercises 11-42.

11) $(5p - 5p^4) - (8p - 8p^4)$
12) $(7m^2 + 5m^3) - (6m^3 - 5m^2)$
13) $(3n^2 + n^3) - (2n^3 - 7n^2)$
14) $(x^2 + 5x^3) + (7x^2 + 3x^3)$
15) $(8n + n^4) - (3n - 4n^4)$
16) $(3v^4 + 1) + (5 - v^4)$
17) $(1 + 5p^3) - (1 - 8p^3)$
18) $(6x^3 + 5x) - (8x + 6x^3)$
19) $(5n^4 + 6n^3) + (8 - 3n^3 - 5n^4)$
20) $(8x^2 + 1) - (6 - x^2 - x^4)$
21) $(3 + b^4) + (7 + 2b + b^4)$
22) $(1 + 6r^2) + (6r^2 - 2 - 3r^4)$
23) $(8x^3 + 1) - (5x^4 - 6x^3 + 2)$
24) $(4n^4 + 6) - (4n - 1 - n^4)$
25) $(2a + 2a^4) - (3a^2 - 5a^4 + 4a)$
26) $(6v + 8v^3) + (3 + 4v^3 - 3v)$
27) $(4p^2 - 3 - 2p) - (3p^2 - 6p + 3)$
28) $(7 + 4m + 8m^4) - (5m^4 + 1 + 6m)$
29) $(4b^3 + 7b^2 - 3) + (8 + 5b^2 + b^3)$

30) $(7n+1-8n^4)-(3n+7n^4+7)$

31) $(3+2n^2+4n^4)+(n^3-7n^2-4n^4)$

32) $(7x^2+2x^4+7x^3)+(6x^3-8x^4-7x^2)$

33) $(n-5n^4+7)+(n^2-7n^4-n)$

34) $(8x^2+2x^4+7x^3)+(7x^4-7x^3+2x^2)$

35) $(8r^4-5r^3+5r^2)+(2r^2+2r^3-7r^4+1)$

36) $(4x^3+x-7x^2)+(x^2-8+2x+6x^3)$

37) $(2n^2+7n^4-2)+(2+2n^3+4n^2+2n^4)$

38) $(7b^3-4b+4b^4)-(8b^3-4b^2+2b^4-8b)$

39) $(8-b+7b^3)-(3b^4+7b-8+7b^2)+(3-3b+6b^3)$

40) $(1-3n^4-8n^3)+(7n^4+2-6n^2+3n^3)+(4n^3+8n^4+7)$

41) $(8x^4+2x^3+2x)+(2x+2-2x^3-x^4)-(x^3+5x^4+8x)$

42) $(6x-5x^4-4x^2)-(2x-7x^2-4x^4-8)-(8-6x^2-4x^4)$

3.4 Answers

1) 3
2) 7
3) -10
4) -6
5) -7
6) 8
7) 5
8) -1
9) 12
10) -1
11) $3p^4 - 3p$
12) $-m^3 + 12m^2$
13) $-n^3 + 10n^2$
14) $8x^3 + 8x^2$
15) $5n^4 + 5n$
16) $2v^4 + 6$
17) $13p^3$
18) $-3x$
19) $3n^3 + 8$
20) $x^4 + 9x^2 - 5$
21) $2b^4 + 2b + 10$
22) $-3r^4 + 12r^2 - 1$
23) $-5x^4 + 14x^3 - 1$
24) $5n^4 - 4n + 7$
25) $7a^4 - 3a^2 - 2a$
26) $12v^3 + 3v + 3$
27) $p^2 + 4p - 6$
28) $3m^4 - 2m + 6$
29) $5b^3 + 12b^2 + 5$
30) $-15n^4 + 4n - 6$
31) $n^3 - 5n^2 + 3$
32) $-6x^4 + 13x^3$
33) $-12n^4 + n^2 + 7$
34) $9x^4 + 10x^2$
35) $r^4 - 3r^3 + 7r^2 + 1$

36) $10x^3 - 6x^2 + 3x - 8$

37) $9n^4 + 2n^3 + 6n^2$

38) $2b^4 - b^3 + 4b^2 + 4b$

39) $-3b^4 + 13b^3 - 7b^2 - 11b + 19$

40) $12n^4 - n^3 - 6n^2 + 10$

41) $2x^4 - x^3 - 4x + 2$

42) $3x^4 + 9x^2 + 4x$