

“I’ve always believed that
if you put in the work,
the results will come.”

MICHAEL JORDAN

CHAPTER 3

Formulas, Proportions, and Percent

Section 3.1 Formulas

Section 3.2 Proportions

Section 3.3 Percent

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Section 3.1 Objectives

- Evaluate formulas.
- Solve formulas for a specified variable.
- Solve real-world formulas for a specified variable.

SECTION 3.1 Formulas

INTRODUCTION

In the last chapter, you worked with equations that contained just one variable, usually x or y . In this section, you will work with equations that contain more than one variable. These kinds of equations are called formulas. A **formula** is an equation that relates two or more variables. For example, $A = LW$ is the formula for the area of a rectangle. The formula specifies that $Area = Length \times Width$. Formulas are very useful in real life applications. In this section, you will learn two major topics dealing with formulas: evaluating a formula and rewriting a formula.

EVALUATING A FORMULA

You have already learned how to evaluate an algebraic expression. Evaluating a formula is very similar. As before, the process involves substituting given values in place of the variables in the equation. The difference is that one variable will remain in the formula after the substitutions are made. You will then use the rules of algebra to solve the equation for the variable that remains.

EVALUATING A FORMULA

1. Substitute the given values in place of the variables in the equation.
2. Use the rules of algebra to solve the equation for the remaining variable.

EXAMPLES: Evaluate each formula.

1. The formula for the volume V of a rectangular solid is $V = LWH$, where L is the length, W is the width, and H is the height. Find V if $L = 10$, $W = 6$, and $H = 12$.

$$V = L \quad W \quad H \quad \text{Substitute 10 for } L, 6 \text{ for } W, \text{ and } 12 \text{ for } H.$$

$$V = 10 \cdot 6 \cdot 12 \quad \text{The remaining variable is } V. \text{ It is alone on the left side of the equation.}$$

$$V = 60 \cdot 12 \quad \text{Simplify the right side of the equation by multiplying.}$$

$$V = 720 \quad \text{This is the answer.}$$

2. The formula for the area of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the perpendicular height. Find A if $b = 10$ and $h = 6$.

$$A = \frac{1}{2} \quad b \quad h \quad \text{Substitute 10 for } b \text{ and } 6 \text{ for } h.$$

$$A = \frac{1}{2} \cdot 10 \cdot 6 \quad \text{The remaining variable is } A. \text{ It is alone on the left side of the equation.}$$

$$A = \frac{1}{\cancel{2}_1} \cdot \frac{\cancel{10}^5}{1} \cdot 6$$

$$A = 5 \cdot 6$$

$$A = 30 \quad \text{This is the answer.}$$

3. A formula used in a constant velocity (speed) problem is $d = rt$, where d is distance, r is rate, and t is time. Find r if $d = 120$ and $t = 2$.

$$\begin{array}{l} d = r t \\ \downarrow \quad \downarrow \\ 120 = r \cdot 2 \end{array}$$

Substitute 120 for d and 2 for t .

The remaining variable is r . We need to solve for r by getting it alone on the right side of the equation.

$$\frac{120}{2} = \frac{r \cdot \cancel{2}}{\cancel{2}}$$

Since r is multiplied by 2, perform the inverse and divide by 2 on both sides of the equation.

$$60 = r$$

$$r = 60$$

This is the answer.

4. A formula from chemistry is $D = \frac{m}{V}$, where D is the density, m is the mass, and V is the volume of a substance. Find m if $D = 2$ and $V = 64$.

$$\begin{array}{l} D = \frac{m}{V} \\ \downarrow \quad \downarrow \\ 2 = \frac{m}{64} \end{array}$$

Substitute 2 for D and 64 for V .

The remaining variable is m . We need to solve for m by getting it alone on the right side of the equation.

$$64 \cdot 2 = \frac{m}{\cancel{64}} \cdot \cancel{64}$$

Since m is being divided by 64, perform the inverse and multiply by 64 on both sides of the equation.

$$128 = m$$

$$m = 128$$

This is the answer.

