"Success is the sum of small efforts,

repeated day in and day out."

ROBERT COLLIER

CHAPTER 2

Algebraic Equations and Inequalities

Fall 2017

Section 2.1 Objectives

- Determine if a given value is a solution of an equation.
- Solve one-step equations using algebra properties of equality.
- Solve two-step equations using algebra properties of equality.
- Translate "number" word problems into algebraic equations and solve the equations.

SECTION 2.1 One and Two Step Equations

INTRODUCTION

In the previous chapter you learned to simplify algebraic ***expressions***. Recall that an algebraic expression does not contain an equal sign. A mathematical statement that does contain an equal sign is called an ***equation***. In this chapter you will learn to solve algebraic equations. Refer to the table below to make sure you understand the differences between expressions and equations.

EXPRESSION	EQUATION
Does not contain an equal sign Example: $6+x+3$	Contains an equal sign Example: $6+x+3=11$
Expressions are **simplified** Answer: $x+9$	Equations are **solved** Answer: $x=2$ *Note: You will learn how we got this answer later in the chapter.*

SOLUTIONS OF EQUATIONS

A ***solution*** of an equation is the number that can be put in place of the variable to make the equation true. For example, the box above says that the answer to the equation $6+x+3=11$ is $x=2$. To show that the number 2 is the solution of the equation, we replace the variable in the equation with the number 2. Then we perform the arithmetic and simplify. If the result shows that the two sides of the equation are equal, then the number 2 is the solution. Let's try this.

$$6+x+3=11$$

$$6+2+3\stackrel{?}{=}11$$ The question mark above the equal sign indicates that we have not determined yet whether or not the two sides of the equation are equal.

$$8+3\stackrel{?}{=}11$$

$$11=11\ \checkmark$$ This final step shows that the two sides of the equation are equal.

The work above confirms that the number 2 is the solution of the equation $6+x+3=11$.

There are some equations that have more than one solution, and there are some equations that have no solution. You will study those types of equations in the next section.

SOLUTION OF AN EQUATION

To determine if a given value is a solution of a given equation:

1. Substitute the value in place of the variable(s) in the equation.
2. Perform the arithmetic on each side of the equation.
   - Be sure to follow the correct order of operations (PEMDAS).
   - Simplify until there is just one number on each side of the equation.
3. If the two sides of the equation are equal, then the given value is a solution. Otherwise, the given value is not a solution.

***EXAMPLES*:** Determine if the given value is a solution of the given equation.

1. Is $x=5$ a solution of $3x+1=16$?

$3x+1=16$	Substitute 5 in place of the variable in the equation.
$3(5)+1\stackrel{?}{=}16$	Perform the multiplication on the left side of the equation.
$15+1\stackrel{?}{=}16$	Perform the addition on the left side of the equation.
$16=16$ ✓	There is one number on each side of the equation. Are the two sides of the equation equal? Yes, 16 = 16.
	Answer: Yes, 5 is a solution of the equation.

2. Is $a=-7$ a solution of $2(a-4)=-12$?

$2(a-4)=-12$	Substitute –7 in place of the variable in the equation.
$2(-7-4)\stackrel{?}{=}-12$	In the parentheses, change the subtraction problem to an addition problem.
$2(-7+-4)\stackrel{?}{=}-12$	Perform the addition problem in the parentheses.
$2(-11)\stackrel{?}{=}-12$	Perform the multiplication on the left side of the equation.
$-22\neq-12$	There is one number on each side of the equation. Are the two sides equal? No, $-22\neq-12$.
	Answer: No, –7 is not a solution of the equation.

3. Is $y=-6$ a solution of $8y+7=4y-3$?

$8y+7=4y-3$ Substitute –6 in place of each variable in the equation.

$8(\mathbf{-6})+7\stackrel{?}{=}4(\mathbf{-6})-3$ Perform the arithmetic on each side of the equation. Begin with the multiplication.

$-48+7\stackrel{?}{=}-24-3$ Continue to perform the arithmetic on each side.

$-41\neq -27$ There is one number on each side of the equation.
Are the two sides equal? No, $-41\neq -27$.

Answer: No, –6 is not a solution of the equation.

4. Is $x=\frac{2}{3}$ a solution of $\frac{19}{15}-x=\frac{9}{10}x$?

$\frac{19}{15}-x=\frac{9}{10}x$ Substitute $\frac{2}{3}$ in place of each variable in the equation.

$\frac{19}{15}-\frac{\mathbf{2}}{\mathbf{3}}\stackrel{?}{=}\frac{9}{10}\left(\frac{\mathbf{2}}{\mathbf{3}}\right)$ On the **left side** of the equation, we need to get a common denominator in order to subtract the fractions. The LCD of 15 and 3 is 15.

$\frac{19}{15}-\frac{2\cdot 5}{3\cdot 5}\stackrel{?}{=}\frac{9}{10}\left(\frac{2}{3}\right)$ Multiply the numerator and denominator of the second fraction by 5 in order to get a denominator of 15.

$\frac{19}{15}-\frac{10}{15}\stackrel{?}{=}\frac{9}{10}\left(\frac{2}{3}\right)$ Subtract the fractions on the left side of the equation since the denominators are the same now.

$\frac{9}{15}\stackrel{?}{=}\frac{9}{10}\left(\frac{2}{3}\right)$ Reduce the fraction on the left side of the equation.

$\frac{3}{5}\stackrel{?}{=}\frac{\overset{\mathbf{3}}{\cancel{9}}}{\underset{\mathbf{5}}{\cancel{10}}}\left(\frac{\overset{\mathbf{1}}{\cancel{2}}}{\underset{\mathbf{1}}{\cancel{3}}}\right)$ On the **right side** of the equation, the fractions are being multiplied. First, divide out the common factors. Then multiply straight across.

$\frac{3}{5}=\frac{3}{5}$ ✓ There is one number on each side of the equation.
Are the two sides equal? Yes, $\frac{3}{5}=\frac{3}{5}$.

Answer: Yes, $\frac{2}{3}$ is a solution of the equation.

PRACTICE: Determine if the given value is a solution of the given equation.

1. Is $x=2$ a solution of $5x-4=3x$?
2. Is $x=-3$ a solution of $-8(x+2)-4=-15$?
3. Is $x=-3$ a solution of $5(4x+10)=9x+17$?
4. Is $y=6$ a solution of $-4y-8=32$?
5. Is $x=-2.5$ a solution of $-2x-7=6x+13$?
6. Is $a=\frac{1}{4}$ a solution of $\frac{8}{5}a=\frac{1}{2}+a$?

