Section 1.3: Factor Trinomials Whose Leading Coefficient is 1

Objective: Factor trinomials when the leading coefficient is 1.

We will now learn a strategy for factoring trinomials (polynomials with three terms). In this section, we will focus on trinomials of the form x^2+bx+c. In this case, the leading coefficient (the coefficient of the first term) is 1.

Since factoring is the reverse of multiplication, we will start with a multiplication problem and look at how we can reverse the process.

Example 1. Multiply and simplify.

$(x+6)(x-4)$	Distribute $(x+6)$ to each term in the second set of parentheses
$=x(x+6)-4(x+6)$	Distribute each monomial through each se of parentheses
$=x^2+6x-4x-24$	Combine like terms
$=x^2+2x-24$	Our Answer

Notice that if you reverse the last three steps, the process is factoring by grouping! The way we will factor trinomials is to make them into a polynomial with four terms and then factor by grouping. This process is shown in the following example, which is this same problem worked backwards:

Example 2. Factor completely.

$x^2+2x-24$	Replace the middle term $+2x$ with $+6x-4x$
$=x^2+6x-4x-24$	Split expression into two pairs of terms; factor the GCF from each pair
$=x(x+6)-4(x+6)$	$(x+6)$ is common to both terms*:* factor this binomial GCF
$=(x+6)(x-4)$	Our Answer

The key to making these problems work is in the way we split the middle term. Why did we choose $+6x-4x$ and not $+5x-3x$? To find the correct way to split the middle term, we will use what is called the *ac method*. The *ac method* works by finding a pair of numbers that multiply to obtain the last number in the trinomial **and** also add up to the coefficient of the middle term of the trinomial. In the previous example, the numbers must multiply to -24 and add to $+2$. The only numbers that can do this are 6 and -4. Notice that $6\cdot\left(-4\right)=-24$ and $6+(-4)=2$.

FACTORING TRINOMIALS OF THE FORM x^2+bx+c

Example 3. Factor completely.

$x^2+9x+18$	Find factors that multiply to 18 and add to 9: Use 6 and 3
	Replace $9x$ with $6x+3x$
$=x^2+6x+3x+18$	Factor by grouping
$=x(x+6)+3(x+6)$	
$=(x+6)(x+3)$	Our Answer

Example 4. Factor completely.

x^2-4x+3	Find factors that multiply to 3 and add to -4: Use -3 and -1
	Replace $-4x$ with $-3x+(-1x)=-3x-1x$
$=x^2-3x-x+3$	Factor by grouping
$=x(x-3)-1(x-3)$	
$=(x-3)(x-1)$	Our Answer

Example 5. Factor completely.

$x^2-8x-20$	Find factors that multiply to -20 and add to -8: Use -10 and 2
	Replace $-8x$ with $-10x+2x$
$=x^2-10x+2x-20$	Factor by grouping
$=x(x-10)+2(x-10)$	
$=(x-10)(x+2)$	Our Answer

FACTORING TRINOMIALS IN TWO VARIABLES

Often we are asked to factor trinomials in two variables. The *ac method* works in much the same way: Find a pair of terms that multiplies to obtain the last term in the trinomial **and** also adds up to the the middle term of the trinomial.

Example 6. Factor completely.

$a^2-9ab+14b^2$	Find factors that multiply to 14 and add to -9: Use -7 and -2
	Replace $-9ab$ with $-7ab-2ab$
$=a^2-7ab-2ab+14b^2$	Factor by grouping
$=a(a-7b)-2b(a-7b)$	
$=(a-7b)(a-2b)$	Our Answer

As the past few examples illustrate, it is very important to find the correct pair of terms we will use to replace the middle term. Consider the following example, done **incorrectly**, where we mistakenly found two factors that multiply to 6 instead of -6:

Warning!

x^2+5x-6	Find factors that multiply to 6 and add to 5:
	Use 2 and 3
	Replace $5x$ with $2x+3x$
$=x^2+2x+3x-6$	Factor by grouping
$=x(x+2)+3(x-2)$	
???	Binomials do not match!

Because we did not use the negative sign with the 6 to find our pair of terms, the binomials did not match and grouping was not able to work at the end. The problem is done correctly below by choosing the correct pair of terms:

Example 7. Factor completely.

x^2+5x-6	Find factors that multiply to -6 and add to 5:
	Use 6 and -1
	Replace $5x$ with $6x-1x$
$=x^2+6x-1x-6$	Factor by grouping
$=x(x+6)-1(x+6)$	
$=(x+6)(x-1)$	Our Answer

FACTORING SHORTCUT

You may have noticed a shortcut for factoring these problems. Once we identify the two numbers that are used to split the middle term, these are the two numbers in our factors! In the previous example, the numbers used to split the middle term were 6 and -1; our factors turned out to be $(x+6)(x-1)$. This pattern does not always work, so be careful getting in the habit of using it. We can use it, however, when the leading coefficient of the trinomial is 1. In all of the problems we have factored in this lesson, the leading coefficient is 1. If this is the case, then we can use this shortcut. This process is shown in the next few examples.

Example 8. Factor completely.

$x^2-7x-18$	Find factors that multiply to -18 and add to -7:
	Use -9 and 2
	Write the binomial factors using -9 and 2
$=(x-9)(x+2)$	Our Answer

Example 9. Factor completely.