5. The formula for the perimeter P of a rectangle is $P = 2L + 2W$, where L and W are the length and width of the rectangle. Find W if $P = 62$ and $L = 20$.

$$P = 2L + 2W$$

Substitute 62 for P and 20 for L .

$$\begin{array}{l} \downarrow \quad \downarrow \\ 62 = 2(20) + 2W \end{array}$$

Simplify the right side by multiplying 2 and 20.

$$\begin{array}{r} 62 = \cancel{40} + 2W \\ -40 \quad -40 \\ \hline 22 = 2W \end{array}$$

To get the variable term $2W$ alone on the right side of the equation, subtract 40 from both sides.

$$22 = 2W$$

$$\frac{22}{2} = \frac{2W}{2}$$

Since W is multiplied by 2, perform the inverse and divide by 2 on both sides of the equation.

$$11 = W$$

$$W = 11$$

This is the answer.

The answers to formula evaluation problems can be checked just like you checked your answers in the last chapter. Substitute the answer in place of the variable to see if it produces a true statement.

Check:

$$62 = 2(20) + 2W$$

In the original equation, replace W with 11.

$$62 = 2(20) + 2(\mathbf{11})$$

Simplify.

$$62 = 40 + 22$$

$$62 = 62 \quad \checkmark$$

Since the two sides are equal, $W = 11$ is the solution.

Remember that the ability to check your answers can be especially helpful on tests.

6. To graph a line, you need to be able to evaluate a formula that represents the equation of the line. For the line $4x + 3y = 32$, find y if $x = 2$.

$4x + 3y = 32$	Substitute 2 for x .
\downarrow $4(2) + 3y = 32$	Simplify the left side by multiplying 4 and 2.
$\cancel{8} + 3y = 32$ <hr style="width: 100px; margin-left: 0;"/> $\cancel{-8} \qquad -8$	To get the variable term $3y$ alone on the left side of the equation, subtract 8 from both sides.
$3y = 24$	
$\frac{3y}{3} = \frac{24}{3}$	Since y is multiplied by 3, perform the inverse and divide by 3 on both sides of the equation.
$y = 8$	This is the answer.

REVIEW: EVALUATING FORMULAS

PRACTICE: Evaluate each formula.

1. The formula to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find P if $W = 5$ and $L = 8$.
2. The equation of a line passing through a point (x, y) with a y -intercept b is described by the equation $y = mx + b$. Find y if $m = 2$, $x = 4$, and $b = 5$.
3. Find P if $P = kVT$ and $k = \frac{1}{3}$, $V = 6$, and $T = 10$.
4. Use the equation $V = \frac{1}{3}bh$ to find V if $b = 18$ and $h = 4$.
5. The formula $D = RT$ relates distance, rate, and time. Find T if $R = 50$ and $D = 100$.
6. In physics, force is measured by the formula $F = ma$ where m is the mass and a is the acceleration. Find m if $F = 18$ and $a = 6$.
7. The formula to determine the volume of a rectangular prism is $V = LWH$. Find H if $V = 100$, $W = 2$, and $L = 10$.
8. The formula $C = \frac{V}{T}$ is used in chemistry to describe how gases expand when heated. Find V if $C = 2.5$ and $T = 310$.
9. Ohm's Law is used to calculate electrical resistance and is defined by the formula $R = \frac{V}{I}$. Find V if $R = 10$ and $I = 3$.
10. Use the linear equation $5x - 9y = 13$ to find x if $y = 3$.
11. Use the linear equation $2x + 3y = 30$ to find y if $x = 9$.
12. The formula used to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find L if $P = 100$ and $W = 10$.

Answers:

1. $P = 26$



7. $H = 5$

2. $y = 13$

8. $V = 775$



3. $P = 20$

9. $V = 30$

4. $V = 24$

10. $x = 8$



5. $T = 2$



11. $y = 4$

6. $m = 3$

12. $L = 40$

SOLVING A FORMULA

We will continue to work with formulas that contain many variables. But now, we will not be given values for any of the variables. Instead, we will solve the equation for a specific variable in the formula. This means we will isolate a specific variable on one side of the equation.

