Section 1.2 Objectives

- Recognize rational numbers.
- Simplify fractions.
- Add, subtract, multiply, and divide with fractions.
- Evaluate fractions raised to a power.
- Use order of operations to simplify arithmetic expressions that contain fractions.



INTRODUCTION

In this section, we will expand the set of numbers that we work with to include more than just the integers. We will work with a set of numbers called the rational numbers. You will learn to perform arithmetic operations with rational numbers. Put simply, the *rational numbers* involve fractions. So before we begin our formal study of the rational numbers, let's review some basic vocabulary and concepts associated with fractions.

FRACTIONS

Fractions are used to indicate how many parts of a whole we have. Fractions contain two numbers separated by a fraction bar as shown below.

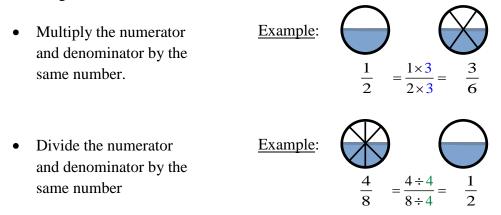
 $\frac{\text{Numerator}}{\text{Denominator}} \leftarrow \text{how many equal parts we have}$

For example, if a pizza is cut into 8 equal pieces and you take 3 pieces, you have $\frac{3}{8}$ of the pizza. The fraction is read "*three eights*" or "3 out of 8."



 $3\frac{2}{5}$

Equivalent fractions have the same value. Equivalent fractions are obtained using one of the following methods.



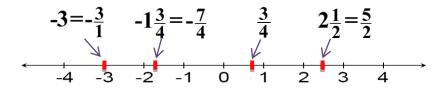
An *improper fraction* is written as a *mixed number* by dividing the denominator into the numerator.

EXAMPLE: Rewrite
$$\frac{17}{5}$$
 as a mixed number.Whole number part $\rightarrow 3$
Denominator $\rightarrow 5$) 17
 $\frac{17}{5} = 3\frac{2}{5}$ Whole number part $\rightarrow 3$
Denominator $\rightarrow 5$) 17
Numerator $\rightarrow 2$ \Rightarrow **PRACTICE:** Rewrite $\frac{21}{8}$ as a mixed number.**Answer:** $2\frac{5}{8}$ **E**

Now we are ready to focus on the set of numbers called the rational numbers.

RATIONAL NUMBERS

An easy way to distinguish the rational numbers from the integers is that the *rational numbers* include fractions whereas the integers do not. Some examples of rational numbers are $\frac{3}{5}$, $-\frac{7}{6}$, and 0. Look at some other rational numbers on the number line below.



The mathematical definition of rational numbers is given below. The definition and the paragraphs that follow explain how to determine whether a number is a rational number.

RATIONAL NUMBERS

Rational numbers are numbers that <u>can be</u> written as one of the following:

- a positive fraction
- a negative fraction
- zero

NOTE: The numerator is any integer. The denominator is any integer <u>except</u> zero.

Can a Rational Number Contain Zero?

The definition above states that 0 is a rational number.

If a fraction contains 0 in the numerator, then the fraction is a rational number.

If a fraction contains 0 in the <u>denominator</u>, then the fraction <u>is not</u> a rational number.

ZERO IN FRACTIONS		
$\frac{0}{n} = 0$	Rational	
$\frac{n}{0} = Undefined$	Not Rational	

EXAMPLES: Are the following numbers rational numbers?

1	0	Yes, this is a rational number	$2 \frac{4}{2}$	No, this is not a rational number
1.	4	since 0 is in the <u>numerator</u> .	2. 0	since 0 is in the <u>denominator</u> .

ARE INTEGERS RATIONAL NUMBERS? Yes, integers are rational numbers since they can be written as fractions with a denominator of 1.

EXAMPLES: Rewrite 3 and –12 as fractions.

$$3 = \frac{3}{1}$$
 $-12 = \frac{-12}{1}$ Write each integer as a fraction with a denominator of 1.

Are Mixed Numbers Rational Numbers? Yes, mixed numbers are rational numbers since they can be written as improper fractions.

EXAMPLE: Rewrite $1\frac{5}{8}$ as an improper fraction.

1^{5}	$(1 \times 8) + 5$	_ 8+5 _	_13
$1\frac{-}{8}$	8	8	8

Multiply the whole number with the denominator, and add that product to the numerator. Place the result in the numerator, and keep the same denominator.

REVIEW: CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION



PRACTICE: Rewrite the mixed number as an improper fraction.

	1.	$3\frac{3}{4}$		2.	$6\frac{2}{5}$
<u>Answers</u> :	1.	$\frac{15}{4}$	**	2.	$\frac{32}{5}$

Are Decimals Rational Numbers? Decimals that terminate (end) and decimals that repeat a pattern are rational numbers since they can be written as fractions.

EXAMPLES: Rewrite the decimal as a fraction in simplest form.

1.	$23.571 = \frac{23571}{1000}$	Since the rightmost decimal place is the thousandths place, the denominator will be 1,000 . (Hint: Since there are three digits to the right of the decimal point, there should be three zeros in the denominator, giving us the denominator 1,000 .)
2.	$3.9 = \frac{39}{10}$	Since the rightmost decimal place is the tenths place, the denominator will be 10 . (Hint: Since there is one digit to the right of the decimal point, there should be one zero in the denominator, giving us the denominator 10 .)

PRACTICE: Rewrite each decimal as a fraction in simplest form.

	1. 3	39.2 01		,	2.	8.17
<u>Answers</u> :	1.	$\frac{39201}{1000}$	**	,	2.	$\frac{817}{100}$

<u>REVIEW</u>: RATIONAL NUMBERS



EXAMPLE: Which of the following is NOT a rational number?

a.	$\frac{7}{3}$	This is a rational number because it is a positive fraction.	e. 3	This is a rational number because the integer 3 can be written as the fraction $\frac{3}{1}$.
b.	$-\frac{2}{5}$	This is a rational number because it is a negative fraction.	f. $4\frac{2}{3}$	This is a rational number because the mixed number $4\frac{2}{3}$ can be written as the fraction $\frac{14}{3}$.
c.	$\frac{0}{6}$	This is a rational number because only the numerator is 0.	g. 3.1	This is a rational number because the decimal 3.1 can be written as the fraction $\frac{31}{10}$.
d.	$\frac{8}{0}$	This is not a rational number because the denominator is 0.		10

The answer is **d**.

