Section 2.4: Add and Subtract Rational Expressions

Objective: Add and subtract rational expressions with like and different denominators.

You will recall that when adding fractions with a common denominator, we add the numerators and keep the denominator. This same process is used with rational expressions. Remember to reduce your sum or difference, if possible, to obtain your final answer.

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH THE *SAME* DENOMINATOR

Example 1. Add the rational expressions, and simplify if possible.

$$\frac{x-4}{x^2-2x-8}+\frac{x+8}{x^2-2x-8}$$ Same denominator*;* add numerators*;* combine like terms

$$=\frac{2x+4}{x^2-2x-8}$$ Factor numerator and denominator

$$=\frac{2(x+2)}{(x+2)(x-4)}$$ Divide out common factor of $(x+2)$ to reduce the fraction to its lowest terms

$$=\frac{2}{x-4}$$ Our Answer

Subtraction with common denominators follows the same pattern. However, subtraction can cause problems if we are not careful to avoid sign errors. Consequently, we will first distribute the subtraction sign to every term in the numerator of the fraction that follows the subtraction sign. Then we can treat it like an addition problem.

Example 2. Subtract the rational expressions, and simplify if possible.

$$\frac{6x-12}{3x-6}-\frac{15x-6}{3x-6}$$ Add the opposite of the second fraction (distribute the subtraction to each term in the second fraction)

$$=\frac{6x-12}{3x-6}+\frac{-15x+6}{3x-6}$$ Add the numerators*;* combine like terms

$$=\frac{-9x-6}{3x-6}$$ Factor numerator and denominator

$= \frac{-3(3x+2)}{3(x-2)}$ Divide out common factor of 3

$= -\frac{(3x+2)}{x-2}$ Our Answer

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH *DIFFERENT* DENOMINATORS

When the denominators of the rational expressions are not like, we find their least common denominator (LCD). Then we build up each fraction to an equivalent one with that LCD as the denominator. The following example shows this process with fractions.

Example 3. Add the fractions, and simplify if possible.

$\frac{5}{6}+\frac{1}{4}$ The LCD is 12. Build up, multiply 6 by 2 and 4 by 3 to get the common denominator;

$= \frac{5}{6}\left(\frac{2}{2}\right)+\frac{1}{4}\left(\frac{3}{3}\right)$ Multiply first fraction by $\left(\frac{2}{2}\right)$ and second by $\left(\frac{3}{3}\right)$

$= \frac{10}{12}+\frac{3}{12}$ Add the numerators

$= \frac{13}{12}$ Our Answer

The same process is used with rational expressions containing variables.

Example 4. Add the rational expressions, and simplify if possible.

$\frac{7a}{3a^2b}+\frac{4b}{6ab^4}$ The LCD is $6a^2b^4$. Build up each expression:

$(3a^2b)(2b^3) = 6a^2b^4$ and $(6ab^4)(a) = 6a^2b^4$

$= \frac{7a}{3a^2b}\left(\frac{2b^3}{2b^3}\right)+\frac{4b}{6ab^4}\left(\frac{a}{a}\right)$ Multiply first fraction by $\left(\frac{2b^3}{2b^3}\right)$ and second by $\left(\frac{a}{a}\right)$

$= \frac{14ab^3}{6a^2b^4} + \frac{4ab}{6a^2b^4}$ Add the numerators, there are no like terms to combine

$= \frac{14ab^3 + 4ab}{6a^2b^4}$ Factor the numerator

$= \frac{2ab(7b^2 + 2)}{6a^2b^4}$ Reduce, dividing out common factors of 2, a, and b

$= \frac{7b^2 + 2}{3ab^3}$ Our Answer

The same process is used for subtraction; we will simply include the first step of changing the subtraction to addition of the opposite value.

Example 5. Subtract the rational expressions, and simplify if possible.

$\frac{4}{5a} - \frac{7b}{4a^2}$ Change subtraction to addition of the opposite

$= \frac{4}{5a} + \frac{-7b}{4a^2}$ The LCD is $20a^2$. Build up each expression:

$(5a)(4a) = 20a^2$ and $(4a^2)(5) = 20a^2$

$= \frac{4}{5a}\left(\frac{4a}{4a}\right) + \frac{-7b}{4a^2}\left(\frac{5}{5}\right)$ Multiply first fraction by $\left(\frac{4a}{4a}\right)$ and second by $\left(\frac{5}{5}\right)$

$= \frac{16a}{20a^2} + \frac{-35b}{20a^2}$ Add numerators; there are no like terms to combine

$= \frac{16a - 35b}{20a^2}$ Factor the numerator, if possible. In this case, the numerator is prime. This is our answer.

If our denominators have more than one term, then we will need to factor first to find the LCD. Next, we build up each fraction using the factors that are missing from each denominator.

Example 6. Add the rational expressions, and simplify if possible.