For example, suppose the problem is to solve the formula $A = LW$ for W . This means we need to get the W alone on one side of the equation. The process used is the same as the process you used to solve algebraic equations that contained just one variable. As before, we will use inverse operations to isolate the variable. So in the formula $A = LW$, since W is being multiplied by L , we divide by L on both sides of the equation. After performing the division, $\frac{A}{L} = \frac{LW}{L}$, we get the equation $\frac{A}{L} = W$. So, the result is $W = \frac{A}{L}$.

As you can see, the only difference in solving formulas is that the result does not give a single number as the value of the variable. Essentially, we have just rearranged or rewritten the formula in a different form.

SOLVING A FORMULA (For a Specific Variable)

1. Identify the variable you are solving for. Let's call it the Target Variable.
 - *Do this by circling the variable.*
2. Get the Target Variable alone on one side of the equation.
 - *Do this by using the rules of algebra and the concept of inverse operations.*
 - *Whatever is done to one side of the equation must also be done to the other side.*
 - *You will not get a single number as the answer.*

LEVEL ONE PROBLEMS

In the following examples, the target variable will be isolated by performing multiplication or division or both.

EXAMPLES: Solve each formula.

1. Solve the formula $U = mgh$ for m .

$$U = \boxed{m}gh$$

We are solving for m . Box (or circle) it.
We need to get m alone on the right side of the equation.

$$\frac{U}{gh} = \frac{\cancel{m} \cancel{g} \cancel{h}}{\cancel{g} \cancel{h}}$$

Since m is being multiplied by g and h , perform the inverse and divide by g and h on both sides of the equation.

$$\frac{U}{gh} = m$$

Simplify the right side of the equation.

Switch the left and right sides of the equation so that m is on the left side.

$$m = \frac{U}{gh}$$

This is the answer.

2. The formula $r = \frac{d}{t}$ relates distance, rate, and time. Solve the formula for d .

$$r = \frac{\boxed{d}}{t}$$

We are solving for d . Box (or circle) it.
We need to get d alone on the right side of the equation.

$$t \cdot r = \frac{d}{t} \cdot t$$

Since d is being divided by t , perform the inverse and multiply by t on both sides of the equation.

$$tr = \frac{d}{\cancel{t}} \cdot \frac{\cancel{t}}{1}$$

Simplify the right side of the equation.

$$tr = d$$

Switch the left and right sides of the equation so that d is on the left side.
Also, rewrite tr as rt .

$$d = rt$$

This is the answer.

3. Solve the formula $P = \frac{k}{V}$ for V .

$$P = \frac{k}{\boxed{V}}$$

We are solving for V . Box (or circle) it.

We solve equations with fractions by multiplying by the LCD.

$$V \cdot P = \frac{k}{V} \cdot V$$

The LCD is the denominator V . Multiply both sides of the equation by V .

$$V \cdot P = \frac{k}{\cancel{V}} \cdot \frac{\cancel{V}}{1}$$

Simplify the right side of the equation.

$$VP = k$$

We need to get V alone on the left side of the equation.

$$\frac{\cancel{V} \cancel{P}}{\cancel{P}} = \frac{k}{P}$$

Since V is being multiplied by P , perform the inverse and divide by P on both sides of the equation.

$$V = \frac{k}{P}$$

This is the answer.

PRACTICE: Solve each formula.

- Solve $z = 2xy$ for y .
- Solve $I = prt$ for r .
- Solve $V = LWH$ for H .
- Solve $m = \frac{F}{a}$ for F .
- Solve $D = \frac{M}{V}$ for V .
- Solve $k = \frac{V}{T}$ for V .
- Solve $W = \frac{A}{L}$ for L .

Answers:

- $y = \frac{z}{2x}$ 
- $r = \frac{I}{pt}$
- $H = \frac{V}{LW}$
- $F = ma$
- $V = \frac{M}{D}$ 
- $V = kT$
- $L = \frac{A}{W}$

LEVEL TWO PROBLEMS

Level One problems involved just multiplication or division. Level Two problems will include addition and subtraction too. To isolate the target variable in the following examples, you will use addition or subtraction as well as multiplication or division.