Answers:

1. Yes 
2. No 
3. Yes 
4. No
5. Yes
6. No

SOLVING EQUATIONS

In the last set of problems you were given both an equation and a value of the variable. You determined if the given value was the solution of the equation. Now you will start with just an equation and you will determine its solution. In other words, you will find the value of the variable that makes the equation true. This process is called ***solving an equation***.

The goal in ***solving an equation*** is to get the variable alone on one side of the equal sign and a number alone on the other side (Example: $x = 7$). To achieve this, you will use ***inverse operations*** to "undo" whatever is being done to the variable.

For example, if 5 is being added to the variable, you will do the inverse and subtract 5.

$$x + 5 = 12$$
$$-5$$

BUT . . . When you solve an equation, it is important to remember that the equal sign represents a balance. So, if you perform an operation on one side of the equation, that same operation must be performed on the other side in order to produce an equivalent equation.

So, if you subtract 5 from one side of an equation, you must also subtract 5 from the other side.

$$x + 5 = 12$$
$$-5 \quad -5$$

You finish by simplifying both sides of the equation. The result is an equation in the form $x = number$.

When the left side is simplified, +5 – 5 is 0. Then x + 0 is x.

When the right side is simplified, 12 – 5 is 7.

$$x + 5 = 12$$
$$-5 \quad -5$$
$$x + 0 = 7$$
$$x = 7$$

Now the equation is ***solved*** because the variable is alone on one side of the equal sign and a number is alone on the other side.

SOLVING AN EQUATION

Meaning: Determine the value of the variable that makes the equation true.

Goal: Get the variable alone on one side of the equation.

Goal: Variable = Number

Example: $x = 2$

Process: Use inverse operations (*opposite operations that undo each other*).

- Addition and Subtraction are inverse operations
- Multiplication and Division are inverse operations

Golden Rule: Perform the same operation on both sides of the equation.

SOLVING ONE-STEP EQUATIONS (with Addition and Subtraction)

We begin with equations that require only one step to solve, either addition or subtraction.

- If a number is being added to the variable, then we will subtract the number from both sides of the equation.
- If a number is being subtracted from the variable, then we will add the number to both sides of the equation.

These problems may seem very easy and you may even be able to solve them in your head. However, it is important that you write out the algebra steps so that you develop the skills needed to solve more complex equations later.

EXAMPLES: Solve each equation.

1. Solve $x+9=16$

The variable x is on the left side of the equal sign. We need to get x alone on that side.

$x+9 = 16$ — 9 is *added* to x. To isolate x, perform the ***inverse*** operation.

$-9 \quad -9$ — *Subtract* 9 from both sides of the equation.

$x+0 = 7$ — On the left side of the equal sign, $9-9$ is 0. Then $x + 0$ is x. On the right side of the equal sign, $16 - 9$ is 7.

$x = 7$ — This is the solution to the original equation.

2. Solve $x-5=-8$

The variable x is on the left side of the equal sign. We need to get x alone on that side.

$x-5 = -8$ — 5 is *subtracted* from x. To isolate x, perform the ***inverse*** operation.

$+5 \quad +5$ — *Add* 5 to both sides of the equation.

$x+0 = -3$ — On the left side of the equal sign, $-5 + 5$ is 0. Then $x + 0$ is x. On the right side of the equal sign, $-8 + 5$ is -3.

$x = -3$ — This is the solution to the original equation.

3. Solve $y+2.6=4$

The variable y is on the left side of the equal sign. We need to get y alone on that side.

$y+2.6 = 4$ — 2.6 is *added* to y. To isolate y, perform the ***inverse*** operation.

$-2.6 \quad -2.6$ — *Subtract* 2.6 from both sides of the equation.

$y + 0 = 1.4$ — On the left side, $2.6 - 2.6$ is 0. Then $y + 0$ is y. On the right side, $4.0 - 2.6$ is 1.4

$y = 1.4$ — This is the solution.

4. Solve $-9 = c - 4$ This time the variable is on the **right** side of the equal sign. So we will get c alone on that side.

$-9 = c - 4$	4 is *subtracted* from c. To isolate c, perform the ***inverse*** operation.
$+4 \quad +4$	*Add* 4 to both sides of the equation.
$-5 = c + 0$	On the left side, –9 + 4 is –5. On the right side, –4 + 4 is 0.
$-5 = c$	This is the solution.

Since the solution is the number that replaces the variable to make the equation true, we can easily check our answer. We show the check below.

Check:

$-9 = c - 4$	In the original equation, replace the variable with –5.
$-9 \stackrel{?}{=} (-5) - 4$	On the right side of the equation, rewrite the problem as an addition problem.
$-9 \stackrel{?}{=} -5 + -4$	Simplify the right side.
$-9 = -9$ ✓	Since the two sides of the equation are equal, $c = -5$ is the solution.

5. Solve $-18 = 2 + x$ Again the variable is on the **right** side of the equal sign. So we will get x alone on that side.

$-18 = 2 + x$	2 is *added* to x. To isolate x, perform the ***inverse*** operation.
$-2 \quad -2$	*Subtract* 2 from both sides of the equation.
$-20 = 0 + x$	On the left side, –18 – 2 is –20. On the right side, 2 – 2 is 0.
$-20 = x$	This is the solution.

Check:

$-18 = 2 + x$	In the original equation, replace x with –20.
$-18 \stackrel{?}{=} 2 + (-20)$	Perform the arithmetic on the right side of the equation.
$-18 = -18$ ✓	Since the two sides of the equation are equal, $x = -20$ is the solution.

6. Solve $y-\frac{2}{3}=\frac{5}{6}$

The variable y is on the left side of the equal sign. So we will get y alone on that side.

$y-\frac{2}{3}=\frac{5}{6}$

$\frac{2}{3}$ is *subtracted* from y. To isolate y, perform the ***inverse*** operation.

$y-\frac{2}{3}+\frac{2}{3}=\frac{5}{6}+\frac{2}{3}$

Add $\frac{2}{3}$ to both sides of the equation.

$y+0=\frac{5}{6}+\frac{2\cdot 2}{3\cdot 2}$

On the left, $-\frac{2}{3}+\frac{2}{3}$ is 0. On the right, we get a common denominator.