$\frac{6}{8a + 4} + \frac{3a}{8}$ Factor denominators to find LCD

$=\dfrac{6}{4(2a+1)}+\dfrac{3a}{8}$ The LCD is $8(2a+1)$. Build up each expression:

$4(2a+1)\cdot 2=8(2a+1)$ and

$(8)(2a+1)=8(2a+1)$

$=\dfrac{6}{4(2a+1)}\left(\dfrac{2}{2}\right)+\dfrac{3a}{8}\left(\dfrac{2a+1}{2a+1}\right)$ Multiply first fraction by $\left(\dfrac{2}{2}\right)$, second by $\left(\dfrac{2a+1}{2a+1}\right)$

$=\dfrac{12}{8(2a+1)}+\dfrac{6a^2+3a}{8(2a+1)}$ Add numerators

$=\dfrac{6a^2+3a+12}{8(2a+1)}$ Factor the numerator

$=\dfrac{3(2a^2+a+4)}{8(2a+1)}$ Cannot be simplified

$=\dfrac{3(2a^2+a+4)}{8(2a+1)}$ Our Answer

Example 7. Add the rational expressions, and simplify if possible.

$\dfrac{3}{5y+20}+\dfrac{y+2}{6y^2+23y-4}$ Factor denominators to find LCD

$=\dfrac{3}{5(y+4)}+\dfrac{y+2}{(6y-1)(y+4)}$ The LCD is $5(y+4)\,(6y-1)$. Build up each expression

$=\dfrac{3}{5(y+4)}+\dfrac{y+2}{(6y-1)(y+4)}$ Multiply first fraction by $\left(\dfrac{6y-1}{6y-1}\right)$ and second by $\left(\dfrac{5}{5}\right)$

$=\dfrac{3}{5(y+4)}\cdot\left(\dfrac{6y-1}{6y-1}\right)+\dfrac{y+2}{(6y-1)(y+4)}\cdot\left(\dfrac{5}{5}\right)$ Multiply the numerators and denominators

$=\dfrac{18y-3}{5(y+4)(6y-1)}+\dfrac{5y+10}{5(6y-1)(y+4)}$ Add numerators, combine like terms

$=\dfrac{23y+7}{5(y+4)(6y-1)}$ Factor the numerator, if possible. In this case, the numerator is prime. This is our answer.

Whenever you encounter a subtraction problem, remember to rewrite the problem using addition of the opposite.

Example 8. Subtract the rational expressions, and simplify if possible.

$\frac{x+1}{x-4}-\frac{x+1}{x^2-7x+12}$	Add the opposite: Distribute the subtraction to each term in the numerator of the second fraction
$=\frac{x+1}{x-4}+\frac{-x-1}{x^2-7x+12}$	Factor denominators to find LCD
$=\frac{x+1}{x-4}+\frac{-x-1}{(x-3)(x-4)}$	The LCD is $(x-4)(x-3)$. Build up each expression
$=\frac{x+1}{x-4}\left(\frac{x-3}{x-3}\right)+\frac{-x-1}{(x-3)(x-4)}$	Only the first fraction needs to be multiplied by $(x-3)$
$=\frac{x^2-2x-3}{(x-3)(x-4)}+\frac{-x-1}{(x-3)(x-4)}$	Add the numerators, combine like terms
$=\frac{x^2-3x-4}{(x-3)(x-4)}$	Factor the numerator
$=\frac{(x-4)(x+1)}{(x-3)(x-4)}$	Divide out the common factor of $(x-4)$
$=\frac{x+1}{x-3}$	Our Answer

Practice Exercises
Section 2.4: Add and Subtract Rational Expressions

Add or subtract, expressing the result in its lowest terms.

1) $\frac{2}{a+3}+\frac{4}{a+3}$

2) $\frac{x^2}{x-2}-\frac{6x-8}{x-2}$

3) $\frac{t^2+4t}{t-1}+\frac{2t-7}{t-1}$

4) $\frac{a^2+3a}{a^2+5a-6}-\frac{4}{a^2+5a-6}$

5) $\frac{2x^2+3}{x^2-6x+5}-\frac{x^2-5x+9}{x^2-6x+5}$

6) $\frac{3}{x}+\frac{4}{x^2}$

7) $\frac{5}{6r}-\frac{5}{8r}$

8) $\frac{7}{xy^2}+\frac{3}{x^2y}$

9) $\frac{8}{9t^3}+\frac{5}{6t^2}$

10) $\frac{x+5}{8}+\frac{x-3}{12}$

11) $\frac{a+2}{2}-\frac{a-4}{4}$

12) $\frac{2a-1}{3a^2}+\frac{5a+1}{9a}$

13) $\frac{x-1}{4x}-\frac{2x+3}{x}$

14) $\frac{2c-d}{c^2d}-\frac{c+d}{cd^2}$

15) $\frac{5x+3y}{2x^2y}-\frac{3x+4y}{xy^2}$

16) $\frac{2}{x-1}+\frac{2}{x+1}$

17) $\frac{2z}{z-1}-\frac{3z}{z+1}$

18) $\frac{2}{x-5}+\frac{3}{4x}$

19) $\frac{8}{x^2-4}-\frac{3}{x+2}$

20) $\frac{4x}{x^2-25}+\frac{x}{x+5}$

21) $\frac{t}{t-3}-\frac{5}{4t-12}$

22) $\frac{2}{x+3}+\frac{4}{(x+3)^2}$

The Practice Exercises are continued on the next page.

