Section 3.4: Multiply and Divide Radical Expressions

Objective: Multiply and divide radical expressions using the product and quotient rules for radicals.

MULTIPLYING RADICAL EXPRESSIONS

The product rule of radicals we used previously can be generalized as follows:

PRODUCT RULE OF RADICALS

For any *nonnegative* real numbers b and d,

$$a\sqrt[n]{b}\cdot c\sqrt[n]{d} = a\cdot c\sqrt[n]{b\cdot d}$$

In words, this rule states that we are allowed to multiply the factors outside the radical and we are allowed to multiply the factors inside the radicals, as long as the indices match.

Example 1. Multiply.

$-5\sqrt{14}\cdot 4\sqrt{6}$	Multiply outside and inside the radical
$=-20\sqrt{84}$	Simplify the radical, divisible by 4
$=-20\sqrt{4\cdot 21}$	Take the square root where possible
$=-20\cdot 2\sqrt{21}$	Multiply coefficients
$=-40\sqrt{21}$	Our Answer

The same process works with higher roots.

Example 2. Multiply.

$2\sqrt[3]{18}\cdot 6\sqrt[3]{15}$	Multiply outside and inside the radical
$=12\sqrt[3]{270}$	Simplify the radical, divisible by 27
$=12\sqrt[3]{27\cdot 10}$	Take the square root where possible
$=12\cdot 3\sqrt[3]{10}$	Multiply coefficients
$=36\sqrt[3]{10}$	Our Answer

When multiplying radical expressions we can still use the distributive property or FOIL just as we could when multiplying polynomials.

Example 3. Multiply.

$7\sqrt{6}(3\sqrt{10}-5\sqrt{15})$	Distribute, following rules for multiplying radicals
$=21\sqrt{60}-35\sqrt{90}$	Simplify radicals, finding perfect square factors
$=21\sqrt{4\cdot 15}-35\sqrt{9\cdot 10}$	Take the square root where possible
$=21\cdot 2\sqrt{15}-35\cdot 3\sqrt{10}$	Multiply coefficients
$=42\sqrt{15}-105\sqrt{10}$	Our Answer

Example 4. Multiply.

$(\sqrt{5}-2\sqrt{3})(4\sqrt{10}+6\sqrt{6})$	FOIL, following rules for multiplying radicals
$=4\sqrt{50}+6\sqrt{30}-8\sqrt{30}-12\sqrt{18}$	Simplify radicals, finding perfect square factors
$=4\sqrt{25\cdot 2}+6\sqrt{30}-8\sqrt{30}-12\sqrt{9\cdot 2}$	Take the square root where possible
$=4\cdot 5\sqrt{2}+6\sqrt{30}-8\sqrt{30}-12\cdot 3\sqrt{2}$	Multiply coefficients
$=20\sqrt{2}+6\sqrt{30}-8\sqrt{30}-36\sqrt{2}$	Combine like radicals
$=-16\sqrt{2}-2\sqrt{30}$	Our Answer

Example 5. Multiply.

$(2\sqrt{5}-3\sqrt{6})(7\sqrt{2}-8\sqrt{7})$	FOIL, following rules for multiplying radicals
$=14\sqrt{10}-16\sqrt{35}-21\sqrt{12}+24\sqrt{42}$	Simplify radicals, finding perfect square factors
$=14\sqrt{10}-16\sqrt{35}-21\sqrt{4\cdot 3}+24\sqrt{42}$	Take the square root where possible
$=14\sqrt{10}-16\sqrt{35}-21\cdot 2\sqrt{3}+24\sqrt{42}$	Multiply coefficients
$=14\sqrt{10}-16\sqrt{35}-42\sqrt{3}+24\sqrt{42}$	Our Answer

The next example shows how to use FOIL to square a radical expression with two terms.

Example 6. Multiply.

$(\sqrt{2}+\sqrt{3})^2$	Write as a product
$=(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})$	FOIL, following rules for multiplying radicals
$=\sqrt{4}+\sqrt{6}+\sqrt{6}+\sqrt{9}$	Take the square root where possible
$=2+\sqrt{6}+\sqrt{6}+3$	Combine like terms
$=5+2\sqrt{6}$	Our Answer

As we are multiplying we always look at our final answer to check if all the radicals are simplified and all like radicals have been combined.

DIVIDING RADICAL EXPRESSIONS

Division with radicals is very similar to multiplication. If we think about division as reducing fractions, we can reduce the coefficients outside the radicals and reduce the values inside the radicals to get our final answer.