$y=\frac{5}{6}+\frac{4}{6}$

Add the fractions.

$y=\frac{9}{6}$

Simplify the answer.

$y=\frac{3}{2}$

This is the solution.

NOTE: After solving each equation, it is a good idea to check the solution as we showed in Examples 4 and 5. The ability to check your answers can be especially helpful on tests.

PRACTICE: Solve each equation and check the solution. Be sure to write out all the algebra steps.

1. $h+8=2$
2. $-8=w-1$
3. $5.6+c=9$
4. $x-13=-7$
5. $-21=a+8$
6. $6.5=m-3.4$
7. $-9+x=-16$
8. $-4=10+p$
9. $x-\frac{8}{9}=\frac{1}{3}$
10. $\frac{3}{4}=a+\frac{1}{12}$

Answers:

1. $h=-6$
2. $w=-7$
3. $c=3.4$
4. $x=6$
5. $a=-29$
6. $m=9.9$
7. $x=-7$
8. $p=-14$
9. $x=\frac{11}{9}$
10. $a=\frac{2}{3}$

SOLVING ONE-STEP EQUATIONS (with Multiplication and Division)

We will continue working with equations that require only one step to solve. These equations will use multiplication or division though.

- If a variable is being multiplied by a number, then we will divide by the number on both sides of the equation.
- If a variable is being divided by a number, then we will multiply by the number on both sides of the equation.

It is important to be familiar with the algebraic notation that represents multiplication and division.

Multiplication is shown by placing a number right next to a variable. For instance, the term $4x$ means that 4 and x are being multiplied. Recall that in terms like $4x$, the 4 is called the ***coefficient*** of the variable.

Division is usually shown in fractional form. For instance, the term $\frac{x}{5}$ means that x is being divided by 5. The fractional bar means division. So, $\frac{x}{5}$ means the same thing as $x \div 5$.

Like the last set of problems, the examples that follow may seem easy and you may be able to solve them in your head. However, it is important that you write out the algebra steps so that you develop the skills needed to solve more complex equations later.

EXAMPLES: Solve each equation.

1. Solve $7x = 84$

We need to get x alone on the left side of the equal sign.

$7x = 84$ The coefficient of x is 7, which means that x is being *multiplied* by 7. To isolate x, perform the ***inverse*** operation.

$\frac{7x}{7} = \frac{84}{7}$ *Divide* by 7 on both sides of the equation.

On the left of the equal sign, $\frac{7}{7}$ is 1. On the right side, $\frac{84}{7}$ is 12.

$1x = 12$ On the left side of the equation, multiplying 1 and x gives x.

$x = 12$ This is the solution.

2. Solve $\frac{x}{6} = 15$

We need to get x alone on the left side of the equal sign.

$\frac{x}{6} = 15$ The variable x is being *divided* by 6. To isolate x, perform the ***inverse*** operation.

$6\left(\frac{x}{6}\right) = 6(15)$ *Multiply* by 6 on both sides of the equation.

$\frac{6}{1} \cdot \frac{x}{6} = 6(15)$ Rewrite the left side as the multiplication of two fractions.

$\frac{{}^{1}\cancel{6}}{1} \cdot \frac{x}{\cancel{6}_{1}} = 6(15)$ On the left side, divide out a 6 in the numerator and denominator. Multiply the fractions on the left side and the integers on the right side.

$\frac{x}{1} = 90$ On the left side, x divided by 1 is x.

$x = 90$ This is the solution.

3. Solve $-x = 24$

Recall that $-x$ means $-1x$. Begin by rewriting the problem with $-1x$.

$-1x = 24$

Now we need to get x alone on the left side of the equal sign.

$-1x = 24$

The coefficient of x is -1, which means that x is being *multiplied* by -1.
To isolate x, perform the ***inverse*** operation.

$\frac{-1x}{\mathbf{-1}} = \frac{24}{\mathbf{-1}}$

Divide by -1 on both sides of the equation.

On the left, $\frac{-1}{-1}$ is 1. On the right, $\frac{24}{-1}$ is -24.

$1x = -24$

On the left side of the equation, multiplying 1 and x gives x.

$x = -24$

This is the solution.

4. Solve $0.5x = -20$

We need to get x alone on the left side of the equal sign.

$0.5x = -20$

The coefficient of x is 0.5, which means that x is being *multiplied* by 0.5.
To isolate x, perform the ***inverse*** operation.

$\frac{0.5x}{\mathbf{0.5}} = \frac{-20}{\mathbf{0.5}}$

Divide by 0.5 on both sides of the equation.

On the left, $\frac{0.5}{0.5}$ is 1. On the right, $\frac{-20}{0.5}$ is -40.

$1x = -40$

Multiplying 1 and x gives x.

$x = -40$

This is the solution.

5. Solve $-8 = \frac{x}{3.7}$

We need to get x alone on the right side of the equal sign.

$-8 = \frac{x}{3.7}$

The variable x is being *divided* by 3.7.
To isolate x, perform the ***inverse*** operation.

$\mathbf{3.7}(-8) = \mathbf{3.7}\left(\frac{x}{3.7}\right)$

Multiply by 3.7 on both sides of the equation.

Rewrite the right side as the multiplication of two fractions.

$3.7(-8) = \frac{\overset{1}{\cancel{3.7}}}{1}\left(\frac{x}{\cancel{3.7}_{\,1}}\right)$

Simplify each side of the equation.

$-29.6 = \frac{x}{1}$

$-29.6 = x$

This is the solution.

Check:

Let's check the answer to this problem.

$-8 = \frac{x}{3.7}$

In the original equation, replace x with -29.6.

$-8 \overset{?}{=} \frac{\mathbf{-29.6}}{3.7}$

Perform the arithmetic on the right side of the equation.

$-8 = -8$ ✓

Since the two sides of the equation are equal, $x = -29.6$ is the solution.

6. Solve $-10 = -6x$

We need to get x alone on the right side of the equal sign.

$-10 = -6x$

x is being *multiplied* by –6.

$\frac{-10}{\mathbf{-6}} = \frac{-6x}{\mathbf{-6}}$

Perform the ***inverse*** operation and *divide* by –6 on both sides of the equation.

$\frac{10}{6} = 1x$

Simplify each side of the equation.

$\frac{5}{3} = x$

This is the solution.