5 6	$\frac{-1}{4}$	9	$\frac{0}{2}$	$\frac{3}{0}$	-8
0.	**				

<u>Answer</u>: $\frac{3}{0}$ is not a rational number because the denominator is 0.

PRACTICE: Which of the following is NOT a rational number? $\frac{11}{8} = \frac{-2}{0} = 1\frac{3}{4} = \frac{-2}{3} = -4 = 8.2 = \frac{0}{7}$

PRACTICE: Which of the following is NOT a rational number?

<u>Answer</u>: $\frac{-2}{0}$ is not a rational number because the denominator is 0.

SIMPLIFYING RATIONAL NUMBERS

An important skill necessary for working with rational numbers is simplifying fractions. In fact, if an answer to any math problem is a fraction, it is important to express the final answer as the fraction in <u>simplified (reduced)</u> form. The procedure for simplifying fractions is given below.

SIMPLIFYING FRACTIONS
To simplify a fraction, divide the numerator and the denominator by the Greatest Common Factor (GCF).
A common factor is a number that divides into both numbers without a remainder.
$\frac{a}{b} = \frac{a \div c}{b \div c} \qquad \text{Note: } b \neq 0, \ c \neq 0$

Remember that a fraction is a division problem. So, when you simplify a fraction, it is important to use the rules for dividing signed numbers.

FRACTIONS AND NEGATIVES			
$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$	If only the numerator is negative or only the denominator is negative, the fraction is negative.		
	NOTE: Generally, we write the negative sign in the numerator or in front of the fraction. The negative sign is usually not written in the denominator because it is easy to miss it there.		
$\frac{-a}{-b} = \frac{a}{b}$	If both the numerator and the denominator are negative, the fraction is positive.		

EXAMPLES: Simplify each fraction.

1.	$\frac{8}{28}$	The Greatest Common Factor (GCF) of 8 and 28 is 4.
	$=\frac{8\div4}{28\div4}$	Divide both the numerator and denominator by 4.
	$= \frac{2}{7}$	This is the final answer in simplified form.
2.	$-\frac{15}{6}$	The Greatest Common Factor (GCF) of -15 and 6 is 3.
	$=\frac{-15\div 3}{6\div 3}$	Divide both the numerator and denominator by 3.
	$= \frac{-5}{2}$	Rewrite the fraction with the negative sign in front of the fraction.
	$= -\frac{5}{2}$	This is the final answer in simplified form.
3.	$\frac{-18}{-10}$	The Greatest Common Factor (GCF) of -18 and -10 is 2.
	$=\frac{-18\div 2}{-10\div 2}$	Divide both the numerator and denominator by 2.
	$= \frac{-9}{-5}$	A negative number divided by a negative number is a positive number.
	$= \frac{9}{5}$	This is the final answer in simplified form.

PRACTICE: Simplify. 3. $\frac{-70}{-45}$ 1. $\frac{24}{36}$ 3. $\frac{-70}{-45}$ 2. $\frac{-28}{12}$ 4. $\frac{0}{6}$ **Answers:** 3. $\frac{14}{9}$ **Solution** 2. $-\frac{7}{3}$ **Solution** 4. 0 **Solution**

ARITHMETIC OPERATIONS ON RATIONAL NUMBERS

Now we will review the procedures for performing the four basic arithmetic operations on rational numbers: addition, subtraction, multiplication, and division.

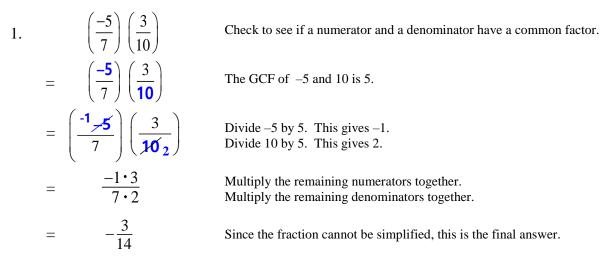
MULTIPLYING RATIONAL NUMBERS

A common denominator is NOT needed to multiply fractions. Instead, we simplify if possible, then multiply straight across.

MULTIPLYING RATIONAL NUMBERS

- 1. Check to see if a numerator and a denominator have a common factor. If so, simplify by dividing out the GCF (Greatest Common Factor).
- 2. Multiply numerators together. Multiply denominators together.

EXAMPLES: Perform the multiplication.



2.	$\frac{-20}{9} \times \frac{-27}{8}$	Check to see if there are any common factors.
	$= \frac{-20}{9} \times \frac{-27}{8}$	The GCF of -20 and 8 is 4.
	$= \frac{-5}{9} \times \frac{-27}{8} \times \frac{-27}{2}$	Divide –20 by 4. This gives –5. Divide 8 by 4. This gives 2.
	$= \frac{\frac{-5}{20}}{9} \times \frac{-27}{8}_{2}$	The GCF of -27 and 9 is 9.
	$= \frac{-5 - 20}{1 9} \times \frac{-3 - 27}{8 2}$	Divide –27 by 9. This gives –3. Divide 9 by 9. This gives 1.
	$= \frac{-5 \times -3}{1 \times 2}$	Multiply the remaining numerators together. Multiply the remaining denominators together.
	$= \frac{15}{2}$	Since the fraction cannot be simplified, this is the final answer.
3.	$\begin{array}{rcl} 0 \cdot \frac{2}{3} \\ = & 0 \end{array}$	0 times any number equals 0. This is the final answer.
4.	$\frac{4}{3} \times 8$	Rewrite 8 as a fraction with a denominator of 1.
	$= \frac{4}{3} \times \frac{8}{1}$	There is no numerator / denominator pair with a GCF. So, multiply the numerators together and multiply the denominators together.
	$=\frac{32}{3}$	Since the fraction cannot be simplified, this is the final answer.

REVIEW: MULTIPLYING RATIONAL NUMBERS

PRACTICE: Perform the multiplication.

