Section 1.6: Factoring Strategy

Objective: Identify and use the correct method to factor various polynomials.

With so many different tools used to factor, it is easy to get lost as to which strategy to use and when. Here, we will organize all the different factoring methods we have seen.

A large part of deciding how to factor a polynomial is based the number of terms in the polynomial.

For all problem types, we will always try to factor out the GCF first.

FACTORING STRATEGY

1. If there is a common factor other than 1, factor the **Greatest Common Factor (GCF)**. Always look for the GCF first!

2. Count the number of terms. Select a method to try based on the number of terms:

    *   **2 terms:** Try one of the special methods:
        Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$
        Sum of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
        Difference of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

    *   **3 terms:** Try the *ac method*. Find two numbers that multiply to $a \cdot c$ and sum to b. Split the middle term and then continue to factor by grouping.
        If the trinomial is recognized as a perfect square trinomial, factor using one of these forms:
        $$a^2 + 2ab + b^2 = (a+b)^2 \quad \text{or} \quad a^2 - 2ab + b^2 = (a-b)^2$$

    *   **4 terms:** Try factoring by **grouping**.

3. Check if the polynomial has been factored **completely**.

We will use the above strategy to factor each of the following examples. Here, the emphasis will be on which strategy to use rather than the steps used in that method.

Example 1. Factor completely.

$x^2 - 23x + 42$

GCF = 1, so no need to divide by GCF
Three terms: try *ac method*
Find factors to multiply to 42 and add to -23:
    Use -2 and -21

$= (x-2)(x-21)$

Our Answer

Example 2. Factor completely.

$z^2 + 6z - 9$

GCF = 1, so no need to divide by GCF
Three terms: try *ac method*
Find factors to multiply to -9 and add to 6
    Try $(-1)(9)$, $(1)(-9)$, $(3)(-3)$; none sum to 6

    prime

Our Answer

Example 3. Factor completely.

$4x^2 + 56xy + 196y^2$

GCF = 4, first factor GCF from each term

$= 4(x^2 + 14xy + 49y^2)$

Three terms inside parentheses*:* try *ac method*
Find factors to multiply to 49 and add to 14
    7 and 7; a perfect square trinomial

$= 4(x+7y)^2$

Our Answer

Example 4. Factor completely.

$5x^2y + 15xy - 35x^2 - 105x$
$= 5x(xy + 3y - 7x - 21)$
$= 5x[y(x+3) - 7(x+3)]$
$= 5x(x+3)(y-7)$

GCF $= 5x$, first factor GCF from each term
Four terms inside parentheses*:* try grouping

Our Answer

Example 5. Factor completely.

$100x^2 - 400$
$= 100(x^2 - 4)$

$= 100(x+2)(x-2)$

GCF = 100, first factor GCF from both terms
Two terms inside parentheses*:* try difference of two
perfect squares
Our Answer

Example 6. Factor completely.

$108x^3y^2 - 39x^2y^2 + 3xy^2$
$= 3xy^2(36x^2 - 13x + 1)$

GCF $= 3xy^2$, first factor GCF from each term
Three terms inside parentheses: try *ac method*
Find factors to multiply to 36 and add to -13"
    Use -9 and -4

$$= 3xy^2(36x^2 - 9x - 4x + 1) \qquad \text{Factor by grouping}$$
$$= 3xy^2[9x(4x-1) - 1(4x-1)]$$
$$= 3xy^2(4x-1)(9x-1) \qquad \text{Our Answer}$$

Example 7. Factor completely.

$$5 + 625y^3 \qquad \text{GCF first, factor out 5 from each term}$$
$$= 5(1 + 125y^3) \qquad \text{Two terms inside parentheses: try sum of two perfect cubes}$$
$$= 5(1 + 5y)(1 - 5y + 25y^2) \qquad \text{Our Answer}$$

It is important to be comfortable and confident not just with using all the factoring methods, but also with deciding on which method to use. Your practice with these problems is very important!

Practice Exercises
Section 1.6: Factoring Strategy

Factor completely.