Practice Exercises: Section 2.4 (continued)

23) $\frac{2}{5x^2+5x}-\frac{4}{3x+3}$

24) $\frac{3a}{4a-20}+\frac{9a}{6a-30}$

25) $\frac{t}{y-t}-\frac{y}{y+t}$

26) $\frac{x}{x-5}+\frac{x-5}{x}$

27) $\frac{x}{x^2+5x+6}-\frac{2}{x^2+3x+2}$

28) $\frac{2x}{x^2-1}-\frac{3}{x^2+5x+4}$

29) $\frac{x}{x^2+15x+56}-\frac{7}{x^2+13x+42}$

30) $\frac{2x}{x^2-9}+\frac{5}{x^2+x-6}$

31) $\frac{5x}{x^2-x-6}-\frac{18}{x^2-9}$

32) $\frac{4x}{x^2-2x-3}-\frac{3}{x^2-5x+6}$

33) $\frac{2x}{x^2-1}-\frac{4}{x^2+2x-3}$

34) $\frac{x-1}{x^2+3x+2}+\frac{x+5}{x^2+4x+3}$

35) $\frac{x+1}{x^2-2x-35}+\frac{x+6}{x^2+7x+10}$

36) $\frac{3x+2}{3x+6}+\frac{x}{4-x^2}$

37) $\frac{4-a^2}{a^2-9}-\frac{a-2}{3-a}$

38) $\frac{4y}{y^2-1}-\frac{2}{y}-\frac{2}{y+1}$

39) $\frac{2z}{1-2z}+\frac{3z}{2z+1}-\frac{3}{4z^2-1}$

40) $\frac{2r}{r^2-s^2}+\frac{1}{r+s}-\frac{1}{r-s}$

41) $\frac{2x-3}{x^2+3x+2}+\frac{3x-1}{x^2+5x+6}$

42) $\frac{x+2}{x^2-4x+3}+\frac{4x+5}{x^2+4x-5}$

43) $\frac{2x+7}{x^2-2x-3}-\frac{3x-2}{x^2+6x+5}$

44) $\frac{3x-8}{x^2+6x+8}+\frac{2x-3}{x^2+3x+2}$

ANSWERS to Practice Exercises
Section 2.4: Add and Subtract Rational Expressions

1) $\dfrac{6}{a+3}$

2) $x-4$

3) $t+7$

4) $\dfrac{a+4}{a+6}$

5) $\dfrac{x+6}{x-5}$

6) $\dfrac{3x+4}{x^2}$

7) $\dfrac{5}{24r}$

8) $\dfrac{7x+3y}{x^2y^2}$

9) $\dfrac{15t+16}{18t^3}$

10) $\dfrac{5x+9}{24}$

11) $\dfrac{a+8}{4}$

12) $\dfrac{5a^2+7a-3}{9a^2}$

13) $\dfrac{-7x-13}{4x}$

14) $\dfrac{-c^2+cd-d^2}{c^2d^2}$

15) $\dfrac{3y^2-3xy-6x^2}{2x^2y^2}$

16) $\dfrac{4x}{(x+1)(x-1)}$

17) $\dfrac{-z^2+5z}{(z+1)(z-1)}$

18) $\dfrac{11x-15}{4x(x-5)}$

19) $\dfrac{14-3x}{(x+2)(x-2)}$

20) $\dfrac{x^2-x}{(x+5)(x-5)}$

21) $\dfrac{4t-5}{4(t-3)}$

22) $\dfrac{2x+10}{(x+3)^2}$

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 2.4 (continued)

23) $\dfrac{6-20x}{15x(x+1)}$

24) $\dfrac{9a}{4(a-5)}$

25) $\dfrac{t^2+2ty-y^2}{(y+t)(y-t)}$

26) $\dfrac{2x^2-10x+25}{x(x-5)}$

27) $\dfrac{x-3}{(x+3)(x+1)}$

28) $\dfrac{2x+3}{(x-1)(x+4)}$

29) $\dfrac{x-8}{(x+8)(x+6)}$

30) $\dfrac{2x-5}{(x-3)(x-2)}$

31) $\dfrac{5x+12}{(x+3)(x+2)}$

32) $\dfrac{4x+1}{(x+1)(x-2)}$

33) $\dfrac{2x+4}{(x+3)(x+1)}$

34) $\dfrac{2x+7}{(x+3)(x+2)}$

35) $\dfrac{2x-8}{(x-7)(x+2)}$

36) $\dfrac{-3x^2+7x+4}{3(x+2)(2-x)}$

37) $\dfrac{a-2}{(a+3)(a-3)}$

38) $\dfrac{2}{y(y-1)}$

39) $\dfrac{z-3}{2z-1}$

40) $\dfrac{2}{r+s}$

41) $\dfrac{5(x-1)}{(x+1)(x+3)}$

42) $\dfrac{5x+5}{(x+5)(x-3)}$

43) $\dfrac{-(x-29)}{(x-3)(x+5)}$

44) $\dfrac{5x-10}{(x+4)(x+1)}$