Check:

Let's check the answer to this problem.

$-10 = -6x$

In the original equation, replace x with $\frac{5}{3}$.

$-10 \stackrel{?}{=} -6\left(\mathbf{\frac{5}{3}}\right)$

Rewrite the right side as the multiplication of two fractions.

$-10 \stackrel{?}{=} \frac{\overset{-2}{\cancel{-6}}}{1} \cdot \frac{5}{\cancel{3}_{1}}$

Divide out 3. Then multiply the fractions.

$-10 \stackrel{?}{=} \frac{-10}{1}$

Simplify.

$-10 = -10$ ✓

Since the two sides of the equation are equal, $x = \frac{5}{3}$ is the solution.

7. Solve $\frac{2}{3}x = 10$

We need to get x alone on the left side of the equal sign.

$\frac{2}{3}x = 10$

x is being *multiplied* by $\frac{2}{3}$.

$\frac{\frac{2}{3}}{\frac{2}{3}}x = \frac{10}{\frac{2}{3}}$

Perform the ***inverse*** operation and *divide* by $\frac{2}{3}$ on both sides of the equation.

Rewrite the right side of the equation as the division of two fractions.

$1x = \frac{10}{1} \div \frac{2}{3}$

Change from dividing to multiplying by the reciprocal of the second fraction.

$x = \frac{10}{1} \cdot \frac{3}{2}$

Divide out common factors in the numerator and denominator. Then multiply.

$x = \frac{\overset{5}{\cancel{10}}}{1} \cdot \frac{3}{\cancel{2}_{1}}$

This is the solution.

$x = 15$

Check:

We leave the check of this problem for you.

NOTE: After solving each equation, it is a good idea to check the solution as we showed in Examples 5 and 6. The ability to check your answers can be especially helpful on tests.

REVIEW: SOLVING EQUATIONS WITH MULTIPLICATION OR DIVISION 

PRACTICE: Solve each equation and check the solution. Be sure to write out all the algebra steps.

1. $9x = 54$
2. $-y = 16$
3. $\frac{a}{5} = 7$
4. $-96 = -4a$
5. $-1.2w = 6$
6. $\frac{x}{8} = -13$
7. $-52 = 8a$
8. $5 = \frac{x}{2.7}$
9. $12x = 3$
10. $12 = \frac{3}{4}x$

Answers:

1. $x = 6$ 
2. $y = -16$ 
3. $a = 35$ 
4. $a = 24$ 
5. $w = -5$ 
6. $x = -104$
7. $a = -6.5$ OR $a = -\frac{13}{2}$
8. $x = 13.5$
9. $x = 0.25$ OR $x = \frac{1}{4}$
10. $x = 16$

SOLVING ONE-STEP EQUATIONS (with any operation)

An important skill in solving equations is identifying which operation to use. So now we will solve some equations that could involve any of the four operations (addition, subtraction, multiplication, or division). Pay special attention to how we determine which operation to use.

EXAMPLES: Solve each equation.

1. Solve $x - 4 = -28$

$$\begin{array}{rl} x - 4 &= -28 \\ +4 & \quad +4 \\ \hline x &= -24 \end{array}$$

What side of the equation contains the variable? *Left*
What operation is shown on that side? *Subtracting 4*

Do the inverse: *Add 4* to both sides of the equation.

2. Solve $126 = -9x$

$126 = -9x$ — What side of the equation contains the variable? *Right*
What operation is shown on that side? *Multiplying by* -9

$\frac{126}{-9} = \frac{-9x}{-9}$ — Do the inverse: *Divide by* -9 on both sides of the equation.

$-14 = x$

3. Solve $-5.7 = x + 0.3$

$-5.7 = x + 0.3$ — What side of the equation contains the variable? *Right*
What operation is shown on that side? *Adding 0.3*

$-0.3 \quad -0.3$ — Do the inverse: *Subtract 0.3* from both sides of the equation.

$-6 = x$

4. Solve $\frac{x}{8} = 2.5$

$\frac{x}{8} = 2.5$ — What side of the equation contains the variable? *Left*
What operation is shown on that side? *Dividing by 8*

$8\left(\frac{x}{8}\right) = 8(2.5)$ — Do the inverse: *Multiply by 8* on both sides of the equation.

$\frac{8}{1}\left(\frac{x}{8}\right) = 8(2.5)$ — Simplify.

$x = 20$

PRACTICE: Solve each equation and check the solution. Be sure to write out all the algebra steps.

1. $x + 13 = -5$
2. $7 = \frac{y}{42}$
3. $-2 = -14 + a$
4. $7a = -161$
5. $x - 6.1 = -8.5$
6. $-3.9 = -0.1x$
7. $9.8 = 3.7 + a$
8. $\frac{x}{2.5} = 0.6$

Answers:

1. $x = -18$
2. $y = 294$
3. $a = 12$
4. $a = -23$
5. $x = -2.4$
6. $x = 39$
7. $a = 6.1$
8. $x = 1.5$

SOLVING TWO-STEP EQUATIONS

In all the problems given so far, you were able to solve the equations by using just one step. That one step involved either addition, subtraction, multiplication, or division. Now you will learn to solve equations that require two steps. For example, an equation might require that you perform a subtraction followed by a division.

The goal remains the same – to get the variable alone on one side of the equal sign and a number alone on the other side. Inverse operations will still be used to achieve this. And the rule about balancing equations still applies – any operation performed on one side of an equation must also be performed on the other side.

Now let's discuss what is new in the process. First, recall that a ***variable term*** refers to a term in which a coefficient (number) and a variable are being multiplied. For example, $8x$ is a ***variable term*** since 8 and x are being multiplied. When we solve two-step equations, we first need to get the ***variable term*** alone on one side of the equal sign. Then we proceed to get the variable itself alone on that same side of the equation.

SOLVING AN EQUATION

Meaning: Determine the value of the variable that makes the equation true.

Steps:

1. Use inverse operations to get the ***variable term*** alone on one side of the equal sign.
2. Use inverse operations to get the ***variable*** alone on one side of the equal sign.
3. Check the answer in the original equation to see if it produces a true statement.

Golden Rule: Perform the same operations on both sides of the equation.

EXAMPLES: Solve each equation.