$m^2 - mn - 30n^2$	Find factors that multiply to -30 and add to -1:
	Use 5 and -6
	Write the binomial factors using 5 and -6
	Don't forget the second variable
$= (m+5n)(m-6n)$	Our Answer

It is possible to have an expression that does not factor. If there is no combination of numbers that multiply and add up to the correct numbers, then we cannot factor the polynomial and we say the polynomial is *prime*. This is shown in the following example.

Example 10. Factor completely.

$x^2 + 2x + 6$	Find factors that multiply to 6 and add to 2:
	$1 \cdot 6$, $2 \cdot 3$, $(-1)\cdot(-6)$, and $(-2)\cdot(-3)$ are the only ways to multiply to 6 but none of these pairs adds to 2
prime	Our Answer

FACTORING USING MORE THAN ONE STRATEGY

When factoring any polynomial, it is important to first factor any GCF (other than 1). If all of the terms in a polynomial have a common factor other than 1, we will want to first factor out the GCF before attempting any other method. The next three examples illustrate this technique.

Example 11. Factor completely.

$3x^2 - 24x + 45$	GCF of all three terms is 3*;* divide each term by 3
$= 3(x^2 - 8x + 15)$	Find factors that multiply to 15 and add to -8:
	Use -5 and -3
	Write the binomial factors using -5 and -3
$= 3(x-5)(x-3)$	Our Answer

Example 12. Factor completely.

$4x^2y - 8xy - 32y$	GCF of all three terms is $4y$*;* divide each terms by $4y$
$= 4y(x^2 - 2x - 8)$	Find factors that multiply to -8 and add to -2:
	Use -4 and 2
	Write the binomial factors using -4 and 2
$= 4y(x-4)(x+2)$	Our Answer

Example 13. Factor completely.

$7a^4b^2+28a^3b^2-35a^2b^2$	GCF of all three terms is $7a^2b^2$*;* divide each term by $7a^2b^2$
$=7a^2b^2(a^2+4a-5)$	Find factors that multiply to -5 and to 4: Use -1 and 5
	Write the binomial factors using -1 and 5
$=7a^2b^2(a-1)(a+5)$	Our Answer

Again it is important to comment on the shortcut of jumping right to the factors. This shortcut only works if the leading coefficient is 1. In the next lesson, we will look at how this process changes slightly when we have a number other than 1 as the leading coefficient.

Practice Exercises
Section 1.3: Factoring Trinomials Whose Leading Coefficient is 1

Factor completely.

1) $x^2+12x+32$
2) $x^2+13x+40$
3) $x^2-7x+10$
4) x^2-9x+8
5) x^2+x-30
6) x^2+x-72
7) $x^2-6x-27$
8) $x^2-9x-10$
9) $y^2-17y+70$
10) $x^2+3x-18$
11) $p^2+17p+72$
12) $x^2+3x-70$
13) $p^2+15p+54$
14) $p^2+7p-30$
15) c^2-4c+9
16) $m^2-15mn+50n^2$
17) $u^2-10uv+21v^2$
18) $m^2-3mn-40n^2$
19) $m^2+2mn-8n^2$
20) $x^2+10xy+16y^2$
21) $x^2-11xy+18y^2$
22) $u^2-10uv-24v^2$
23) $x^2+xy-12y^2$
24) $x^2+14xy+45y^2$
25) $x^2+4xy-12y^2$
26) $4x^2+52x+168$
27) $5a^2+60a+100$
28) $7w^2+35w-63$
29) $6a^2+24a-192$
30) $-5v^2-20v+25$
31) $6x^2+18xy+12y^2$
32) $5m^2+30mn-80n^2$
33) $6x^2+96xy+378y^2$
34) $-n^2+15n-56$
35) $-16t^2+48t+64$

ANSWERS to Practice Exercises
Section 1.3: Factoring Trinomials Whose Leading Coefficient is 1

1) $(x+8)(x+4)$

2) $(x+8)(x+5)$

3) $(x-2)(x-5)$

4) $(x-1)(x-8)$

5) $(x-5)(x+6)$

6) $(x+9)(x-8)$

7) $(x-9)(x+3)$

8) $(x-10)(x+1)$

9) $(y-10)(y-7)$

10) $(x+6)(x-3)$

11) $(p+9)(p+8)$

12) $(x+10)(x-7)$

13) $(p+6)(p+9)$

14) $(p+10)(p-3)$

15) prime

16) $(m-5n)(m-10n)$

17) $(u-7v)(u-3v)$

18) $(m+5n)(m-8n)$

19) $(m+4n)(m-2n)$

20) $(x+8y)(x+2y)$

21) $(x-9y)(x-2y)$

22) $(u-12v)(u+2v)$

23) $(x-3y)(x+4y)$

24) $(x+5y)(x+9y)$

25) $(x+6y)(x-2y)$

26) $4(x+7)(x+6)$

27) $5(a+10)(a+2)$

28) $7(w^2+5w-9)$

29) $6(a-4)(a+8)$

30) $-5(v-1)(v+5)$

31) $6(x+2y)(x+y)$

32) $5(m-2n)(m+8n)$

33) $6(x+9y)(x+7y)$

34) $-(n-8)(n-7)$

35) $-16(t-4)(t+1)$