# Section 4.1: Square Root Property

**Objective: Solve quadratic equations by applying the square root property.**

In an earlier chapter, we learned how to solve equations by factoring. The next example reviews how we solved a quadratic equation $ax^2+bx+c=0$ by factoring.

**Example 1**. Solve the equation.

$$\begin{array}{rcrl} & & x^2+5x+6=0 & \text{Factor using } \textit{ac method} \\ & & (x+3)(x+2)=0 & \text{Set each factor equal to zero} \\ x+3=0 & \text{or} & x+2=0 & \\ \underline{-3=-3} & & \underline{-2=-2} & \\ x=-3 & \text{or} & x=-2 & \text{Our Solutions} \end{array}$$

However, not every quadratic equation can be solved by factoring. For example, consider the equation $x^2-2x-7=0$. The trinomial on the left side, $x^2-2x-7$, cannot be factored; however, we will see in a later section that the equation $x^2-2x-7=0$ has two solutions: $1+2\sqrt{2}$ and $1-2\sqrt{2}$.

## SQUARE ROOT PROPERTY

In this chapter, we will learn additional methods besides factoring for solving quadratic equations. We will start with a method that makes use of the following property:

> **SQUARE ROOT PROPERTY:**
>
> If $k$ is a real number and $x^2=k$,
> then $x=\sqrt{k}$ or $x=-\sqrt{k}$
>
> Often this property is written using shorthand notation:
>
> If $x^2=k$, then $x=\pm\sqrt{k}$.

To solve a quadratic equation by applying the square root property, we will first need to isolate the squared expression on one side of the equation and the constant term on the other side.

# SOLVING QUADRATIC EQUATIONS BY APPLYING THE SQUARE ROOT PROPERTY

**Example 2**. Solve the equation.

| | |
|---|---|
| $x^2 = 16$ | The squared term is already isolated; Apply the square root property $(\pm)$ |
| $\sqrt{x^2} = \pm\sqrt{16}$ | Simplify radicals |
| $x = \pm 4$ | Our Solutions |

**Example 3**. Solve the equation.

| | |
|---|---|
| $x^2 - 7 = 0$ | Isolate the squared term |
| $x^2 = 7$ | Apply the square root property $(\pm)$ |
| $\sqrt{x^2} = \pm\sqrt{7}$ | Simplify radicals |
| $x = \pm\sqrt{7}$ | Our Solutions |

**Example 4**. Solve the equation.

| | |
|---|---|
| $2x^2 + 36 = 0$ | Isolate the squared term: |
| $\dfrac{2x^2}{2} = \dfrac{-36}{2}$ | Subtract 36 from both sides<br>Divide by 2 |
| $x^2 = -18$ | Apply the square root property $(\pm)$ |
| $\sqrt{x^2} = \pm\sqrt{-18}$ | Simplify radicals: $\pm\sqrt{-18} = \pm\sqrt{9}\cdot\sqrt{-1}\cdot\sqrt{2} = \pm 3i\sqrt{2}$ |
| $x = \pm 3i\sqrt{2}$ | Our Solutions |

**Example 5**. Solve the equation.

| | | | |
|---|---|---|---|
| | | $(2x+4)^2 = 36$ | A squared expression is already isolated on the left side; Apply the square root property $(\pm)$ |
| | | $\sqrt{(2x+4)^2} = \pm\sqrt{36}$ | Simplify radicals |
| | | $2x+4 = \pm 6$ | To avoid sign errors, separate into two equations |
| $2x+4 = 6$ | or | $2x+4 = -6$ | with one equation for $+$, one equation for $-$ |
| $-4 = -4$ | | $-4 = -4$ | Subtract $4$ from both sides |
| $\dfrac{2x}{2} = \dfrac{2}{2}$ | or | $\dfrac{2x}{2} = \dfrac{-10}{2}$ | Divide both sides by $2$ |
| $x = 1$ | or | $x = -5$ | Our Solutions |

In the previous example we used two separate equations to simplify, because when we took the root, our solutions were two rational numbers, $6$ and $-6$. If the roots do not simplify to rational numbers, we may keep the $\pm$ in the equation.

**Example 6**. Solve the equation.

| | |
|---|---|
| $(6x-9)^2 = 45$ | A squared expression is already isolated on the left side; Apply the square root property $(\pm)$ |
| $\sqrt{(6x-9)^2} = \pm\sqrt{45}$ | Simplify radicals: $\pm\sqrt{45} = \pm\sqrt{9}\cdot\sqrt{5} = \pm\,3\sqrt{5}$ |
| $6x-9 = \pm 3\sqrt{5}$ <br> $+9 = +9$ | Use one equation because radical did not simplify to a rational number |
| $\dfrac{6x}{6} = \dfrac{9 \pm 3\sqrt{5}}{6}$ | Divide both sides by $6$ |
| $x = \dfrac{9 \pm 3\sqrt{5}}{6}$ | Factor numerator and denominator |
| $x = \dfrac{3\left(3 \pm \sqrt{5}\right)}{3\cdot 2}$ | Divide out common factor of 3 |
| $x = \dfrac{3 \pm \sqrt{5}}{2}$ | Our Solutions |