1. Solve $4x+3=11$

First, we need to get the ***variable term*** ($4x$) alone on the left side of the equal sign.

$$\begin{array}{r} 4x+3 = 11 \\ \underline{-3 \quad -3} \\ 4x \quad = \quad 8 \end{array}$$

3 is being added to the variable term.
Do the inverse: subtract 3 from both sides of the equation.

Next, we need to get the ***variable*** (x) alone on the left side of the equal sign.

$$\frac{4x}{4} = \frac{8}{4}$$

x is being multiplied by 4.
Do the inverse: divide by 4 on both sides of the equation.

$$x = 2$$

Last, we check the answer.

Check: $4x+3=11$

In the original equation, replace x with 2.

$4(2)+3\stackrel{?}{=}11$

Perform the arithmetic on the left side of the equation.

$8+3\stackrel{?}{=}11$

$11=11$ ✓

Since the two sides of the equation are equal, $x = 2$ is the solution.

2. Solve $-3x-7=5$

First, we need to get the ***variable term*** ($-3x$) alone on the left side of the equal sign.

$$\begin{array}{rcl} -3x \cancel{-7} & = & 5 \\ \cancel{+7} & & +7 \end{array}$$

7 is being subtracted from the variable term.
Do the inverse: add 7 to both sides of the equation.

$$-3x = 12$$

Next, we need to get the ***variable*** (x) alone on the left side of the equal sign.

x is being multiplied by –3.
Do the inverse: divide by –3 on both sides of the equation.

$$\frac{\cancel{-3}x}{\cancel{-3}} = \frac{12}{-3}$$

$$x = -4$$

Check:

Last, we check the answer.

$$-3x-7=5$$

In the original equation, replace x with –4.

$$-3(-4)-7\stackrel{?}{=}5$$

Perform the arithmetic on the left side of the equation.

$$12-7\stackrel{?}{=}5$$

$$5=5\checkmark$$

Since the two sides of the equation are equal, $x = -4$ is the solution.

3. Solve $-4=2x+1$

First, we need to get the ***variable term*** ($2x$) alone on the right side of the equal sign.

$$\begin{array}{rcl} -4 & = & 2x \cancel{+1} \\ -1 & & \cancel{-1} \end{array}$$

1 is being added to the variable term.
Do the inverse: subtract 1 from both sides of the equation.

$$-5 = 2x$$

Next, we need to get the ***variable*** (x) alone on the right side of the equal sign.

x is being multiplied by 2.
Do the inverse: divide by 2 on both sides of the equation.

$$\frac{-5}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$-\frac{5}{2} = x$$

Check:

Last, we check the answer.

$$-4=2x+1$$

In the original equation, replace x with $-\frac{5}{2}$.

$$-4\stackrel{?}{=}2\left(-\frac{5}{2}\right)+1$$

Perform the arithmetic on the right side of the equation.

$$-4\stackrel{?}{=}\frac{\cancel{2}}{1}\left(-\frac{5}{\cancel{2}}\right)+1$$

$$-4\stackrel{?}{=}-5+1$$

$$-4=-4\ \checkmark$$

Since the two sides of the equation are equal, $x=-\frac{5}{2}$ is the solution.

4. Solve $0.2x-1.7=2.4$

First, we need to get the ***variable term*** ($0.2x$) alone on the left side of the equal sign.

$$\begin{array}{rcl} 0.2x-1.7 & = & 2.4 \\ +1.7 & & +1.7 \end{array}$$

1.7 is being subtracted from the variable term.
Do the inverse: add 1.7 to both sides of the equation.

$$0.2x = 4.1$$

Next, we need to get the ***variable*** (x) alone on the left side of the equal sign.

x is being multiplied by 0.2.
Do the inverse: divide by 0.2 on both sides of the equation.

$$\frac{0.2x}{0.2} = \frac{4.1}{0.2}$$

$$x = 20.5$$

Check: Last, we check the answer.

$0.2x - 1.7 = 2.4$ In the original equation, replace x with 20.5.

$0.2(20.5) - 1.7 \stackrel{?}{=} 2.4$ Perform the arithmetic on the left side of the equation.

$4.1 - 1.7 \stackrel{?}{=} 2.4$

$2.4 = 2.4$ ✓ Since the two sides of the equation are equal, $x = 20.5$ is the solution.

5. Solve $2 = 1.5 - 0.5x$ First, we need to get the ***variable term*** ($-0.5x$) alone on the right side of the equal sign.

$2 = 1.5 - 0.5x$ 1.5 is being added to the variable term.

$-1.5 \quad -1.5$ Do the inverse: subtract 1.5 from both sides of the equation.

$0.5 = -0.5x$ Next, we need to get the ***variable*** (x) alone on the right side of the equal sign.

$\frac{0.5}{-0.5} = \frac{-0.5x}{-0.5}$ x is being multiplied by –0.5.

Do the inverse: divide by –0.5 on both sides of the equation.

$-1 = x$

Check: Last, we check the answer.

$2 = 1.5 - 0.5x$ In the original equation, replace x with –1.

$2 \stackrel{?}{=} 1.5 - 0.5(-1)$ Perform the arithmetic on the right side of the equation.

$2 \stackrel{?}{=} 1.5 + 0.5$

$2 = 2$ ✓ Since the two sides of the equation are equal, $x = -1$ is the solution.

6. Solve $5x + \frac{2}{9} = \frac{7}{9}$ First, we need to get the ***variable term*** ($5x$) alone on the left side of the equal sign.

$5x + \frac{2}{9} - \frac{2}{9} = \frac{7}{9} - \frac{2}{9}$ $\frac{2}{9}$ is being added to the variable term.

Do the inverse: subtract $\frac{2}{9}$ from both sides of the equation.

$5x + 0 = \frac{5}{9}$

$5x = \frac{5}{9}$ Next, we need to get the ***variable*** (x) alone on the left side of the equal sign.

$\frac{5x}{5} = \frac{\frac{5}{9}}{5}$ x is being multiplied by 5.

Do the inverse: divide by 5 on both sides of the equation.

$x = \frac{5}{9} \div \frac{5}{1}$ Rewrite the right side as the division of two fractions.

Change dividing to multiplying by the reciprocal of the second fraction.

$x = \frac{5}{9} \cdot \frac{1}{5}$ Divide out common factors, then multiply.

$x = \frac{1}{9}$ This is the answer.

Check: Last, we check the answer.

$5x+\frac{2}{9}=\frac{7}{9}$ In the original equation, replace x with $\frac{1}{9}$.

