Section 5.2: Operations on Functions

Objective: Combine functions using sum, difference, product, and quotient of functions.

We can combine functions using four common operations. The four basic operations on functions are addition, subtraction, multiplication, and division. The notation for these functions is as follows.

Addition	$(f+g)(x) = f(x)+g(x)$
Subtraction	$(f-g)(x) = f(x)-g(x)$
Multiplication	$(f \cdot g)(x) = f(x)\cdot g(x)$
Division	$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$ for $g(x) \neq 0$

EVALUATING FUNCTIONS

When we evaluate the sum/difference/product/quotient of two functions, we evaluate each function independently and then perform the given operation with both results.

Example 1. Perform the indicated operation.

Let $f(x) = x^2 - x - 2$ and $g(x) = x+1$.

Find $(f+g)(-3)$	Evaluate $(f+g)$ at $x=-3$
$= f(-3)+g(-3)$	Evaluate f at $x=-3$; $f(-3) = (-3)^2-(-3)-2$ $f(-3) = 9+3-2$ $f(-3) = 10$ Evaluate g at $x=-3$: $g(-3) = (-3)+1$ $g(-3) = -2$
$= 10 + (-2)$	Add the two results together
$= 8$	Our Answer

The process is the same regardless of the operation being performed.

Example 2. Perform the indicated operation.

Let $h(x) = 2x - 4$ and $k(x) = -3x + 1$.	
Find $(h \cdot k)(5)$	Evaluate $(h \cdot k)$ at $x = 5$
$= h(5) \cdot k(5)$	Evaluate h at $x = 5$; $h(5) = 2(5) - 4$ $h(5) = 10 - 4$ $h(5) = 6$ Evaluate k at $x = 5$: $k(5) = -3(5) + 1$ $k(5) = -15 + 1$ $k(5) = -14$
$= (6)(-14)$	Multiply the two results together
$= -84$	Our Answer

COMBINING FUNCTIONS

In the examples above, we evaluated the sum and quotient of two functions at a given value of the variable. Often, instead of evaluating, we are asked to create a new function by performing an operation on the two given functions. We combine the two functions using the indicated operation, writing each in parentheses, and then simplifying the expression.

Example 3. Perform the indicated operation.

Let $f(x) = 2x - 4$ and $g(x) = x^2 - x + 5$.	
Find $(f - g)(x)$	Find the difference of two functions
$= f(x) - g(x)$	Replace $f(x)$ with $(2x - 4)$ and $g(x)$ with $(x^2 - x + 5)$
$= (2x - 4) - (x^2 - x + 5)$	Subtract by distributing the negative
$= 2x - 4 - x^2 + x - 5$	Combine like terms
$= -x^2 + 3x - 9$	Our Answer

The parentheses are very important when we are combining $f(x)$ and $g(x)$ using a given operation. In the previous example, we needed the parentheses to know to distribute the negative.

Example 4. Perform the indicated operation.

Let $f(x) = x^2 - 4x - 5$ and $g(x) = x - 5$.	
Find $\left(\dfrac{f}{g}\right)(x)$.	Find the quotient of two functions
$= \dfrac{f(x)}{g(x)}$	Replace $f(x)$ with $(x^2 - 4x - 5)$ and $g(x)$ with $(x - 5)$
$= \dfrac{(x^2 - 4x - 5)}{(x - 5)}$	Simplify the fraction; we must first factor
$= \dfrac{(x-5)(x+1)}{(x-5)}$	Divide out common factor of $x - 5$
$= x + 1$	Our Answer

In the examples below, we will combine and evaluate functions. Notice the input value in these examples is a variable expression.

Example 5. Perform the indicated operation.

Let $f(x) = 2x - 1$ and $g(x) = x + 4$.	
Find $(f + g)(x^2)$.	Find the sum of two functions
$= f(x^2) + g(x^2)$	Evaluate $f(x)$ at x^2 and evaluate $g(x)$ at x^2
$= [2(x^2) - 1] + [(x^2) + 4]$	Distributing the $+$ does not change the problem
$= 2x^2 - 1 + x^2 + 4$	Combine like terms
$= 3x^2 + 3$	Our Answer

Example 6. Perform the indicated operation.

Let $f(x) = 2x - 1$ and $g(x) = x + 4$.	
Find $(f \cdot g)(3x)$.	Find the product of two functions
$= f(3x) \cdot g(3x)$	Evaluate $f(x)$ at $3x$ and evaluate $g(x)$ at $3x$
$= [2(3x) - 1] \cdot [(3x) + 4]$	Multiply $2(3x)$
$= (6x - 1)(3x + 4)$	FOIL
$= 18x^2 + 24x - 3x - 4$	Combine like terms
$= 18x^2 + 21x - 4$	Our Answer

APPLICATIONS OF OPERATIONS ON FUNCTIONS

Example 7. A college has two campuses that opened at the same time. The function $A(x)=200x+500$ gives the enrollment at campus A x years after opening. The function $B(x)=100x+1000$ gives the enrollment at campus B x years after opening.

Find $(A+B)(x)$.	Find the sum of two functions
$=A(x)+B(x)$	Replace $A(x)$ with $(200x+500)$ and $B(x)$ with $(100x+1000)$
$=(200x+500)+(100x+1000)$	Combine like terms
$=300x+1500$	Our Answer

This combined function $(A+B)(x)=300x+1500$ gives the total enrollment at both campuses of the college x years after opening.

