Section 2.5: Proportions

Objectives: Solve proportions using the cross product.
Use proportions to solve application problems.

When two fractions are equal, they are said to be in proportion. This definition can be generalized for two equal rational expressions.

The following principle is true for any proportion and will be useful when solving proportions.

CROSS PRODUCT:

If $\frac{a}{b}=\frac{c}{d}$ **, then** $ad=bc$**.**

SOLVING A PROPORTION USING THE CROSS PRODUCT

To solve a proportion $\frac{a}{b}=\frac{c}{d}$, set the cross products ad and bc equal and solve the resulting equation.

Example 1. Solve the proportion for x.

$\frac{20}{6}=\frac{x}{9}$ Set the cross products equal

$(20)(9)=6x$ Multiply

$\frac{180}{6}=\frac{6x}{6}$ Divide both sides by 6

$30=x$ Our Solution

If the proportion has more than one term in either the numerator or denominator, we distribute when calculating the cross product.

Example 2. Solve the proportion for x.

$$\frac{x+3}{4}=\frac{2}{5}$$ Set the cross products equal

$$5(x+3)=(4)\,(2)$$ Multiply and distribute

$$5x+15=8$$ Solve

$$\underline{\quad -15 \quad -15}$$ Subtract 15 from both sides

$$\frac{5x}{5}=\frac{-7}{5}$$ Divide both sides by 5

$$x=-\frac{7}{5}$$ Our Solution

Example 3. Solve the proportion for x.

$$\frac{4}{x}=\frac{6}{3x+2}$$ Set the cross products equal

$$4(3x+2)=6x$$ Distribute

$$12x+8=6x$$ Move variables to one side of the equation

$$\underline{-12x \quad -12x}$$ Subtract $12x$ from both sides

$$\frac{8}{-6}=\frac{-6x}{-6}$$ Divide both sides by -6, simplify fraction

$$-\frac{4}{3}=x$$ Our Solution

Example 4. Solve the proportion for x.

$$\frac{2x-3}{7x+4}=\frac{2}{5}$$ Set the cross products equal

$$5(2x-3)=2(7x+4)$$ Distribute

$10x-15=14x+8$ Move variables to one side of the equation
$-10x \quad -10x$ Subtract $10x$ from both sides
$-15=4x+8$
$-8 \quad -8$ Subtract 8 from both sides
$\frac{-23}{4}=\frac{4x}{4}$ Divide both sides by 4

$-\frac{23}{4}=x$ Our Solution

When solving a proportion, we may end up with a quadratic equation to solve. In this section, we will solve the quadratic equations in the same way we solved quadratics previously - by factoring. Other methods, such as completing the square or utilizing the quadratic formula, will be discussed in a later chapter. As before, we will generally end up with two solutions.

Example 5. Solve the proportion for k.

$\frac{k+3}{3}=\frac{8}{k-2}$ Set the cross products equal

$(k+3)(k-2)=(8)(3)$ FOIL and multiply

$k^2+k-6=24$ Set the equation equal to zero
$-24 \quad -24$ Subtract 24 from both sides
$k^2+k-30=0$ Factor completely

$(k+6)(k-5)=0$ Set each factor equal to zero

$k+6=0$ or $k-5=0$ Solve each equation
$-6 \quad -6$ $+5 \quad +5$ Add or subtract
$k=-6$ or $k=5$ Our Solutions

SOLVING APPLICATION PROBLEMS USING PROPORTIONS

Proportions are very useful in that they can be used in many different types of applications. We can use them to compare different quantities and make conclusions about how quantities are related.

Proportions can be used in situations where multiplying one variable by a value k results in the other variable also being multiplied by k. For example, suppose that a person gets an hourly wage. If that person works twice as long they would make twice as much money. A contrasting example is someone who has a normal work week of 40 hours and will get an

overtime bonus for working extra hours. Working twice as much would get that person more than twice as much money. Another example of proportional reasoning is if something has a set price, then buying twice as much would cost twice as much. A contrasting example is if there is a "buy 12, get one free" deal. Buying twice as much then would not always cost twice as much. In the following examples, assume that the variables involved are indeed proportional.

As we set up these problems using proportions, it is important to stay organized. For example, if we are comparing dogs and cats and the number of dogs is in the numerator of the first fraction, then the numerator of the second fraction must also refer to the number of dogs. This consistency of the numerator and denominator is essential in setting up proportions.

Example 6. Solve.

A six foot tall man casts a shadow that is 3.5 feet long. If the shadow of a flag pole is 8 feet long, how tall is the flag pole? Round the answer to the tenths place.

$\frac{\text{shadow}}{\text{height}}$ We will put shadows in numerator; heights in denominator

The man has a shadow of 3.5 feet and a height of 6 feet: Write $\frac{3.5}{6}$

The flag pole has a shadow of 8 feet, but the height is unknown: Write $\frac{8}{x}$

Set up the proportion

$\frac{3.5}{6}=\frac{8}{x}$ Set the cross products equal

$3.5x=(8)(6)$ Multiply

$\frac{3.5x}{3.5}=\frac{48}{3.5}$ Divide both sides by 3.5 and round to the tenths place.

$x\approx 13.7$ feet Our Solution

Example 7. Solve.

In a basketball game, the home team was down by 9 points at the end of the game. They only scored 6 points for every 7 points the visiting team scored. What was the final score of the game?

$\frac{\text{home}}{\text{visitor}}$ We will put the home team in numerator, visitors in denominator

The solution is continued on the next page.