$5\left(\frac{\mathbf{1}}{\mathbf{9}}\right)+\frac{2}{9}=\frac{7}{9}$ Write 5 as $\frac{5}{1}$.

$\left(\frac{5}{1}\right)\left(\frac{1}{9}\right)+\frac{2}{9}=\frac{7}{9}$ Perform the multiplication on the left side of the equation.

$\frac{5}{9}+\frac{2}{9}=\frac{7}{9}$ Perform the addition on the left side of the equation.

$\frac{7}{9}=\frac{7}{9}$ ✓ Since the two sides of the equation are equal, $x=\frac{1}{9}$ is the solution.

PRACTICE: Solve each equation and check the solution. Be sure to write out all the algebra steps.

1. $7x+2=23$
2. $5x-9=-24$
3. $-8x-4=52$
4. $-10=6x+4$
5. $0.5x+5.2=3.6$
6. $-2.3=1.7+0.2x$
7. $1.6x-7.8=9$
8. $-12.3-0.9x=0.3$
9. $2x-\frac{5}{7}=\frac{3}{7}$
10. $\frac{3}{4}x+6=15$

Answers:

1. $x=3$
2. $x=-3$
3. $x=-7$
4. $x=-\frac{7}{3}$
5. $x=-3.2$
6. $x=-20$
7. $x=10.5$
8. $x=-14$
9. $x=\frac{4}{7}$
10. $x=12$

TRANSLATING WORDS TO MATH

In this section you will solve basic word problems. This means that you will translate English words into algebraic equations and then solve the equations using the procedures you already learned. Writing the equation is the new part. It is important to properly translate the words of the problem into numbers and math symbols. The chart below lists key words often used to represent the basic operations. You should become familiar with these key words.

KEY WORDS				
Addition +	**Subtraction** −	**Multiplication** ×	**Division** ÷	**Equals** =
Sum Add Plus Increased by More than Total	Difference Subtract Minus Decreased by Less than * Subtracted from *	Product Multiply Times Twice	Quotient Divide Divided by Per	Is Will be Gives Results in

* **Warning**: Be very careful of the order in which two numbers are subtracted or divided.

For example, "*the difference of 10 and x*" is written as $10-x$.

Now consider the phrase "*10 less than x.*" Your first instinct may be to write this as $10-x$ also.

But think about the phrase "*less than*" used in real life.

Suppose you scored 90 on a test, and a classmate says, "I scored 10 points ***less than*** you." What was your classmate's score? 80
To determine this, you would compute $90-10$ (not $10-90$).

So a word phrase like "*10 less than x"* is written in math as $x-10$. It is important to notice that the number 10 is written first in the word phrase, but last in the math problem.

This pattern applies to the word phrase "*subtracted from*" as well.

SUBTRACTION: ORDER MATTERS	
To translate the phrases "***less than***" and "***subtracted from***" into math, reverse the order of the parts in the Word Phrase to get the correct Math Phrase.	
Example: Word Phrase: 2 *less than* 8 Math Phrase: $8-2$	Example: Word Phrase: 2 *subtracted from* 8 Math Phrase: $8-2$

Solving word problems involves writing a mathematical equation to represent the problem and then solving the equation. The following approach is suggested.

SOLVING WORD PROBLEMS

1. Read the problem carefully.
2. Choose a variable to represent the unknown quantity.
3. Break the sentence down and translate one part at a time into math.
4. Use the key words to identify the math operations involved.
5. Write the equation.
6. Solve the equation.
7. Check the answer.

EXAMPLES: Translate the words into an algebraic equation, then solve the equation.

1. *The sum of 15 and a number is 7. Determine the number.*

Translate: The sum of 15 and a number is 7. Sum means to add.

$15 + \quad x \quad = \quad 7$

Solve:
$$\begin{array}{r} 15 + x = 7 \\ -15 \qquad -15 \\ \hline x = -8 \end{array}$$
To get the variable alone on the left side of the equal sign, subtract 15 from both sides of the equation.

Check:
$$\begin{array}{l} 15 + x = 7 \\ 15 + (-8) \stackrel{?}{=} 7 \\ 7 = 7 \ \checkmark \end{array}$$
In the original equation, replace x with –8.
Simplify.
Since the check works, the solution is $x = -8$.

2. *The product of 25 and a number is 170. Determine the number.*

Translate: The product of 25 and a number is 170. Product means to multiply.

$25 \cdot \quad x \quad = 170$

Solve:
$$25 \cdot x = 170$$
$$\frac{25x}{25} = \frac{170}{25}$$
$$x = 6.8$$
To get the variable alone on the left side of the equal sign, divide by 25 on both sides of the equation.

Check:
$$\begin{array}{l} 25x = 170 \\ 25(6.8) \stackrel{?}{=} 170 \\ 170 = 170 \ \checkmark \end{array}$$
In the original equation, replace x with 6.8.
Simplify.
Since the check works, the solution is $x = 6.8$

3. *Nine less than a number is 38. Determine the number.*

Translate: $\underbrace{\text{Nine less than a number}}_{x-9}$ is 38. $\quad = 38$

Less than means to subtract.

Remember that the phrase "***less than***" requires that you reverse the order of the parts in the math phrase.

Solve:

$$\begin{array}{r} x-9 = 38 \\ +9 \quad +9 \\ \hline x \quad = 47 \end{array}$$

To get the variable alone on the left side of the equal sign, add 9 to both sides of the equation.

Check:

$$x-9=38$$
$$47-9 \stackrel{?}{=} 38$$
$$38=38 \ \checkmark$$

In the original equation, replace x with 47.
Simplify.
Since the check works, the solution is $x = 47$.

4. *The quotient of a number and 2 is –14. Determine the number.*

Translate: The $\underbrace{\text{quotient of a number and 2}}_{\frac{x}{2}}$ is -14. $\quad = -14$

Quotient means to divide.

Solve:

$$\frac{x}{2}=-14$$
$$2\left(\frac{x}{2}\right)=2(-14)$$
$$\frac{\cancel{2}}{1}\left(\frac{x}{\cancel{2}}\right)=2(-14)$$
$$x=-28$$

To get the variable alone on the left side of the equal sign, multiply by 2 on both sides of the equation.

Check:

$$\frac{x}{2}=-14$$
$$\frac{-28}{2} \stackrel{?}{=} -14$$
$$-14=-14 \ \checkmark$$

In the original equation, replace x with –28.
Simplify.
Since the check works, the solution is $x = -28$.