1.

$$\frac{5}{6} \times \frac{7}{9}$$
 5.
 $\frac{7}{9} \times 5$
 9.
 $\frac{3}{4} \times \frac{4}{16}$

 2.
 $\left(\frac{2}{3}\right)\left(-\frac{2}{3}\right)$
 6.
 $\frac{6}{7} \times \frac{2}{3}$
 10.
 $\frac{4}{65} \times \frac{-13}{100}$

 3.
 $\frac{4}{-11} \times \frac{-3}{7}$
 7.
 $5 \times \frac{4}{15}$
 11.
 $\left(-\frac{4}{5}\right)\left(-\frac{25}{16}\right)$

 4.
 $\frac{2}{13} \cdot 0$
 8.
 $\frac{3}{4} \times \frac{8}{15}$
 12.
 $\frac{12}{15} \times \frac{10}{9}$

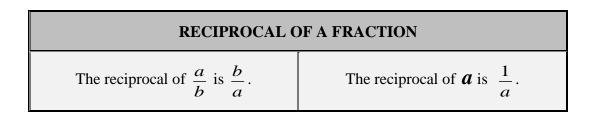
Answers:



DIVIDING RATIONAL NUMBERS

A common denominator is NOT needed to divide fractions. We will divide fractions by performing a procedure similar to the procedure used to subtract integers. Recall how we changed a <u>subtraction</u> problem into an <u>addition</u> problem and added the <u>opposite</u>. Similarly, we will change a <u>division</u> problem into a <u>multiplication</u> problem and multiply by the <u>reciprocal</u>.

First you need to understand what is meant by a reciprocal. The *reciprocal* of a fraction is formed by switching the positions of the numerator (top) and the denominator (bottom). In other words, a reciprocal is a fraction flipped upside down!



KEEP – CHANGE – FLIP METHOD

Now you will learn how to change a division problem into a multiplication problem.

- KEEP: keep the first fraction the same
- CHANGE: change the operation to multiplication
- FLIP: flip the second fraction to get its reciprocal

Numerator Denominator

The division $\frac{a}{b} \div \frac{c}{d}$ equals the multiplication $\frac{a}{b} \cdot \frac{d}{c}$.

KEEP – CHANGE – FLIP		
$\frac{a}{b}$	•	$\frac{c}{d}$
₽	₽	₽
KEEP	CHANGE	FLIP
₽	₽	₽
$\frac{a}{b}$	×	$\frac{d}{c}$
Note: b,	c, and d cannot	t equal 0.

So, to divide fractions, you must multiply the first fraction by the reciprocal of the second fraction. Review the entire procedure for dividing fractions below.

DIVIDING RATIONAL NUMBERS	
 Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP. 	$\frac{a}{b} \div \frac{c}{d}$
 Simplify by dividing out common factors if possible. <u>Note</u>: Never do this before Step 1. 	$= \frac{a}{b} \times \frac{d}{c}$
3. Multiply the numerators together. Multiply the denominators together.	$= \frac{ad}{bc}$

EXAMPLE: Perform the division.

$\frac{2}{5}$ ÷ $\frac{2}{7}$	Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP.
 ↓ ↓	
$=\frac{2}{5}$ \times $\frac{7}{2}$	The first fraction is being multiplied by the reciprocal of the second fraction.
$= \frac{\cancel{2}^{1}}{5} \times \frac{7}{\cancel{2}_{1}}$	Simplify by dividing out common factors. NEVER do this until the problem has been changed to multiplication.
$= \frac{1 \times 7}{5 \times 1}$	Multiply the numerators together. Multiply the denominators together.
$= \frac{7}{5}$	This is the final answer. It can be left as an improper fraction.

<u>REVIEW</u>: Dividing Rational Numbers



PRACTICE: Perform the division.

1.	$-\frac{1}{3} \div \frac{4}{5}$	4.	$\frac{14}{3} \div \left(-\frac{21}{5}\right)$
2.	$\frac{5}{7} \div \frac{10}{11}$	5.	$\left(-\frac{7}{9}\right) \div \left(-\frac{7}{36}\right)$
3.	$\frac{11}{24} \div \frac{55}{36}$	6.	$\frac{15}{6} \div 20$

Answers:



DIVISION WITH ZERO

When you studied integers, you learned the rules for dividing when zero was in the problem. Those same rules apply to division problems that contain fractions. Review the rules below.

DIVISION INVOLVING ZERO	
$0 \div n = 0$	If 0 is divided by any number (except 0), the <u>answer is 0</u> .
$n \div 0 =$ undefined	If any number is divided by 0, the <i>answer is undefined</i> . In other words, there is no numerical answer.

EXAMPLES: Perform the division.

- $0 \div \frac{3}{5}$ 1. Zero divided by any number is 0. = 0
- 2. $\frac{2}{9} \div 0$ Any number divided by 0 is undefined.

<u>REVIEW</u>: Division Involving Zero



PRACTICE: Perform the division.

2. $\frac{1}{2} \div 0$ 1. $0 \div \frac{4}{7}$

Answers:

1. 0

2. Undefined

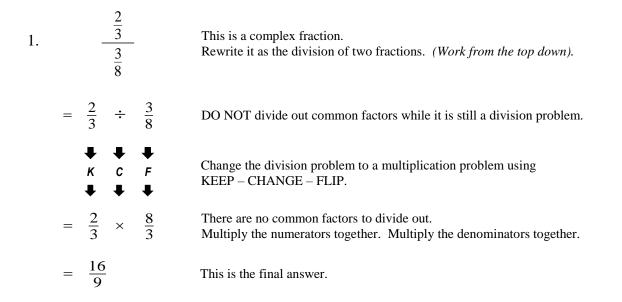


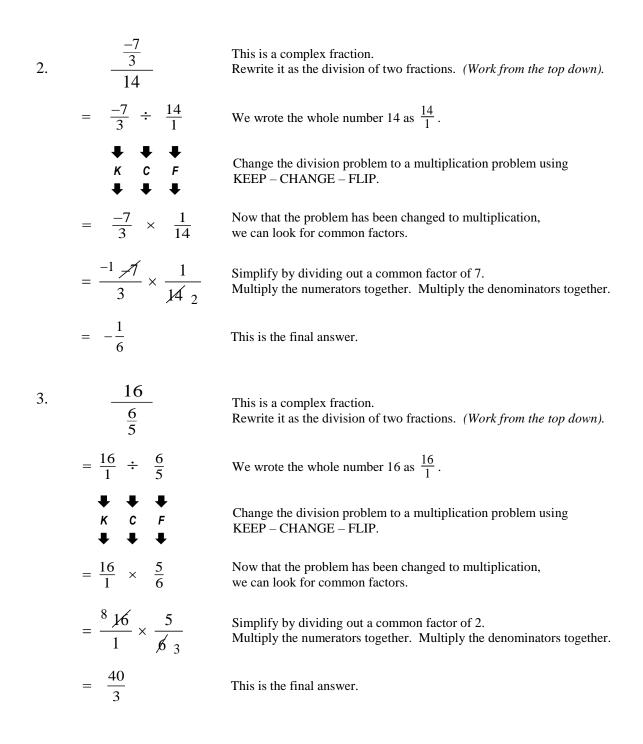
COMPLEX FRACTIONS

A complex fraction has one or two fractions within a fraction. While simplifying this type of problem may seem like it is going to be "complex," it really is not. We will simply rewrite a complex fraction as the division of two fractions and then proceed as we did in the problems above.

