Section 2.1: Reduce Rational Expressions

Objective: Reduce rational expressions by dividing out common factors.

A **rational expression** is a quotient of polynomials. Examples of rational expressions include:

$$\frac{x^2-x-12}{x^2-9x+20} \quad \text{and} \quad \frac{3}{x-2} \quad \text{and} \quad \frac{a-b}{b-a} \quad \text{and} \quad \frac{3}{2}$$

DETERMINING EXCLUDED VALUES FOR A RATIONAL EXPRESSION

It is important to remember that the denominator of a fraction cannot have a value of zero. A rational expression is undefined when its denominator equals 0. W must exclude all value(s) of the variable that make a denominator zero. We must determine the value(s) that will give us zero in the denominator, and then exclude those values as possible values of the variable.

Example 1. State the excluded value(s).

$\frac{4x}{x+2}$	Find values of x for which denominator is 0
	Set the denominator equal to 0
$x+2=0$	Solve the equation
$\underline{-2 \quad -2}$	
$x=-2$	
Excluded value: $x=-2$	Our Answer

We can evaluate the rational expression for any other value of x except for -2.

Example 2. State the excluded value(s).

$\frac{x^2-1}{3x^2+5x}$	Find values of x for which denominator is 0
	Set the denominator equal to 0
$3x^2+5x=0$	Factor the left side
$x(3x+5)=0$	Set each factor equal to zero
$x=0$ or $3x+5=0$	Solve each equation
$\underline{-5 \quad -5}$	
$\frac{3x}{3}=-\frac{5}{3}$	
$x=-\frac{5}{3}$	Second equation solved
Excluded values: $x=0$ and $-\frac{5}{3}$	Our Answer

EVALUATING A RATIONAL EXPRESSION

We evaluate rational expressions by substituting the given value for the variable and then simplifying using the order of operations.

Example 3. Evaluate the expression for the given value of the variable.

$\dfrac{x^2-4}{x^2+6x+8}$ when $x=-6$	Substitute -6 in for each variable
$\dfrac{(-6)^2-4}{(-6)^2+6(-6)+8}$	Exponents first
$=\dfrac{36-4}{36+6\,(-6)+8}$	Multiply
$=\dfrac{36-4}{36-36+8}$	Add and subtract
$=\dfrac{32}{8}$	Reduce
$=4$	Our Answer

SIMPLIFYING A RATIONAL EXPRESSION

Just as we reduced the fraction in the previous example, often a rational expression can be reduced. When we reduce, we divide out common factors. A rational expression is reduced, or simplified, if the numerator and denominator have no common factors other than 1 or -1.

If the rational expression only has monomials, we can reduce the coefficients, and divide out common factors of the variables.

Example 4. Simplify the expression.

$\dfrac{15x^4y^2}{25x^2y^6}$	Reduce 15 and 25 by dividing out the common factor of 5; Subtract the exponents: $x^{4-2}=x^2$ and $y^{2-6}=y^{-4}$; Negative exponents move to denominator because $y^{-4}=\dfrac{1}{y^4}$
$=\dfrac{3x^2}{5y^4}$	Our Answer

If the rational expression has more than one term in either the numerator or denominator, we need to first factor the numerator or denominator and then divide out common factors.

Example 5. Simplify the expression.

$\dfrac{28}{8x^2-16}$	Denominator has a common factor of 8; factor.
$=\dfrac{28}{8(x^2-2)}$	Reduce by dividing 28 and 8 by 4.
$=\dfrac{7}{2(x^2-2)}$	Our Answer

Example 6. Simplify the expression.

$\dfrac{9x-3}{18x-6}$	Numerator has a common factor of 3; denominator has a common factor of 6; factor both the numerator and denominator
$=\dfrac{3(3x-1)}{6(3x-1)}$	Divide out common factor of $(3x-1)$, and divide both 3 and 6 by 3
$=\dfrac{1}{2}$	Our Answer

Example 7. Simplify the expression.

$\dfrac{x^2-25}{x^2+8x+15}$	Factor both the numerator and the denominator; numerator is the difference of two squares; denominator can be factored using the *ac method*;
$=\dfrac{(x+5)\,(x-5)}{(x+5)\,(x+3)}$	Divide out common factor of $(x+5)$
$=\dfrac{x-5}{x+3}$	Our Answer

It is important to remember that we cannot divide *terms*, only *factors*. This means if there are any $+$ or $-$ signs between the parts we want to reduce, we cannot reduce. In the previous example, we had the solution $\dfrac{x-5}{x+3}$ we cannot divide out the x s because they are terms (separated by $+$ or $-$), not factors (separated by multiplication).

FACTORS THAT ARE OPPOSITES

Sometimes, factors in the numerator and denominator are *opposites*. To simplify, factor -1 from the numerator or denominator.

Example 8. Simplify the expression.

$\dfrac{5-x}{x-5}$	Rewrite the numerator
$=\dfrac{-x+5}{x-5}$	Factor -1 from the numerator
$=\dfrac{-1(x-5)}{x-5}$	Divide out common factor of $(x-5)$
$=-1$	Our Answer

Example 9. Simplify the expression.

$\dfrac{7-x}{x^2-49}$	Rewrite the numerator
$=\dfrac{-x+7}{x^2-49}$	Factor -1 from the numerator; Factor denominator as difference of two squares
$=\dfrac{-1\,(x-7)}{(x+7)\,(x-7)}$	Divide out common factor of $(x-7)$
$=-\dfrac{1}{x+7}$	Our Answer

Practice Exercises
Section 2.1: Reduce Rational Expressions

Evaluate.