5. *The difference of 9 and a number is 15. Determine the number.*

Translate: The $\underbrace{\text{difference of 9}}_{9-}$ and $\underbrace{\text{a number}}_{x}$ is 15. $\quad = 15$

Difference means to subtract.

Solve:

$$\begin{array}{r} 9-x = 15 \\ -9 \quad -9 \\ \hline -x = 6 \end{array}$$

To get the ***variable term*** ($-x$) alone on the left side of the equal sign, subtract 9 from both sides of the equation.

$$\frac{\cancel{-1}x}{\cancel{-1}}=\frac{6}{-1}$$
$$x=-6$$

To get the ***variable*** (x) alone on the left side of the equal sign, divide by –1 on both sides of the equation.

Check:

$$9-x=15$$
$$9-(-6) \stackrel{?}{=} 15$$
$$9+6 \stackrel{?}{=} 15$$
$$15=15 \ \checkmark$$

In the original equation, replace x with –6.
Simplify.
Since the check works, the solution is $x = -6$

6. *If the product of 6 and a number is decreased by 18, the result is –42. Determine the number.*

Translate: If the product of 6 and a number is decreased by 18, the result is -42.

$6x \quad -18 \quad = \quad -42$

Solve:

$6x - 18 = -42$

$+18 \quad +18$

$6x = -24$

$\frac{6x}{6} = \frac{-24}{6}$

$x = -4$

To get the ***variable term*** ($6x$) alone on the left side of the equal sign, add 18 to both sides of the equation.

To get the ***variable*** (x) alone on the left side of the equal sign, divide by 6 on both sides of the equation.

Check:

$6x - 18 = -42$

$6(-4) - 18 \stackrel{?}{=} -42$

$-24 - 18 \stackrel{?}{=} -42$

$-42 = -42$ ✓

In the original equation, replace x with –4.

Simplify.

Since the check works, the solution is $x = -4$

7. *If 5 is subtracted from three times a number, the result is 10. Determine the number.*

Translate: If 5 is subtracted from 3 times a number, the result is 10.

Skip this for now $3x \quad = \quad 10$

If 5 is **subtracted from** $3x$ the result is 10

$3x - 5 \quad = \quad 10$

Remember that the phrase "***subtracted from***" requires that you reverse the order of the parts in the math phrase.

Solve:

$3x - 5 = 10$

$+5 \quad +5$

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

To get the ***variable term*** ($3x$) alone on the left side of the equal sign, add 5 to both sides of the equation.

To get the ***variable*** (x) alone on the left side of the equal sign, divide by 3 on both sides of the equation.

Check:

$3x - 5 = 10$

$3(5) - 5 \stackrel{?}{=} 10$

$15 - 5 \stackrel{?}{=} 10$

$10 = 10$ ✓

In the original equation, replace x with 5.

Simplify.

Since the check works, the solution is $x = 5$.

REVIEW: SOLVING WORD PROBLEMS 

PRACTICE: Translate the words into an algebraic equation, then solve the equation.

1. Four more than a number is -18. Determine the number.
2. The difference of a number and 23 is 4. Determine the number.
3. The product of 2.8 and a number is -84. Determine the number.
4. The sum of a number and 67 is 52. Determine the number.
5. If 9 is subtracted from a number, the result is -29. Determine the number.
6. If six times a number is decreased by 14, the result is 64. Determine the number.
7. The quotient of 90 and 5 is equal to the difference of 16 and a number. Determine the number.
8. If the product of -4 and a number is increased by 12, the result is 84. Determine the number.
9. Ten less than three times a number is 11. Determine the number.
10. The sum of five times a number and 8 is -17. Determine the number.

Answers:

1. $4+x=-18$
   $x=-22$ 
2. $x-23=4$
   $x=27$ 
3. $2.8x=-84$
   $x=-30$
4. $x+67=52$
   $x=-15$
5. $x-9=-29$
   $x=-20$
6. $6x-14=64$
   $x=13$
7. $\frac{90}{5}=16-x$
   $x=-2$
8. $-4x+12=84$
   $x=-18$
9. $3x-10=11$
   $x=7$ 
10. $5x+8=-17$
    $x=-5$

SECTION 2.1 SUMMARY
One and Two Step Equations

SOLUTION OF AN EQUATION

The solution is the number that can replace the variable to make the equation true.

1. Replace the variable with the given value.
2. Simplify each side of the equation using the order of operations (PEMDAS).
3. If the two sides of the equation are equal, then the given value is a solution.

Example: Is $x = 4$ a solution of $5x - 2 = -3x + 30$?

$$5x - 2 = -3x + 30$$
$$5(4) - 2 \stackrel{?}{=} -3(4) + 30$$
$$20 - 2 \stackrel{?}{=} -12 + 30$$
$$18 = 18$$

SOLVING ONE-STEP EQUATIONS

Determine the value of the variable that makes the equation true.

1. Get the variable alone on one side of the equation by using inverse operations. IMPORTANT: Perform the same operation on both sides of the equation.
2. Check the answer by substituting it in the original equation. Simplify to see if it produces a true statement.

Example: Solve $x - 8 = 14$

$$\begin{array}{r} x - 8 = 14 \\ +8 \quad +8 \\ \hline x = 22 \end{array}$$

Check: $x - 8 = 14$
$22 - 8 \stackrel{?}{=} 14$
$14 = 14$ ✓

SOLVING TWO-STEP EQUATIONS

Determine the value of the variable that makes the equation true.

1. Get the ***variable term*** (x *term*) alone on one side of the equation by using inverse operations to get rid of the constant.
2. Get the ***variable*** (x) alone on one side of the equation by using inverse operations to get rid of the coefficient.
3. Check the answer by substituting it in the original equation. Simplify to see if it produces a true statement.

Example: Solve $-7x + 2 = -12$

$$\begin{array}{r} -7x + 2 = -12 \\ -2 \quad -2 \\ \hline -7x = -14 \end{array}$$
$$\frac{-7x}{-7} = \frac{-14}{-7}$$
$$x = 2$$

Check: $-7x + 2 = -12$
$-7(2) + 2 \stackrel{?}{=} -12$
$-14 + 2 \stackrel{?}{=} -12$
$-12 = -12$ ✓

TRANSLATING WORDS TO MATH

Use a variable for the unknown.