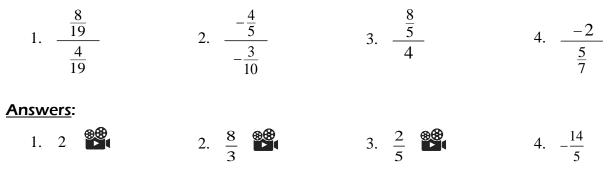
COMPLEX FRACTIONS	
A complex fraction has a fraction in its numerator, denominator, or	both.
1. The fraction to the right is a complex fraction. The numerator is the fraction $\frac{a}{b}$. The denominator is the fraction $\frac{c}{d}$. NOTE: <i>b</i> , <i>c</i> , and <i>d</i> cannot equal 0.	$\frac{\frac{a}{b}}{\frac{c}{d}}$
2. Rewrite the complex fraction as the division of two fractions. (<i>Work from the top down</i>).	$=\frac{a}{b}\div\frac{c}{d}$
 Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP. 	$=\frac{a}{b} \times \frac{d}{c}$
4. Simplify by dividing out common factors if possible. Then multiply numerators together and denominators together.	$= \frac{ad}{bc}$

EXAMPLES: Perform the division.





PRACTICE: Perform the division.



EXPONENTS AND FRACTIONS

In the previous section, exponents were used with integers. Now, exponents will be used with fractions. If a fraction is raised to a power, the fraction is placed in parentheses and the exponent is written on the outside. The fraction in parentheses is called the *base*. Recall that the *exponent* expresses a repeated multiplication of the *base*. More specifically, the *exponent* specifies how many times the *base* is used in the multiplication.

EXAMPLES: Evaluate.

1.
$$\left(\frac{2}{5}\right)^3$$

 $\left(\frac{2}{5}\right)^3$
 $= \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)$
 $= \frac{2 \times 2 \times 2}{5 \times 5 \times 5}$

 $\frac{8}{125}$

=

The base is the fraction $\frac{2}{5}$ inside the parentheses, and the **exponent is 3**.

Multiply $\frac{2}{5}$ three times.

Multiply the numerators together. Multiply the denominators together.

This is the answer.

$$\begin{pmatrix} -\frac{3}{4} \end{pmatrix}^2 = \begin{pmatrix} -\frac{3}{4} \end{pmatrix}^2 = \begin{pmatrix} -\frac{3}{4} \end{pmatrix} \begin{pmatrix} -\frac{3}{4} \end{pmatrix} = \frac{(-3)(-3)}{4 \times 4} = \frac{9}{16}$$

The base is the fraction $-\frac{3}{4}$ inside the parentheses, and the **exponent is 2**. Multiply $-\frac{3}{4}$ **two times**.

Multiply the numerators together. Multiply the denominators together.

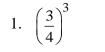
4. $\left(-\frac{3}{5}\right)^{3}$

3. $\left(\frac{6}{5}\right)^2$

This is the answer.

2. $\left(-\frac{2}{3}\right)^2$

PRACTICE: Evaluate.



Answers:

2



ADDING AND SUBTRACTING RATIONAL NUMBERS

It is important to remember that two fractions must have the same denominator in order to add or subtract them. If the denominators <u>are not</u> the same, you will need to find a common denominator.

A *common denominator* (also called *common multiple*) is a number that both of the denominators can divide into without a remainder. You can find a common denominator of two numbers by multiplying them together. However, when adding or subtracting fractions it is best to use the *LCD - Least Common Denominator* (also called *LCM – Least Common Multiple*). A procedure for finding the *Least Common Denominator* is given below.

LEAST COMMON DENOMINATOR (LCD)

To determine the *LCD*, multiply the larger denominator by 1, by 2, by 3, and so on until one of the answers is divisible by the other denominator with no remainder.

EXAMPLE: Determine the LCD needed to add $\frac{2}{3} + \frac{4}{5}$.

The larger denominator is 5.

$5 \times 1 = 5$	5 is not evenly divisible by 3 (the other denominator)
$5 \times 2 = 10$	10 is not evenly divisible by 3 (the other denominator)
5×3=15	15 \underline{is} evenly divisible by 3 (the other denominator)
LCD = 15	Therefore, the LCD is 15.

So, if you are adding or subtracting two fractions that do not have the same denominator, you will begin by finding the LCD. Then you will rewrite the fractions as equivalent fractions with the LCD. Once the two fractions have the same denominator, you simply add or subtract the fractions by combining the numerators.

ADDING AND SUBTRACTING RATIONAL NUMBERS

- 1. If the fractions have the same denominator, add the *numerators* by following the rules for adding and subtracting integers. Keep the denominator the same.
- 2. If the fractions do not have the same denominator, rewrite the fractions with the Least Common Denominator (LCD). Then follow the procedure in Step 1 above.
- * To add or subtract rational numbers, the fractions must have the same denominator.
- * Remember to simplify your final answer if possible.

EXAMPLES: Perform the indicated operation.