1) $24az - 18ah + 60yz - 45hy$

2) $2x^2 - 11x + 15$

3) $5u^2 - 9uv + 4v^2$

4) $16x^2 + 48xy + 36y^2$

5) $-2x^3 + 128y^3$

6) $20uv - 60u^3 - 5xv + 15xu^2$

7) $5n^3 + 7n^2 - 6n$

8) $2x^3 + 5x^2y + 3y^2x$

9) $54u^3 - 16$

10) $54 - 128x^3$

11) $n^2 - n$

12) $5x^2 - 22x - 15$

13) $x^2 - 4xy + 3y^2$

14) $45u^2 - 150uv + 125v^2$

15) $64x^2 + 49y^2$

16) $x^3 - 27y^3$

17) $m^2 - 4n^2$

18) $12ab - 18a + 6bn - 9n$

19) $36b^2c - 16dx - 24b^2d + 24cx$

20) $3m^3 - 6m^2n - 24mn^2$

21) $128 + 54x^3$

22) $64m^3 + 27n^3$

23) $2x^3 + 6x^2y - 20xy^2$

24) $3ac + 15ad^2 + cx^2 + 5d^2x^2$

25) $n^3 + 7n^2 + 10n$

26) $64m^3 - n^3$

27) $27x^3 - 64$

28) $16a^2 - 9b^2$

29) $5x^2 + 2x$

30) $2x^2 - 10x + 12$

31) $-3k^3 + 27k^2 - 60k$

32) $75x^2 - 12y^2$

33) $mn - 12x + 3m - 4nx$

34) $2k^2 + k - 10$

35) $16x^2 - 8xy + y^2$

36) $v^2 + v$

37) $27m^2 - 48n^2$

38) $x^3 + 4x^2$

39) $9x^3 + 21x^2y - 60xy^2$

40) $9n^3 - 3n^2$

41) $2m^2 + 6mn - 20n^2$

42) $2u^2v^2 - 11uv^3 + 15v^4$

43) $5x^2 - 6x + 7$

44) $9x^2 - 25y^2$

45) $2x^2 - 2x + 14$

46) $x^2 - 100$

ANSWERS to Practice Exercises
Section 1.6: Factoring Strategy

1) $3(2a+5y)(4z-3h)$

2) $(2x-5)(x-3)$

3) $(5u-4v)(u-v)$

4) $4(2x+3y)^2$

5) $-2(x-4y)(x^2+4xy+16y^2)$

6) $5(4u-x)(v-3u^2)$

7) $n(5n-3)(n+2)$

8) $x(2x+3y)(x+y)$

9) $2(3u-2)(9u^2+6u+4)$

10) $2(3-4x)(9+12x+16x^2)$

11) $n(n-1)$

12) $(5x+3)(x-5)$

13) $(x-3y)(x-y)$

14) $5(3u-5v)^2$

15) prime

16) $(x-3y)(x^2+3xy+9y^2)$

17) $(m+2n)(m-2n)$

18) $3(2a+n)(2b-3)$

19) $4(3b^2+2x)(3c-2d)$

20) $3m(m+2n)(m-4n)$

21) $2(4+3x)(16-12x+9x^2)$

22) $(4m+3n)(16m^2-12mn+9n^2)$

23) $2x(x+5y)(x-2y)$

24) $(3a+x^2)(c+5d^2)$

25) $n(n+2)(n+5)$

26) $(4m-n)(16m^2+4mn+n^2)$

27) $(3x-4)(9x^2+12x+16)$

28) $(4a+3b)(4a-3b)$

29) $x(5x+2)$

30) $2(x-2)(x-3)$

31) $-3k(k-5)(k-4)$

32) $3(5x+2y)(5x-2y)$

33) $(m-4x)(n+3)$

34) $(2k+5)(k-2)$

35) $(4x-y)^2$

36) $v(v+1)$

37) $3(3m+4n)(3m-4n)$

38) $x^2(x+4)$

39) $3x(3x-5y)(x+4y)$

40) $3n^2(3n-1)$

41) $2(m-2n)(m+5n)$

42) $v^2(2u-5v)(u-3v)$

43) prime

44) $(3x+5y)(3x-5y)$

45) $2(x^2-x+7)$

46) $(x+10)(x-10)$