**Example 7**. Solve the equation.

| | |
|---|---|
| $2(3x-1)^2 + 7 = 23$ <br> $-7 \quad = -7$ | Isolate the squared expression on the left side; Subtract 7 from both sides |
| $2\,(3x-1)^2 = 16$ | Divide both sides by 2 |
| $\dfrac{2\,(3x-1)^2}{2} = \dfrac{16}{2}$ | |
| $(3x-1)^2 = 8$ | Apply the square root property $(\pm)$ |
| $\sqrt{(3x-1)^2} = \pm\sqrt{8}$ | Simplify radicals: $\pm\sqrt{8} = \pm\sqrt{4}\cdot\sqrt{2} = \pm\,2\sqrt{2}$ |
| $3x-1 = \pm 2\sqrt{2}$ | Use one equation because radical did not simplify to a rational |
| $3x-1 = \pm 2\sqrt{2}$ <br> $+1 = +1$ | Add $1$ to both sides <br> Divide both sides by 3 |
| $\dfrac{3x}{3} = \dfrac{1 \pm 2\sqrt{2}}{3}$ | |
| $x = \dfrac{1 \pm 2\sqrt{2}}{3}$ | Our Solutions |

**Example 8**. Solve the equation.

| | |
|---|---|
| $(x+3)^2+9=7$ <br> $\underline{\quad -9 = -9}$ | Isolate the squared expression on the left side; Subtract 9 from both sides |
| $(x+3)^2=-2$ | Apply the square root property ( $\pm$ ) |
| $\sqrt{(x+3)^2}=\pm\sqrt{-2}$ | Simplify radicals: $\pm\sqrt{-2}=\pm\sqrt{-1}\cdot\sqrt{2}=\pm i\sqrt{2}$ |
| $x+3=\pm i\sqrt{2}$ | Use one equation because radical did not simplify to a rational |
| $x+3=\pm i\sqrt{2}$ <br> $\underline{\quad -3=-3}$ | Subtract 3 from both sides |
| $x=-3\pm i\sqrt{2}$ | Our Solutions |

**Example 9**. Solve the equation.

| | |
|---|---|
| $\left(x+\frac{1}{3}\right)^2=\frac{2}{9}$ | Apply the square root property ( $\pm$ ) |
| $\sqrt{\left(x+\frac{1}{3}\right)^2}=\pm\sqrt{\frac{2}{9}}$ | Simplify radicals: $\pm\sqrt{\frac{2}{9}}=\pm\frac{\sqrt{2}}{\sqrt{9}}=\pm\frac{\sqrt{2}}{3}$ |
| $x+\frac{1}{3}=\pm\frac{\sqrt{2}}{3}$ | Use one equation because radical did not simplify to a rational |
| $x+\frac{1}{3}=\pm\frac{\sqrt{2}}{3}$ | Subtract $\frac{1}{3}$ from both sides |
| $x=-\frac{1}{3}\pm\frac{\sqrt{2}}{3}$ | Add fractions |
| $x=\frac{-1\pm\sqrt{2}}{3}$ | Our Solutions |

# Practice Exercises
## Section 4.1: Square Root Property

**Solve each equation using the square root property.**

1) $x^2 = 64$

2) $x^2 = 75$

3) $x^2 + 5 = 13$

4) $x^2 - 7 = 20$

5) $x^2 + 50 = 0$

6) $5x^2 - 7 = 18$

7) $(x-4)^2 = 9$

8) $(2x+1)^2 = 25$

9) $(x+1)^2 = 3$

10) $(x-3)^2 = 12$

11) $(x+2)^2 = -9$

12) $(2x+1)^2 + 3 = 21$

13) $(9x-3)^2 = 72$

14) $(2x-8)^2 - 5 = 15$

15) $-2(x-6)^2 - 13 = 7$

16) $-3(4x-5)^2 + 8 = -19$

17) $\left(x - \frac{5}{2}\right)^2 = \frac{81}{4}$

18) $\left(x + \frac{3}{4}\right)^2 = \frac{10}{16}$

# ANSWERS to Practice Exercises
## Section 4.1: Square Root Property

1) $\pm 8$
2) $\pm 5\sqrt{3}$
3) $\pm 2\sqrt{2}$
4) $\pm 3\sqrt{3}$
5) $\pm 5i\sqrt{2}$
6) $\pm\sqrt{5}$
7) $1, 7$
8) $-3, 2$
9) $-1 \pm \sqrt{3}$
10) $3 \pm 2\sqrt{3}$
11) $-2 \pm 3i$
12) $\dfrac{-1 \pm 3\sqrt{2}}{2}$
13) $\dfrac{1 \pm 2\sqrt{2}}{3}$
14) $4 \pm \sqrt{5}$
15) $6 \pm i\sqrt{10}$
16) $\dfrac{1}{2}, 2$
17) $7, -2$
18) $\dfrac{-3 \pm \sqrt{10}}{4}$