1.	$\frac{2}{3} + \frac{4}{5}$	The fractions cannot be added because they do not have the same denominator. We must find the LCD, a number that both denominators will divide into without a remainder. The LCD for 3 and 5 is 15. (<i>Work shown in Example on last page.</i>)
		Rewrite each fraction so the denominator will be 15 (the LCD).
	$=\frac{2\cdot 5}{3\cdot 5}+\frac{4\cdot 3}{5\cdot 3}$	<u>1st Fraction</u> : Multiply the denominator by 5 in order to get 15 (the LCD). Multiply the numerator by 5 also so we get an equivalent fraction.
		2^{nd} Fraction: Multiply the denominator by 3 in order to get 15 (the LCD). Multiply the numerator by 3 also so we get an equivalent fraction.
	$= \frac{10}{15} + \frac{12}{15}$	The fractions have a common denominator now.
	$= \frac{10+12}{15}$	Add the numerators. Keep the denominator the same.
	$=$ $\frac{22}{15}$	This fraction cannot be simplied. This is the final answer.
2.	$\frac{-5}{6} - \frac{1}{2}$	The fractions cannot be subtracted because they do not have the same denominator. We must find the LCD, a number that both denominators will divide into evenly. The LCD for 6 and 2 is 6. (<i>Use the procedure on the previous page to find the LCD</i> .)
	-5 1· 3	Rewrite each fraction so the denominator will be 6 (the LCD).
	$=\frac{-5}{6}-\frac{1\cdot 3}{2\cdot 3}$	<u>1^{st} Fraction</u> : The denominator is already 6.
		<u>2nd Fraction</u> : Multiply the denominator by 3 in order to get 6 (the LCD). Multiply the numerator by 3 also so we get an equivalent fraction.
	$=\frac{-5}{6}-\frac{3}{6}$	The fractions have a common denominator now.
	$=\frac{-5-3}{6}$	Combine the numerators. Keep the denominator the same.
	$=\frac{-5+(-3)}{6}$	In the numerator, change the subtraction to adding the opposite. Then add.
	$= \frac{-8}{6}$	This fraction can be simplified because the numerator and denominator have a common factor of 2.
	$=\frac{-8\div 2}{6\div 2}$	Divide the numerator and denominator by 2.
	$=\frac{-4}{3}$	Move the negative sign in front of the fraction.
	$= -\frac{4}{3}$	This is the final answer.

3.	$\frac{3}{8} - \frac{5}{6}$	In order to subtract the fractions, we must get a common denominator. The LCD for 8 and 6 is 24. (<i>Use the procedure you learned earlier to find the LCD</i> .)
		Rewrite each fraction so the denominator will be 24 (the LCD).
	$=\frac{3\cdot 3}{8\cdot 3}-\frac{5\cdot 4}{6\cdot 4}$	<u>1st Fraction</u> : Multiply the denominator by 3 in order to get 24 (the LCD). Multiply the numerator by 3 also.
		$\frac{2^{nd} \text{ Fraction}}{1000}$: Multiply the denominator by 4 in order to get 24 (the LCD). Multiply the numerator by 4 also.
	$= \frac{9}{24} - \frac{20}{24}$	The fractions have a common denominator now.
	$=$ $\frac{9-20}{24}$	Combine the numerators. Keep the denominator the same.
	$=\frac{9+(-20)}{24}$	In the numerator, change the subtraction to adding the opposite. Then add.
	$= \frac{-11}{24}$	This fraction cannot be simplified. We will just move the negative sign in front of the fraction.
	$= -\frac{11}{24}$	This is the final answer.
4.	$5 - \frac{1}{2}$	Rewrite the whole number 5 a a fraction with a denominator of 1.
	$=$ $\frac{5}{1} - \frac{1}{2}$	In order to subtract the fractions, we must get a common denominator. The LCD for 1 and 2 is 2.
		Rewrite each fraction so the denominator will be 2 (the LCD).
	$=\frac{5\times 2}{1\times 2}-\frac{1}{2}$	<u>1st Fraction</u> : Multiply the denominator by 2 in order to get 2 (the LCD). Multiply the numerator by 2 also.
		2^{nd} Fraction: The denominator is already 2.
	$= \frac{10}{2} - \frac{1}{2}$	The fractions have a common denominator now.
	$= \frac{10-1}{2}$	Subtract the numerators. Keep the denominator the same.
	$=$ $\frac{9}{2}$	This fraction cannot be simplified. This is the final answer as an improper fraction.

REVIEW: ADDING RATIONAL NUMBERS (LIKE DENOMINATORS)

REVIEW: Adding Rational Numbers (Unlike Denominators)



	······································		
1.	$\frac{-1}{6} + \frac{-2}{6}$	9.	$-\frac{3}{11}-\frac{1}{3}$
2.	$\frac{3}{5} + \frac{1}{10}$	10.	$-\frac{8}{9}-4$
3.	$\frac{1}{6} + \frac{5}{9}$	11.	$\frac{1}{9} - \frac{4}{3}$
4.	$-\frac{1}{6}+\frac{3}{4}$	12.	$2 + \frac{7}{8}$
5.	$\frac{5}{9} + \frac{11}{12}$	13.	$\frac{5}{12} + \frac{1}{3}$
6.	$6 - \frac{2}{3}$	14.	$\frac{4}{3} + \left(-\frac{9}{5}\right)$
7.	$\frac{3}{5} - \frac{1}{10}$	15.	$\frac{2}{3} - \frac{1}{6}$
8.	$\frac{1}{4} - \frac{7}{10}$	16.	$-\frac{4}{5}-\frac{3}{7}$
<u>Ansv</u>	<u>vers</u> :		
1.	$-\frac{1}{2}$	9.	$-\frac{20}{33}$
2.	$\frac{7}{10}$	10.	$-\frac{44}{9}$
3.	$\frac{13}{18}$	11.	$-\frac{11}{9}$
4.	$\frac{7}{12}$	12.	$\frac{23}{8}$
5.	$\frac{53}{36}$	13.	$\frac{3}{4}$

PRACTICE: Perform the indicated operation.

36

 $\frac{16}{3}$

 $\frac{1}{2}$

8. $-\frac{9}{20}$

6.

7.

⊜⊛ ►

⊛⊛ ►

4

14.

15.

16.

7

15

 $\frac{1}{2}$

 $\frac{43}{35}$

ORDER OF OPERATIONS WITH RATIONAL NUMBERS

Now that you have studied how to add, subtract, multiply, and divide fractions, we can evaluate more complicated expressions where fractions are involved. These expressions will involve more than one operation. The operations must be performed in the same order that you learned when you studied integers. Review the *Order of Operation* rules listed below.

ORDER OF OPERATIONS

Step 1: Parentheses

If there are any operations in parentheses, those computations should be performed first.