EXAMPLES: Solve each formula.

- Solve the formula $10x + 5y = 30$ for y .

$$10x + 5\boxed{y} = 30$$

We are solving for the variable y .

$$\begin{array}{r} \cancel{10x} + 5y = 30 \\ \underline{-\cancel{10x} \quad -10x} \end{array}$$

First we need to get the variable term $5y$ alone on the left side of the equation.

Since $10x$ is being added to that variable term, we perform the inverse and subtract $10x$ from both sides of the equation.

$$5y = 30 - 10x$$

We cannot simplify $30 - 10x$ because they are not *like* terms.

$$\frac{\cancel{5}y}{\cancel{5}} = \frac{30 - 10x}{5}$$

Now we need to get y alone on the left side of the equation.

Since y is being multiplied by 5, we perform the inverse and divide by 5 on both sides of the equation.

$$y = \frac{30}{5} - \frac{10x}{5}$$

On the right side of the equation, we must divide every term by 5.

$$y = 6 - 2x$$

Last, we rearrange the terms on the right side so the constant is last.

$$y = -2x + 6$$

This is the answer.

2. Solve the formula $M = \frac{X+Y}{N}$ for Y .

$$M = \frac{X + \boxed{Y}}{N}$$

We are solving for Y .

First eliminate the fraction by multiplying by the LCD.

$$N \cdot M = \frac{X+Y}{N} \cdot N$$

The LCD is the denominator N .
Multiply both sides of the equation by N .

$$NM = \frac{X+Y}{\cancel{N}} \cdot \frac{\cancel{N}}{1}$$

Divide out the common factor N .

We cannot simplify $X+Y$ because they are not *like* terms.

$$\begin{array}{r} NM = \cancel{X} + Y \\ -X \quad \cancel{-X} \\ \hline \end{array}$$

Now we need to get Y (the target variable) alone on the right side of the equation.

Since X is being added to the target variable, we perform the inverse and subtract X from both sides of the equation.

$$NM - X = Y$$

We cannot simplify $NM - X$ because they are not *like* terms.

Switch the left and right sides of the equation so that Y is on the left side.

$$Y = NM - X$$

Rewrite NM as MN .

$$Y = MN - X$$

This is the answer.

PRACTICE: Solve each formula.

1. Solve $6x + 3y = 12$ for y .

4. Solve $E = \frac{A+B}{N}$ for A .

2. Solve $2x + 8y = 16$ for x .

5. Solve $A = \frac{X-Y}{2}$ for X .

3. Solve $4x - 9y = 36$ for x .

6. Solve $A = \frac{D+E}{4}$ for E .

Answers:


1. $y = -2x + 4$


4. $A = EN - B$

2. $x = -4y + 8$

5. $X = 2A + Y$

3. $x = \frac{36+9y}{4}$ or $x = \frac{9y+36}{4}$

6. $E = 4A - D$ 

or $x = 9 + \frac{9}{4}y$ or $x = \frac{9}{4}y + 9$ 

LEVEL THREE PROBLEMS (WITH PARENTHESES)

Now that you can isolate a target variable using all four operations, let's make it more interesting by solving formulas that contain parentheses. The first step is to clear the parentheses from the equation by using the Distributive Property.

EXAMPLES: Solve each formula.