1) $\frac{4v+2}{6}$ when $v=4$

2) $\frac{b-3}{3b-9}$ when $b=-2$

3) $\frac{x-3}{x^2-4x+3}$ when $x=-4$

4) $\frac{a+2}{a^2+3a+2}$ when $a=-1$

5) $\frac{b+2}{b^2+4b+4}$ when $b=0$

6) $\frac{n^2-n-6}{n-3}$ when $n=4$

State the excluded value(s).

7) $\frac{3k^2+30k}{k+10}$

8) $\frac{27p}{18p^2-36p}$

9) $\frac{15n^2}{10n+25}$

10) $\frac{x+10}{8x^2+80x}$

11) $\frac{10m^2+8m}{10m}$

12) $\frac{10x+16}{6x+20}$

13) $\frac{r^2+3r+2}{5r+10}$

14) $\frac{6n^2-21n}{6n^2+3n}$

15) $\frac{b^2+12b+32}{b^2+4b-32}$

16) $\frac{10v^2-30v}{35v^2-5v}$

Simplify each expression.

17) $\frac{21x^2}{18x}$

18) $\frac{24a}{40a^2}$

19) $\frac{32x^3}{8x^4}$

20) $\frac{90x^2}{20x}$

The Practice Exercises are continued on the next page.

Practice Exercises: Section 2.1 (continued)

Simplify each expression.

21) $\dfrac{18m-24}{60}$

22) $\dfrac{20}{4p+2}$

23) $\dfrac{x+3}{3+x}$

24) $\dfrac{5-x}{x-5}$

25) $\dfrac{9-n}{9n-81}$

26) $\dfrac{x+7}{7-x}$

27) $\dfrac{x+1}{x^2+8x+7}$

28) $\dfrac{28m+12}{36}$

29) $\dfrac{32x^2}{28x^2+28x}$

30) $\dfrac{49r+56}{56r}$

31) $\dfrac{n^2+4n-12}{n^2-7n+10}$

32) $\dfrac{b^2+14b+48}{b^2+15b+56}$

33) $\dfrac{9v+54}{v^2-4v-60}$

34) $\dfrac{30x-90}{50x+40}$

35) $\dfrac{12x^2-42x}{30x^2-42x}$

36) $\dfrac{k^2-12k+32}{k^2-64}$

37) $\dfrac{6a-10}{10a+4}$

38) $\dfrac{9p+18}{p^2+4p+4}$

39) $\dfrac{2n^2+19n-10}{9n+90}$

40) $\dfrac{3x^2-29x+40}{5x^2-30x-80}$

41) $\dfrac{3-x}{5x^2-45}$

42) $\dfrac{9r^2+63r}{5r^2+40r+35}$

43) $\dfrac{2x^2-10x+8}{3x^2-7x+4}$

44) $\dfrac{50b-80}{50b+20}$

45) $\dfrac{7n^2-32n+16}{4n-16}$

46) $\dfrac{35v+35}{21v+7}$

47) $\dfrac{n^2-2n+1}{6n+6}$

48) $\dfrac{56x-48}{24x^2+56x+32}$

49) $\dfrac{7a^2-26a-45}{6a^2-34a+20}$

50) $\dfrac{4k^3-2k^2-2k}{9k^3-18k^2+9k}$

ANSWERS to Practice Exercises
Section 2.1: Reduce Rational Expressions

1) 3

2) $\frac{1}{3}$

3) $-\frac{1}{5}$

4) undefined

5) $\frac{1}{2}$

6) 6

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7) -10

8) $0, 2$

9) $-\frac{5}{2}$

10) $0, -10$

11) 0

12) $-\frac{10}{3}$

13) -2

14) $0, -\frac{1}{2}$

15) $-8, 4$

16) $0, \frac{1}{7}$

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17) $\frac{7x}{6}$

18) $\frac{3}{5a}$

19) $\frac{4}{x}$

20) $\frac{9x}{2}$

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 2.1 (continued)

21) $\frac{3m-4}{10}$

22) $\frac{10}{2p+1}$

23) 1

24) -1

25) $-\frac{1}{9}$

26) cannot be simplified

27) $\frac{1}{x+7}$

28) $\frac{7m+3}{9}$

29) $\frac{8x}{7(x+1)}$

30) $\frac{7r+8}{8r}$

31) $\frac{n+6}{n-5}$

32) $\frac{b+6}{b+7}$

33) $\frac{9}{v-10}$

34) $\frac{3(x-3)}{5x+4}$

35) $\frac{2x-7}{5x-7}$

36) $\frac{k-4}{k+8}$

37) $\frac{3a-5}{5a+2}$

38) $\frac{9}{p+2}$

39) $\frac{2n-1}{9}$

40) $\frac{3x-5}{5(x+2)}$

41) $-\frac{1}{5(x+3)}$

42) $\frac{9r}{5(r+1)}$

43) $\frac{2(x-4)}{3x-4}$

44) $\frac{5b-8}{5b+2}$

45) $\frac{7n-4}{4}$

46) $\frac{5(v+1)}{3v+1}$

47) $\frac{(n-1)^2}{6(n+1)}$

48) $\frac{7x-6}{(3x+4)(x+1)}$

49) $\frac{7a+9}{2(3a-2)}$

50) $\frac{2(2k+1)}{9(k-1)}$