Step 2: Exponents and Roots

Simplify any numbers being raised to a power and any numbers under the $\sqrt{}$ symbol.

Step 3: Multiplication and Division

Do these two operations in the order in which they appear, working from *left to right*.

Step 4: Addition and Subtraction

Do these two operations in the order in which they appear, working from *left to right*.

EXAMPLES: Evaluate.

1.	$\frac{2}{3} + \frac{3}{4} \times \frac{1}{2}$	Step 1: Parentheses – There are none.Step 2: Exponents and Roots – There are none.
	$=\frac{2}{3}+\frac{3}{4}\times\frac{1}{2}$	Step 3: Multiplication and Division Since there are no common factors to divide out, just multiply straight across
	$=\frac{2}{3}+\frac{3}{8}$	Step 4: Addition and Subtraction To add these fractions, we need a common denominator. The LCD for 3 and 8 is 24.
	$=\frac{2\times8}{3\times8}+\frac{3\times3}{8\times3}$	 To get 24 as the common denominator: Multiply the numerator and denominator of the first fraction by 8. Multiply the numerator and denominator of the second fraction by 3.
	$=\frac{16}{24}+\frac{9}{24}$	The denominators are the same now. Add the numerators and keep the same denominator.
	$=\frac{25}{24}$	Answer: This fraction cannot be simplified. This is the final answer.

2.	$\frac{6}{7} \div \frac{11}{14} \times \frac{3}{5}$	Step 1: Parentheses – There are none.
	7 14 5	Step 2: Exponents and Roots – There are none.
	$=\frac{6}{7}\div\frac{11}{14}\times\frac{3}{5}$	Step 3: Multiplication and Division There is a multiplication and a division. We will do the division first since it is the leftmost operation.
	$=\frac{6}{7}\times\frac{14}{11}\times\frac{3}{5}$	To change the division problem to a multiplication problem, multiply the 1^{st} fraction by the reciprocal of the 2^{nd} fraction (<i>Keep-Change-Flip</i>)
	$=\frac{6}{1^{1}}\times\frac{14^{2}}{11}\times\frac{3}{5}$	Simplify by dividing 7 and 14 by the common factor 7.
	$=\frac{6\times 2}{1\times 11}\times \frac{3}{5}$	Multiply the first two fractions together.
	$= \frac{12}{11} \times \frac{3}{5}$	Multiply the result by the third fraction. Since there are no common factors to divide out, just multiply straight across.
	$= \frac{36}{55}$	Answer: This fraction cannot be simplified. This is the final answer.
3.	$\frac{4}{7} \times \left(\frac{2}{3} - \frac{1}{9}\right)$	Step 1: Parentheses – Begin with the operation inside the parentheses. To subtract the fractions, we need a common denominator. The LCD for 3 and 9 is 9.
	$=\frac{4}{7} \times \left(\frac{2 \times 3}{3 \times 3} - \frac{1}{9}\right)$	 To get 9 as the common denominator: Multiply the numerator and denominator of the first fraction in the parentheses by 3. The second fraction in the parentheses does not need to be changed.
	$=\frac{4}{7} \times \left(\frac{6}{9} - \frac{1}{9}\right)$	The fractions in the parentheses have the same denominator now. Subtract the numerators and keep the same denominator.
		Step 2: Exponents and Roots – There are none.
	$=\frac{4}{7}\times\frac{5}{9}$	Step 3: Multiplication and Division Since there are no common factors to divide out, just multiply straight across.
	$=\frac{20}{63}$	Answer: This fraction cannot be simplified. This is the final answer.

4.	$3\left(\frac{1}{4}\right)^2 - 5^0$	Step 1: Parentheses – There is nothing to simplify inside the parentheses.Step 2: Exponents and Roots – There are two exponents to evaluate.
	$= 3\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) - 5^0$	$\left(\frac{1}{4}\right)^2 \rightarrow \text{Multiply } \frac{1}{4} \text{ two times.}$ $5^0 \rightarrow Recall that any number (other than 0) raised to the 0 power is 1.$
	$=3\left(\frac{1}{16}\right)-1$	Step 3: Multiplication and Division Express 3 as a fraction in order to multiply.
	$=\frac{3}{1}\left(\frac{1}{16}\right)-1$	Since there are no common factors to divide out, just multiply straight across.
	$=\frac{3}{16}-1$	Step 4: Addition and Subtraction Write 1 as a fraction.
	$= \frac{3}{16} - \frac{1}{1}$	To subtract the fractions, we need a common denominator. The LCD for 16 and 1 is 16.
	$= \frac{3}{16} - \frac{1 \times 16}{1 \times 16}$	To get 16 as the common denominator:Leave the first fraction the same.Multiply the numerator and denominator of the second fraction by 16.
	$= \frac{3}{16} - \frac{16}{16}$	Combine the numerators. Keep the denominator the same.
	$= \frac{3-16}{16}$	Change the subtraction to adding the opposite.
	$= \frac{3+(-16)}{16}$	Simplify the numerator.
	$= \frac{-13}{16}$	Answer: This fraction cannot be simplified. We will rewrite the answer with the negative sign in front of the fraction.
	$= -\frac{13}{16}$	This is the final answer.

5.
$$\frac{5(3-6)+3\cdot4}{2^3-12}$$
 In this problem, the fraction bar acts as parentheses, grouping the numerator together and grouping the denominator together.

First, simplify the numerator.

 $= \frac{5(3-6)+3\cdot 4}{2^3-12}$ Step 1: Parentheses Change the subtraction in the parentheses to adding the opposite. $= \frac{5(3+-6)+3\cdot 4}{2^3-12}$ Perform the addition inside the parentheses.

Step 2: Exponents and Roots – There are none in the numerator.

 $= \frac{5(-3) + 3 \cdot 4}{2^3 - 12}$ Step 3: Multiplication and Division – Perform the two multiplications in the numerator.

Step 4: Addition and Subtraction – Add the integers in the numerator.

$$=\frac{-3}{2^3-12}$$

 $=\frac{-15+12}{2^3-12}$

This is the simplified numerator.

Next, simplify the denominator.