Example 8. Use the functions from Example 7 to answer the question.

Find $(A+B)(10)$.	Since we already combined $A(x)+B(x)$ to get $(A+B)(x)=300x+1500$ in Example 7, evaluate $(A+B)(x)$ when x = 10.
$=300(10)+1500$	Multiply
$=3000+1500$	Add
$=4500$	Our Answer

This answer tells us that the total enrollment at both campuses 10 years after opening is 4500 students.

Practice Exercises
Section 5.2: Operations on Functions

Perform the indicated operations.

1) Let $f(x) = -4x + 1$ and $g(x) = -2x - 1$. Find $(f + g)(5)$.

2) Let $g(x) = 3x + 3$ and $f(x) = 2x - 2$. Find $(g + f)(9)$.

3) Let $f(x) = x^3 + 5x^2$ and $g(x) = 2x + 4$. Find $(f + g)(3)$.

4) Let $g(x) = 3x + 1$ and $f(x) = x^3 + 3x^2$. Find $(g \cdot f)(2)$.

5) Let $f(x) = -3x^2 + 3x$ and $g(x) = 2x + 5$. Find $\left(\frac{f}{g}\right)(-4)$.

6) Let $g(x) = 4x + 3$ and $h(x) = x^3 - 2x^2$. Find $(g - h)(-1)$.

7) Let $g(x) = x + 3$ and $f(x) = -x + 4$. Find $(g - f)(3)$.

8) Let $g(x) = x^2 + 2$ and $f(x) = 2x + 6$. Find $(g - f)(0)$.

9) Let $g(t) = t - 3$ and $h(t) = -3t^3 + 6t$. Find $(g + h)(1)$.

10) Let $f(n) = n - 5$ and $g(n) = 4n + 2$. Find $(f + g)(-8)$.

11) Let $h(t) = t + 5$ and $g(t) = 3t - 5$. Find $(h \cdot g)(5)$.

12) Let $g(a) = 3a - 2$ and $h(a) = 4a - 2$. Find $(g + h)(-10)$.

13) Let $h(n) = 2n - 1$ and $g(n) = 3n - 5$. Find $\left(\frac{h}{g}\right)(0)$.

14) Let $g(x) = x^2 - 2$ and $h(x) = 2x + 5$. Find $(g + h)(-6)$.

15) Let $f(a) = -2a - 4$ and $g(a) = a^2 + 3$. Find $\left(\frac{f}{g}\right)(7)$.

The Practice Exercises are continued on the next page.

Practice Exercises: Section 5.2 (continued)

Perform the indicated operations.

16) Let $f(x) = x^2 - 5x$ and $g(x) = x + 5$. Find $(f + g)(x)$.

17) Let $f(x) = 4x - 4$ and $g(x) = 3x^2 - 5$. Find $(f + g)(x)$.

18) Let $f(x) = -3x + 2$ and $g(x) = x^2 + 5x$. Find $(f - g)(x)$.

19) Let $g(n) = n^2 - 3$ and $h(n) = 2n - 3$. Find $(g - h)(n)$.

20) Let $g(x) = 2x - 3$ and $h(x) = x^3 - 2x^2 + 2x$. Find $(g - h)(x)$.

21) Let $g(t) = t - 4$ and $h(t) = 2t$. Find $(g \cdot h)(t)$.

22) Let $g(x) = 4x + 5$ and $h(x) = x^2 + 5x$. Find $(g \cdot h)(x)$.

23) Let $g(n) = n^2 + 5$ and $f(n) = 3n + 5$. Find $\left(\frac{g}{f}\right)(n)$.

24) Let $f(x) = 2x + 4$ and $g(x) = 4x - 5$. Find $(f - g)(x)$.

25) Let $g(a) = -2a + 5$ and $f(a) = 3a + 5$. Find $\left(\frac{g}{f}\right)(a)$.

26) Let $g(t) = t^3 + 3t^2$ and $h(t) = 3t - 5$. Find $(g - h)(t)$.

27) Let $h(n) = n^3 + 4n$ and $g(n) = 4n + 5$. Find $(h + g)(n)$.

28) Let $f(x) = 4x + 2$ and $g(x) = x^2 + 2x$. Find $\left(\frac{f}{g}\right)(x)$.

ANSWERS to Practice Exercises
Section 5.2: Operations on Functions

1) -30

2) 46

3) 82

4) 140

5) 20

6) 2

7) 5

8) -4

9) 1

10) -43

11) 100

12) -74

13) $\frac{1}{5}$

14) 27

15) $-\frac{9}{26}$

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 5.2 (continued)

16) $x^2 - 4x + 5$

17) $3x^2 + 4x - 9$

18) $-x^2 - 8x + 2$

19) $n^2 - 2n$

20) $-x^3 + 2x^2 - 3$

21) $2t^2 - 8t$

22) $4x^3 + 25x^2 + 25x$

23) $\dfrac{n^2 + 5}{3n + 5}$

24) $-2x + 9$

25) $\dfrac{-2a + 5}{3a + 5}$

26) $t^3 + 3t^2 - 3t + 5$

27) $n^3 + 8n + 5$

28) $\dfrac{4x + 2}{x^2 + 2x}$