$\frac{\text{home}}{\text{visitor}}$ Visitor's score is unknown so label as x; the home team scored 9 points less than the visitors or $x-9$:

Write $\frac{x-9}{x}$

Home team scored 6 points for every 7 points the visiting team scored:

Write $\frac{6}{7}$

Set up the proportion

$\frac{x-9}{x}=\frac{6}{7}$ Set the cross products equal

$7(x-9)=6x$ Distribute

$7x-63=6x$ Move variables to one side

$-7x \quad -7x$ Subtract $7x$ from both sides

$\frac{-63}{-1}=-\frac{-x}{-1}$ Divide both sides by -1

$63=x$ We used x for the visiting team's score.

$63-9=54$ Subtract 9 to get the home team's score

63 to 54 Our Solution — The visiting team scored 63 points and the home team scored 54 points

Practice Exercises
Section 2.5: Proportions

Solve each proportion.

1) $\frac{10}{a}=\frac{6}{8}$

2) $\frac{7}{9}=\frac{n}{6}$

3) $\frac{7}{6}=\frac{2}{k}$

4) $\frac{8}{x}=\frac{4}{8}$

5) $\frac{6}{x}=\frac{8}{2}$

6) $\frac{n-10}{8}=\frac{9}{3}$

7) $\frac{m-1}{5}=\frac{8}{2}$

8) $\frac{8}{5}=\frac{3}{x-8}$

9) $\frac{2}{9}=\frac{10}{p-4}$

10) $\frac{9}{n+2}=\frac{3}{9}$

11) $\frac{b-10}{7}=\frac{b}{4}$

12) $\frac{9}{4}=\frac{r}{r-4}$

13) $\frac{x}{5}=\frac{x+2}{9}$

14) $\frac{n}{8}=\frac{n-4}{3}$

15) $\frac{3}{10}=\frac{a}{a+2}$

16) $\frac{x+1}{9}=\frac{x+2}{2}$

17) $\frac{v-5}{v+6}=\frac{4}{9}$

18) $\frac{n+8}{10}=\frac{n-9}{4}$

19) $\frac{7}{x-1}=\frac{4}{x-6}$

20) $\frac{k+5}{k-6}=\frac{8}{5}$

21) $\frac{x+5}{5}=\frac{6}{x-2}$

22) $\frac{4}{x-3}=\frac{x+5}{5}$

23) $\frac{m+3}{4}=\frac{11}{m-4}$

24) $\frac{x-5}{8}=\frac{4}{x-1}$

25) $\frac{2}{p+4}=\frac{p+5}{3}$

26) $\frac{5}{n+1}=\frac{n-4}{10}$

27) $\frac{n+4}{3}=\frac{-3}{n-2}$

28) $\frac{1}{n+3}=\frac{n+2}{2}$

29) $\frac{3}{x+4}=\frac{x+2}{5}$

30) $\frac{x-5}{4}=\frac{-3}{x+3}$

The Practice Exercises are continued on the next page.

Practice Exercises: Section 2.5 (continued)

Answer each question. Round your answer to the nearest tenth. Round dollar amounts to the nearest cent.

31) The currency in Western Samoa is the Tala. The exchange rate is approximately \$0.70 to 1 Tala. At this rate, how many dollars would you get if you exchanged 13.3 Tala?

32) If you can buy one plantain for \$0.49 then how many plantains can you buy with \$7.84?

33) Kali reduced the size of a painting to a height of 1.3 inches. What is the new width if it was originally 5.2 inches tall and 10 inches wide?

34) A model train has a scale of 1.2 inches : 2.9 feet. If the model train is 5 inches tall, how tall is the real train?

35) A bird bath that is 5.3 feet tall casts a shadow that is 25.4 feet long. Find the length of the shadow that an 8.2 feet adult elephant casts.

36) Victoria and Georgetown are 36.2 miles from each other. How far apart would the cities be on a map that has a scale of 0.9 inches : 10.5 miles?

37) The Vikings led the Timberwolves by 19 points at the half. If the Vikings scored 3 points for every 2 points the Timberwolves scored, what was the half time score?

38) Sarah worked 10 more hours than Josh. If Sarah worked 7 hours for every 2 hours Josh worked, how long did they each work?

39) Computer Services Inc. charges \$8 more for a repair than Low Cost Computer Repair. If the ratio of the costs is 3 : 6, what will it cost for the repair at Low Cost Computer Repair?

40) Kelsey's commute is 15 minutes longer than Christina's. If Christina drives 12 minutes for every 17 minutes Kelsey drives, how long is each commute?

ANSWERS to Practice Exercises
Section 2.5: Proportions

1) $a = \frac{40}{3}$

2) $n = \frac{14}{3}$

3) $k = \frac{12}{7}$

4) $x = 16$

5) $x = \frac{3}{2}$

6) $n = 34$

7) $m = 21$

8) $x = \frac{79}{8}$

9) $p = 49$

10) $n = 25$

11) $b = -\frac{40}{3}$

12) $r = \frac{36}{5}$

13) $x = \frac{5}{2}$

14) $n = \frac{32}{5}$

15) $a = \frac{6}{7}$

16) $v = -\frac{16}{7}$

17) $v = \frac{69}{5}$

18) $n = \frac{61}{3}$

19) $x = \frac{38}{3}$

20) $k = \frac{73}{3}$

21) $x = -8, 5$

22) $x = -7, 5$

23) $m = -7, 8$

24) $x = -3, 9$

25) $p = -7, -2$

26) $n = -6, 9$

27) $n = -1$

28) $x = -4, -1$

29) $x = -7, 1$

30) $x = -1, 3$

ANSWERS to Practice Exercises: Section 2.5 (continued)

31) $9.31

32) 16

33) 2.5 inches

34) 12.1 feet

35) 39.3 feet

36) 3.1 inches

37) T: 38, V: 57

38) J: 4 hours, S: 14 hours

39) $8

40) C: 36 minutes, K: 51 minutes