 $= \frac{-3}{2^{3}-12}$ Step 1: Parentheses – There are none in the denominator. $= \frac{-3}{2 \times 2 \times 2 - 12}$ Step 2: Exponents and Roots – To evaluate 2³, multiply 2 three times. $= \frac{-3}{8-12}$ Step 3: Multiplication and Division – There are none in the denominator. Step 4: Addition and Subtraction In the denominator, change the subtraction to adding the opposite. Add the integers in the denominator. $= \frac{-3}{-4}$ Simplify the fraction. A negative divided by a negative is a positive. $= \frac{3}{4}$ Answer: This is the final answer.

PRACTICE: Evaluate.

1. $\frac{1}{2} + \frac{3}{5} \times \frac{5}{7}$ 2. $\frac{5}{8} \div \frac{9}{10} \times \frac{3}{7}$ 3. $\frac{3}{8} \times \left(\frac{5}{6} - \frac{1}{4}\right)$ 5. $6\left(\frac{3}{4}\right)^2 - \left(\frac{7}{8}\right)^0$ 6. $\frac{9}{4}\left(-\frac{2}{3}\right) + \left(\frac{1}{2}\right)^3$ 7. $\frac{12 + 2^3}{3 \cdot 7 - (4 - 2)}$

$$4. \quad \left(-\frac{2}{9} + -\frac{1}{6}\right) \div 14$$

8.
$$\frac{(-9+5)\div 4-2}{(-4)^2-6}$$

Answers:



SECTION 1.2 SUMMARY Rational Numbers							
	$\frac{\text{Numerator}}{\text{Denominator}} \rightarrow \# \text{ of equal parts used}$ $\frac{1}{\text{Denominator}} \rightarrow \# \text{ of equal parts that}$ $\frac{1}{\text{make a whole}}$	<u>Example</u> : $\frac{3}{5}$ means					
Fractions	Zero in Fractions: $\frac{0}{n} = 0$ $\frac{n}{0} =$ Undefined	<u>Examples</u> : $\frac{0}{2} = 0$ $\frac{2}{0} =$ Undefined					
	Write a Mixed Number as an Improper Fraction	<u>Example</u> : $6\frac{2}{7} = \frac{(6 \times 7) + 2}{7} = \frac{42 + 2}{7} = \frac{44}{7}$					
	Write a Decimal as a Fraction	<u>Example</u> : $26.31 = \frac{2631}{100}$					
Rational Numbers	A rational number is any number that can be written as a fraction in the form: $\frac{\text{Numerator}}{\text{Denominator}} \rightarrow any integer$	Example: Which of the following is NOT a rational number? $1\frac{2}{3} \frac{-7}{9} 5.1 \frac{0}{3} \frac{2}{0} -6$ $\frac{2}{0} \text{ is not a rational number}$ because the denominator is 0.					
Simplifying	GCF – divide the numerator and denominator by the Greatest Common Factor	<u>Example</u> : $\frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}$					
RATIONAL NUMBERS	Negatives: $\frac{-a}{-b} = \frac{a}{b}$ $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$	<u>Examples</u> : $\frac{-5}{-7} = \frac{5}{7}$ $\frac{-5}{7} = \frac{5}{-7} = -\frac{5}{7}$					
Multiplying Rational Numbers	 Divide out common factors in the numerator and denominator. Multiply numerators and multiply denominators. Simplify if possible. 	$\underline{Example}: \frac{5}{6} \times \frac{7}{15}$ $= \frac{1}{5} \times \frac{7}{15}$ $= \frac{1}{6} \times \frac{7}{15}$ $= \frac{1 \times 7}{6 \times 3}$ $= \frac{7}{18}$					

	Change the division problem to a multiplication problem using <i>KEEP</i> – <i>CHANGE</i> – <i>FLIP</i> . Multiply the first fraction by the reciprocal of the second fraction.	$ \underline{Example}: \begin{array}{cccc} \frac{5}{6} & \div & \frac{2}{7} \\ \bullet & \bullet & \bullet \\ K & C & F \\ \bullet & \bullet & \bullet \\ = \frac{5}{6} & \times & \frac{7}{2} \\ = & \frac{35}{12} \end{array} $
Dividing Rational Numbers	Division with Zero: $0 \div n = 0$ $n \div 0 =$ undefined	<u>Examples</u> : $0 \div \frac{8}{9} = 0$ $\frac{8}{9} \div 0 = $ undefined
	 Division with Complex Fractions Rewrite the problem as the division of two fractions. Change to a multiplication problem using <i>KEEP – CHANGE – FLIP</i>. 	$ \underline{Example}: \frac{\frac{3}{5}}{\frac{4}{9}} = \frac{3}{5} \div \frac{4}{9} \\ = \frac{3}{5} \times \frac{9}{4} \\ = \frac{27}{20} $
Exponents	The exponent specifies how many times to use the base in a repeated multiplication.	<u>Example</u> : $\left(\frac{8}{9}\right)^2 = \left(\frac{8}{9}\right)\left(\frac{8}{9}\right) = \frac{8 \times 8}{9 \times 9} = \frac{64}{81}$
Adding and Subtracting Rational Numbers	 Determine the LCD. Rewrite the fractions by multiplying the numerator and denominator of each fraction by a number to obtain the LCD. Add or subtract the numerators as indicated and keep the same denominator. Simplify if possible. 	$\underline{Example}: \frac{1}{4} + \frac{5}{6} \qquad LCD = 12$ $= \frac{1 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2}$ $= \frac{3}{12} + \frac{10}{12}$ $= \frac{13}{12}$
Order of Operations	 Parentheses Exponents and Roots Multiplication and Division (work from left to right whichever operation comes first) Addition and Subtraction (work from left to right whichever operation comes first) 	$ \underline{Example}: = \frac{7}{10} + \frac{3}{5} \times \left(\frac{1}{2}\right)^2 \\ = \frac{7}{10} + \frac{3}{5} \times \left(\frac{1}{2} \times \frac{1}{2}\right) \\ = \frac{7}{10} + \frac{3}{5} \times \frac{1}{4} \\ = \frac{7}{10} + \frac{3}{20} \\ = \frac{7 \times 2}{10 \times 2} + \frac{3}{20} \\ = \frac{14}{20} + \frac{3}{20} \\ = \frac{17}{20} $

SECTION 1.2 EXERCISES

Rational Numbers

Write each number as indicated.

Perform the indicated operation.