1. Solve the formula $A = 2(S + T)$ for T .

$$\begin{array}{l}
 A = 2(S + \boxed{T}) \quad \text{Clear the parentheses by distributing the 2.} \\
 A = \cancel{2S} + 2T \quad \text{We are solving for the variable } T. \\
 \begin{array}{r}
 \cancel{-2S} \quad \cancel{-2S} \\
 \hline
 A - 2S = 2T \quad \text{First we need to get the variable term } 2T \text{ alone on the right side of the equation.} \\
 \text{Since } 2S \text{ is being added to that term, do the inverse and subtract } 2S \text{ from both sides.} \\
 \hline
 \frac{A - 2S}{2} = \frac{\cancel{2}T}{\cancel{2}} \quad \text{We cannot simplify } A - 2S \text{ because they are not } \textit{like} \text{ terms.} \\
 \text{Now we need to get } T \text{ alone on the right side of the equation.} \\
 \text{Since } T \text{ is multiplied by 2, do the inverse and divide by 2 on both sides.} \\
 \hline
 \frac{A}{2} - \frac{\cancel{2}S}{\cancel{2}} = T \quad \text{On the left side, divide every term in the numerator by 2.} \\
 \frac{A}{2} - S = T \quad \text{Switch the left and right sides of the equation so that } T \text{ is on the left side.} \\
 T = \frac{A}{2} - S \quad \text{This is the answer.}
 \end{array}
 \end{array}$$

2. Solve the formula $C = 2(A - 20)$ for A .

$$\begin{array}{l}
 C = 2(\boxed{A} - 20) \quad \text{Clear the parentheses by distributing the 2.} \\
 C = 2A - 40 \quad \text{We are solving for the variable } A. \\
 \begin{array}{r}
 +40 \quad +40 \\
 \hline
 C + 40 = 2A \quad \text{First we need to get the variable term } 2A \text{ alone on the right side of the equation.} \\
 \text{Since } 40 \text{ is being subtracted from that term, do the inverse and add } 40 \text{ to both sides.} \\
 \hline
 \frac{C + 40}{2} = \frac{\cancel{2}A}{\cancel{2}} \quad \text{We cannot simplify } C + 40 \text{ because they are not } \textit{like} \text{ terms.} \\
 \text{Now we need to get } A \text{ alone on the right side of the equation.} \\
 \text{Since } A \text{ is multiplied by 2, do the inverse and divide by 2 on both sides.} \\
 \hline
 \frac{C}{2} + \frac{40}{2} = A \quad \text{On the left side, divide every term in the numerator by 2.} \\
 \frac{C}{2} + 20 = A \quad \text{Switch the left and right sides of the equation so that } A \text{ is on the left side.} \\
 A = \frac{C}{2} + 20 \quad \text{This is the answer.}
 \end{array}
 \end{array}$$

PRACTICE: Solve each formula.

- Solve $x = 7(y + z)$ for z .
- Solve $V = 6(X + 5)$ for X .
- Solve $Z = 3(X - M)$ for X .

Answers:

- $z = \frac{x-7y}{7}$ or $z = \frac{x}{7} - y$
- $X = \frac{V-30}{6}$ or $X = \frac{V}{6} - 5$
- $X = \frac{Z+3M}{3}$ or $X = \frac{Z}{3} + M$



SECTION 3.1 EXERCISES**Formulas**

Evaluate each formula.

1. The formula for the volume V of a rectangular solid is $V = LWH$, where L is the length, W is the width, and H is the height. Find V if $L = 2$, $W = 8.5$, and $H = 7$.
2. Using the equation $T = \frac{1}{4}xy$, find T if $x = 2$ and $y = 6$.
3. The equation of a line passing through a point (x, y) with a y -intercept b is described by the equation $y = mx + b$. Find y when $m = 8$, $x = 3$, and $b = 1$.
4. The formula used to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find P if $L = 9$ and $W = 5$.
5. The formula used to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find P if $L = \frac{5}{2}$ and $W = \frac{1}{2}$.
6. In physics, force is measured by the formula $F = ma$ where m is the mass and a is the acceleration. Find a when $F = 135$ and $m = 9$.
7. The volume of a rectangular prism is $V = LWH$. Find W if $V = 200$, $L = 10$, and $H = 5$.
8. Ohm's Law is used to calculate electrical resistance and is defined by the formula $R = \frac{V}{I}$. Find V if $R = 20$ and $I = 4$.
9. In the equation $Z = \frac{x}{y}$, find x if $Z = 25$ and $y = 3$.
10. Use the linear equation $6x + 2y = 48$ to find y when $x = 7$.
11. Use the linear equation $8x - 3y = 16$ to find x when $y = 8$.
12. The formula used to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find W if $P = 94$ and $L = 6$.