QUOTIENT RULE OF RADICALS

For any *nonnegative* real numbers b and d,

$$\frac{a\sqrt[n]{b}}{c\sqrt[n]{d}}=\frac{a}{c}\sqrt[n]{\frac{b}{d}}$$

Example 7. Divide.

$\frac{15\sqrt[3]{108}}{20\sqrt[3]{2}}$	Reduce $\frac{15}{20}$ by dividing common factor of 5; reduce $\frac{\sqrt[3]{108}}{\sqrt[3]{2}}$ by dividing 108 by 2
$=\frac{3\sqrt[3]{54}}{4}$	Simplify radical, 54 is divisible by 27
$=\frac{3\sqrt[3]{27\cdot 2}}{4}$	Take the cube root of 27
$=\frac{3\cdot 3\sqrt[3]{2}}{4}$	Multiply coefficients
$=\frac{9\sqrt[3]{2}}{4}$	Our Answer

Example 8. Divide.

$\dfrac{\sqrt{50x^3y^7}}{\sqrt{2xy^2}}$	Divide $\dfrac{50}{2}, \dfrac{x^3}{x}$, and $\dfrac{y^7}{y^2}$
$=\sqrt{25x^2y^5}$	Simplify radical, 25 is a perfect square, divide exponents by 2
$=5xy^2\sqrt{y}$	Our Answer

There is one catch to dividing radical expressions. It is considered bad practice to have a radical in the denominator of our final answer. We will see how to handle this situation in the next section.

Practice Exercises
Section 3.4: Multiply and Divide Radical Expressions

Perform the indicated operation.

1) $3\sqrt{5}\cdot -4\sqrt{16}$

2) $-5\sqrt{10}\cdot\sqrt{15}$

3) $\sqrt{12m}\cdot\sqrt{15m}$

4) $\sqrt{5r^3}\cdot -5\sqrt{10r^2}$

5) $\sqrt[3]{4x^3}\cdot\sqrt[3]{2x^4}$

6) $\sqrt{3}(4-\sqrt{6})$

7) $\sqrt{6}(\sqrt{2}+2)$

8) $\sqrt{10}(\sqrt{5}+\sqrt{2})$

9) $-5\sqrt{15}(3\sqrt{3}+2)$

10) $\sqrt{7}(\sqrt{3}+5\sqrt{14})$

11) $6\sqrt{10}(5n+\sqrt{2})$

12) $\sqrt{15}(\sqrt{5}-3\sqrt{3v})$

13) $(2+2\sqrt{2})(-3+\sqrt{2})$

14) $(-2+\sqrt{3})(-5+2\sqrt{3})$

15) $(\sqrt{5}-5)(2\sqrt{5}-1)$

16) $(2\sqrt{3}+\sqrt{5})(2\sqrt{3}-\sqrt{5})$

17) $(\sqrt{2a}+2\sqrt{3a})(3\sqrt{2a}+\sqrt{5a})$

18) $(2\sqrt{2p}+\sqrt{q})(3\sqrt{2p}+\sqrt{q})$

19) $(-5-4\sqrt{3})(-3-4\sqrt{3})$

20) $(7-\sqrt{3})^2$

21) $\frac{\sqrt{12}}{5\sqrt{100}}$

22) $\frac{\sqrt{15}}{2\sqrt{4}}$

23) $\frac{\sqrt{5}}{4\sqrt{125}}$

24) $\frac{\sqrt{12}}{\sqrt{3}}$

ANSWERS to Practice Exercises
Section 3.4: Multiply and Divide Radical Expressions

1) $-48\sqrt{5}$

2) $-25\sqrt{6}$

3) $6m\sqrt{5}$

4) $-25r^2\sqrt{2r}$

5) $2x^2\sqrt[3]{x}$

6) $4\sqrt{3}-3\sqrt{2}$

7) $2\sqrt{3}+2\sqrt{6}$

8) $5\sqrt{2}+2\sqrt{5}$

9) $-45\sqrt{5}-10\sqrt{15}$

10) $\sqrt{21}+35\sqrt{2}$

11) $30n\sqrt{10}+12\sqrt{5}$

12) $5\sqrt{3}-9\sqrt{5v}$

13) $-2-4\sqrt{2}$

14) $16-9\sqrt{3}$

15) $15-11\sqrt{5}$

16) 7

17) $6a+a\sqrt{10}+6a\sqrt{6}+2a\sqrt{15}$

18) $12p+5\sqrt{2pq}+q$

19) $63+32\sqrt{3}$

20) $52-14\sqrt{3}$

21) $\frac{\sqrt{3}}{25}$

22) $\frac{\sqrt{15}}{4}$

23) $\frac{1}{20}$

24) 2