1.	Write $2\frac{7}{9}$ as an improper fraction.	16.	$\frac{7}{12} \div \frac{7}{4}$
2.	Write 11.23 as a fraction.	17.	$\frac{3}{2} \div 24$
	of the following is NOT a rational number.	18.	$\frac{44}{15} \div -\frac{11}{9}$
3.	$-5 \frac{0}{8} 6.3 \frac{25}{3} 4\frac{1}{5} \frac{2}{0} -\frac{9}{7}$		$-20 \div \frac{4}{5}$
Simplif		20.	$\left(-\frac{5}{8}\right) \div \left(-\frac{30}{12}\right)$
4.	$\frac{30}{48}$		$\left(\begin{array}{c} 8 \end{array}\right) \left(\begin{array}{c} 12 \end{array}\right)$ $-\frac{5}{9} \div 0$
5.	$\frac{120}{80}$)
6.	$-\frac{12}{40}$		$0 \div \frac{6}{11}$
	$\frac{-15}{-35}$	23.	$\frac{\frac{2}{3}}{\frac{4}{5}}$
8.	$\frac{15}{0}$	24	5 -5
9.	$\frac{0}{4}$	24.	$\frac{-5}{\frac{-2}{3}}$
Perform	n the indicated operation.	25.	$\frac{\frac{7}{2}}{-14}$
10	14 8		-14

10.	$\frac{14}{12} \times \frac{8}{21}$
11.	$\frac{2}{3}$ ×15
12.	$\frac{4}{5} \cdot \frac{-6}{7}$
13.	$\left(-\frac{3}{8}\right)\left(\frac{5}{12}\right)$
14.	$\frac{-15}{2} \cdot \frac{8}{-25}$
15.	$0 \times \frac{11}{43}$

Evaluate.

26.
$$\left(\frac{3}{7}\right)^2$$

27. $\left(-\frac{2}{5}\right)^2$
28. $\left(\frac{2}{3}\right)^3$
29. $\left(-\frac{1}{2}\right)^3$

Evaluate each expression.

Perform the indicated operation.

30.	$\left(-\frac{3}{14}\right) + \left(-\frac{5}{14}\right)$	41.	$\frac{5}{6} \div \left(\frac{2}{3}\right)^2$
31.	$\frac{3}{8} + \frac{1}{7}$	42.	$\frac{1}{3} \div \frac{5}{6} \times \frac{1}{2}$
32.	$9 + \frac{3}{4}$	43.	$\left(-\frac{2}{3}\right)^2 - 4$
33.	$\frac{-2}{5} + \frac{-4}{15}$	44.	$\frac{3}{8} - \frac{5}{6} \times \frac{12}{7}$
34.	$-\frac{3}{4}+\frac{5}{6}$	45.	$\left(\frac{4}{15} + \frac{-5}{3}\right) - \left(\frac{2}{5}\right)^2$
35.	$\frac{5}{8} + \left(-\frac{11}{12}\right)$	46.	$9 + \frac{3}{4} \div 8$
36.	$\frac{1}{4} - \frac{3}{5}$	47.	$2 - \frac{1}{3} \times \frac{1}{2}$
37.	$\frac{-3}{4} - \frac{2}{3}$	48.	$\left(\frac{2}{9}-\frac{4}{9}\right)$ ÷3
38.	$\frac{3}{8} - \frac{5}{12}$		$\frac{14+3^3}{4\cdot 5-(6-10)}$
39.	$-7 - \frac{2}{3}$		1 0 (0 10)
40.	$\frac{4}{9} - 6$	50.	$\frac{2(-9+7)+8\cdot 4}{5^2-3}$

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Answers to Section 1.2 Exercises

1.	$\frac{25}{9}$	16.	$\frac{1}{3}$
2.	$\frac{1123}{100}$	17.	$\frac{1}{16}$
3.	$\frac{2}{0}$		$-\frac{12}{5}$
4.	$\frac{5}{8}$	19.	
5.	$\frac{5}{8}$ $\frac{3}{2}$	20.	$\frac{1}{4}$
	$-\frac{3}{10}$	21.	Undefined
		22.	0
	$\frac{3}{7}$	23.	$\frac{5}{6}$
	Undefined		
9.	0 4	24.	$\frac{15}{2}$
10.	$\frac{1}{9}$		1
11.	10	25.	$-\frac{1}{4}$
12.	$-\frac{24}{35}$	26.	$\frac{9}{49}$
13.	$-\frac{5}{32}$	27.	$\frac{4}{25}$
14.		28.	$\frac{8}{27}$
15.		29.	$-\frac{1}{8}$

30.	$-\frac{4}{7}$	41.	$\frac{15}{8}$
31.	$\frac{29}{56}$	42.	$\frac{1}{5}$
32.	$\frac{39}{4}$	43.	$-\frac{32}{9}$
33.	$-\frac{2}{3}$	44.	$-\frac{59}{56}$
34.	$\frac{1}{12}$	45.	$-\frac{39}{25}$
35.	$-\frac{7}{24}$	46.	$\frac{291}{32}$
36.	$-\frac{7}{20}$	47.	$\frac{11}{6}$
37.	$-\frac{17}{12}$	48.	$-\frac{2}{27}$
38.	$-\frac{1}{24}$	49.	$\frac{41}{24}$
39.	$-\frac{23}{3}$	50.	$\frac{14}{11}$
40.	$-\frac{50}{9}$		

Mixed Review

Simplify.

SCUUTIT = 1.2	Section	1.1	_	1.2
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1.	- - 8	6.	$80 \div (-4)$	11.	$30-6^2+\frac{1}{3}$
2.	-7+5	7.	6÷0	12.	$(15+-3)$ ÷ $\sqrt{16}$ ×8
3.	-4-6	8.	5 ³	13.	$- 3-7 +5\times(-6+2)$
4.	-8-(-9)	9.	$(-9)^2$	14.	$-3^2 + 4\left(\frac{1}{3} \div \frac{1}{9}\right) - \frac{2}{3}$
5.	$-7 \cdot (-8)$	10.	3√8	15.	$-\left 6\right \div \left(\frac{11}{4} - \frac{3}{4}\right) + \sqrt{25} \times \frac{2}{5}$

Answers to Mixed Review

1.	-8	6.	-20	11.	$-\frac{17}{3}$
2.	-2	7.	Undefined	12.	24
3.	-10	8.	125	13.	-24
4.	1	9.	81	14.	$\frac{7}{3}$
5.	56	10.	2	15.	-1