Solve each formula.

13. In the equation $A = 2LW$, solve for W .

14. In the equation $B = 3CD$, solve for C .

15. In the equation $I = prt$, solve for p .

16. In the equation $k = \frac{y}{x}$, solve for y .

17. In the equation $v = \frac{p}{m}$, solve for m .

18. In the equation $12x + 6y = 24$, solve for y .

19. In the equation $3x + 24y = 6$, solve for x .

20. In the equation $8x - 4y = 20$, solve for y .

21. In the equation $T = \frac{a+b}{c}$, solve for a .

22. In the equation $R = \frac{M+N}{3}$, solve for N .

23. In the equation $M = \frac{x-y}{A}$, solve for x .

24. In the equation $P = 2(L+5)$, solve for L .

25. In the equation $Z = 4(x+y)$, solve for y .

26. In the equation $x = 5(A-B)$, solve for A .

Answers to Section 3.1 Exercises


1. $V = 119$
2. $T = 3$
3. $y = 25$
4. $P = 28$
5. $P = 6$
6. $a = 15$
7. $W = 4$
8. $V = 80$
9. $x = 75$
10. $y = 3$
11. $x = 5$
12. $W = 41$
13. $W = \frac{A}{2L}$
14. $C = \frac{B}{3D}$
15. $p = \frac{I}{rt}$
16. $y = kx$
17. $m = \frac{P}{v}$
18. $y = 4 - 2x$ **or** $y = -2x + 4$
19. $x = 2 - 8y$ **or** $x = -8y + 2$
20. $y = 2x - 5$
21. $a = Tc - b$
22. $N = 3R - M$
23. $x = AM + y$
24. $L = \frac{P-10}{2}$ **or** $L = \frac{P}{2} - 5$
25. $y = \frac{Z-4x}{4}$ **or** $y = \frac{Z}{4} - x$
26. $A = \frac{x+5B}{5}$ **or** $A = \frac{x}{5} + B$

Mixed Review

Sections 1.1 – 3.1

- Simplify $(-19+4^2)-\sqrt{64}+|5-17|$.
- Evaluate $-\frac{a}{b} \div \frac{b}{a} - \left(\frac{1}{b}\right)^2$ if $a=2$ and $b=3$.
- Simplify $7a+5-6b+4a-3-2b$.
- Simplify $-\frac{1}{2}\left(-\frac{6}{5}x+40\right)$.
- Solve $-(4x+9)=8+19$.
- Solve $-3(2x-5)+1=5x-6$.
- Solve $\frac{3}{2}x-\frac{5}{4}=\frac{2}{3}x+1$.
- Translate the word problem into an algebraic equation. Then solve the equation.
If 4 times a number is decreased by 5, the result is 31. Determine the number.
- Write an algebraic equation for the word problem. Then solve the equation to answer the question.
Ian wants to run the same distance on Monday, Wednesday, and Friday. But on Tuesday and on Thursday, he wants to run 5 miles more. If Ian wants to run a total of 50 miles all five days, how many miles should he run on Tuesday and on Thursday and how many miles should he run on Monday, Wednesday, and Friday?
- Solve $-7x+6 > -22$, graph the solution, and write the solution in interval notation.

Answers to Mixed Review

- | | |
|----------------------|--|
| 1. 1 | 8. $4n-5=31$ |
| 2. $-\frac{5}{9}$ | $n=9$ |
| 3. $11a-8b+2$ | 9. $d+d+d+(d+5)+(d+5)=50$ |
| 4. $\frac{3}{5}x-20$ | $5d+10=50$ |
| 5. $x=-9$ | Mon, Wed, Fri: 8 miles |
| 6. $x=2$ | Tues, Thurs: 13 miles |
| 7. $x=\frac{27}{10}$ | 10. $x < 4$ $(-\infty, 4)$ |
| |  |