Use key words to identify the math operations.

- [+] sum, increased by, more than
- [−] difference, decreased by, less than
- [×] product, times
- [÷] quotient

Example: Translate the words below into an algebraic equation, then solve the equation.

If three times a number is increased by 5, the result is 47. Determine the number.

1. Translate: If $\underbrace{\text{three times a number}}_{3x}$ $\underbrace{\text{is increased by 5}}_{+5}$ $\underbrace{\text{the result is}}_{=}$ $\underbrace{47}_{47}$.

2. Solve: $3x + 5 = 47$

$$\begin{array}{r} 3x + 5 = 47 \\ -5 \quad -5 \\ \hline 3x = 42 \end{array}$$
$$\frac{3x}{3} = \frac{42}{3}$$
$$x = 14$$

3. Check: as shown in the last problem

SECTION 2.1 EXERCISES

One and Two Step Equations

Determine if the given value is a solution of the given equation.

1. Is $x=8$ a solution of $-15=-24+x$?
2. Is $x=2$ a solution of $8x-2=14$?
3. Is $x=5$ a solution of $7x-1=-3x$?
4. Is $x=-2$ a solution of $-2x-7=6x+9$?
5. Is $y=-4$ a solution of $2(y-6)=-18$?
6. Is $x=-8$ a solution of $7(x+5)-15=-36$?
7. Is $x=3.5$ a solution of $-4(x-2)=-3x+5.5$?
8. Is $x=\frac{3}{2}$ a solution of $x-\frac{1}{8}=\frac{11}{12}x$?

Solve each equation.

9. $x+7=-5$
10. $-x=10$
11. $5x=115$
12. $23+x=8$
13. $4=16+a$
14. $-12=y-3$
15. $-9+y=-4$
16. $-4.8=-0.4x$
17. $-38=2y$
18. $10=-7+x$
19. $x+5.9=13.7$
20. $8x=0$
21. $\frac{x}{8}=-26$
22. $-6y=84$
23. $\frac{5}{8}=x+\frac{2}{3}$
24. $4=16x$
25. $-12=\frac{x}{3.2}$
26. $x-\frac{2}{5}=\frac{4}{15}$
27. $-25=y+15$
28. $20.5=4.3+a$
29. $x-7=-21$
30. $\frac{4}{5}x=6$

Solve each equation.

31. $2a+3=5$
32. $-10x-12=63$
33. $3x-26=-5$
34. $-6=14-4x$
35. $9=7y+51$
36. $-25=3x-4$
37. $12=-36-4x$
38. $0.02x+7=2$
39. $-5x+7=-3$
40. $\frac{2}{3}x+1=-7$

Translate the words into an algebraic equation. Then solve the equation.

41. The sum of a number and 12 is 30. Determine the number.
42. The quotient of a number and 3 is 10. Determine the number.
43. The difference of a number and 4 is -16. Determine the number.
44. The product of a number and 2.4 is 0.48. Determine the number.
45. If three times a number is increased by 4, the result is -8 . Determine the number.
46. When 6 is subtracted from five times a number, the result is 9. Determine the number.
47. The sum of three times a number and 4 is 19. Determine the number.
48. Five less than 2 times a number is 7. Determine the number.
49. Two more than the product of 3 and a number is -10. Determine the number.
50. If five times a number is decreased by 6, the result is 29. Determine the number.

Answers to Section 2.1 Exercises

1. No
2. Yes
3. No
4. Yes
5. No
6. Yes
7. No
8. Yes
9. $x=-12$
10. $x=-10$
11. $x=23$
12. $x=-15$
13. $a=-12$
14. $y=-9$
15. $y=5$
16. $x=12$
17. $y=-19$
18. $x=17$
19. $x=7.8$
20. $x=0$
21. $x=-208$
22. $y=-14$
23. $x=-\frac{1}{24}$
24. $x=\frac{1}{4}$
25. $x=-38.4$
26. $x=\frac{2}{3}$
27. $y=-40$
28. $a=16.2$
29. $x=-14$
30. $x=\frac{15}{2}$ OR $x=7.5$
31. $a=1$
32. $x=-\frac{15}{2}$ OR $x=-7.5$
33. $x=7$
34. $x=5$
35. $y=-6$
36. $x=-7$
37. $x=-12$
38. $x=-250$
39. $x=2$
40. $x=-12$
41. $x+12=30$ $x=18$
42. $\frac{x}{3}=10$ $x=30$
43. $x-4=-16$ $x=-12$
44. $2.4x=0.48$ $x=0.2$
45. $3x+4=-8$ $x=-4$
46. $5x-6=9$ $x=3$
47. $3x+4=19$ $x=5$
48. $2x-5=7$ $x=6$
49. $3x+2=-10$ $x=-4$
50. $5x-6=29$ $x=7$

Mixed Review

Sections 1.1 – 2.1

1. Simplify $\sqrt{81} - \left|-8-(-3)\right| \div (-6+11)$.
2. Simplify $\dfrac{-2(-1+-3)-7\cdot 2}{(2)^3 \div (-2)}$.
3. Simplify $\left(\dfrac{2}{3}\right)^2 \div \dfrac{2}{15} \times \dfrac{6}{25}$.
4. Evaluate $-5x^2 - 4x$ if $x = -1$.
5. Evaluate $7x + 3y$ if $x = 4$ and $y = -2$.
6. Simplify $6x - 4y - 9x - 2y$.
7. Simplify $-\dfrac{3}{4}x - \dfrac{3}{8} + \dfrac{1}{6}x + \dfrac{3}{4}$.
8. Simplify $12 - 5(x+2)$.
9. Simplify $3(5x-8) - 6(x-1)$.
10. Simplify $\dfrac{1}{3}(12x-6) + \dfrac{4}{5}(20x+10)$.
11. Simplify $-5x - 3y + 6x + 4(1-2y)$.
12. Simplify $-3(5x-2) + 8y + 7x - 6y$.

Answers to Mixed Review

1. 8
2. $\dfrac{3}{2}$
3. $\dfrac{4}{5}$
4. -1
5. 22
6. $-3x - 6y$
7. $-\dfrac{7}{12}x + \dfrac{3}{8}$
8. $-5x + 2$
9. $9x - 18$
10. $20x + 6$
11. $x - 11y + 4$
12. $-8x + 2y + 6$