Section 3.2 Objectives

- Determine if a proportion is true or false.
- Solve proportions for an unknown.
- Solve unit conversion problems using proportions.
- Solve application word problems involving proportions.

SECTION 3.2 Proportions

INTRODUCTION

In this section you will study a special type of equation called a ***proportion***. Learning about proportions is important because they have numerous real-life applications. You will examine some applications yourself as you learn to set up proportions that model real-world scenarios. You will also learn a special technique for solving proportions. But first, let's begin with the basics – what is a ***proportion***? Think of taking two fractions and inserting an equal sign between them. That's a proportion!

PROPORTION		
Definition	**Math Statement**	**Example**
A proportion is a statement that two ratios (fractions) are equal.	$\frac{a}{b}=\frac{c}{d}$ Read "***a*** is to ***b*** as ***c*** is to ***d***" Means (*Middle Terms*) Extremes (*Outer Terms*)	$\frac{2}{6}=\frac{1}{3}$ Read "2 is to 6 as 1 is to 3" Means: 6 and 1 Extremes: 2 and 3

PROPORTIONS – TRUE OR FALSE

Proportions are either true or false. We show an example of each below.

$\frac{4}{8}=\frac{1}{2}$ True. If we reduce $\frac{4}{8}$ we get $\frac{1}{2}$. The two fractions are equal.

$\frac{5}{10}=\frac{4}{9}$ False. If we reduce $\frac{5}{10}$ we do not get $\frac{4}{9}$. The fractions are not equal.

Sometimes it is difficult to determine whether a proportion is true or false by reducing and comparing the fractions. For instance, it would be difficult with the proportion $\frac{7.5}{17.5}=\frac{3}{7}$.

For that reason, there is another method that can be used to determine if a proportion is true or false. The method is shown in the following box.

PROPERTY OF PROPORTIONS

A proportion is true if and only if the product of the extremes equals the product of the means.

$\frac{a}{b}=\frac{c}{d}$ is true only if $a \cdot d = b \cdot c$

This can be stated more simply in the following way:

A proportion is true only if its cross products are equal.

$\frac{a}{b} \overset{\times}{=} \frac{c}{d}$ is true only if $ad = bc$

EXAMPLES: Determine if the proportion is true.

1. Is $\frac{5}{6}=\frac{15}{18}$ a true statement?

$\frac{5}{6} \overset{\times}{=} \frac{15}{18}$ Multiply diagonally to get the cross products.

$5 \cdot 18 \overset{?}{=} 6 \cdot 15$ Perform the multiplications on each side.

$90 = 90$ Yes, since the cross products are equal, the proportion is true.

2. Is $\frac{1.2}{7}=\frac{0.9}{5}$ a true statement?

$\frac{1.2}{7} \overset{\times}{=} \frac{0.9}{5}$ Multiply diagonally to get the cross products.

$(1.2)(5) \overset{?}{=} (7)(0.9)$ Perform the multiplications on each side.

$6 \neq 6.3$ No, since the cross products are not equal, the proportion is not true.

PRACTICE: Determine if the proportion is true.

1. Is $\frac{7}{12}=\frac{28}{36}$ a true statement?

2. Is $\frac{1.5}{3}=\frac{8.5}{17}$ a true statement?

Answers:

1. No

2. Yes

SOLVING A PROPORTION

A proportion contains four numbers. But in some proportions, only three of the numbers are given and a variable represents the fourth. In these types of problems, our goal will be to determine the value of the variable that makes the proportion true. We will solve for the variable using a technique involving the cross products. The complete procedure is explained below.

SOLVING A PROPORTION

To solve a proportion for a variable:

1. Set the cross products equal to each other by cross multiplying.
2. Perform the multiplication on both sides of the equation.
3. To isolate the variable, divide both sides of the equation by the coefficient.

***EXAMPLES*:** Solve each proportion.

1. Solve for n: $\frac{17}{51}=\frac{100}{n}$ This equation is a proportion.

$\frac{17}{51}=\frac{100}{n}$ Set the cross products equal to each other by cross multiplying.

$17\cdot n=51\cdot 100$ Perform the multiplication on both sides of the equation.

$17n=5100$

$\frac{17n}{17}=\frac{5100}{17}$ To isolate the variable, divide by 17 on both sides of the equation.

$n=300$ This is the answer.

2. Solve for x: $\frac{x}{7.5}=\frac{1.5}{2.5}$ This equation is a proportion.

$\frac{x}{7.5}=\frac{1.5}{2.5}$ Set the cross products equal to each other by cross multiplying.

$(x)(2.5)=(7.5)(1.5)$ Perform the multiplication on both sides of the equation.

$2.5x=11.25$

$\frac{2.5x}{2.5}=\frac{11.25}{2.5}$ To isolate the variable, divide by 2.5 on both sides of the equation.

$x=4.5$ This is the answer.

3. Solve for n: $\frac{-3}{n}=\frac{5}{4}$

This equation is a proportion.

$\frac{-3}{n}=\frac{5}{4}$

Set the cross products equal to each other by cross multiplying.

$(-3)(4)=(n)(5)$

Perform the multiplication on both sides of the equation.

$-12=5n$

$\frac{-12}{5}=\frac{5n}{5}$

To isolate the variable, divide by 5 on both sides of the equation.

$-\frac{12}{5}=n$

$n=-\frac{12}{5}$ OR $n=-2.4$

This is the answer.

4. Solve for n: $\frac{\frac{1}{2}}{\frac{3}{5}}=\frac{n}{1\frac{4}{5}}$

This equation is a proportion.

Begin by changing the mixed number $1\frac{4}{5}$ to an improper fraction.

$\frac{\frac{1}{2}}{\frac{3}{5}}=\frac{n}{\frac{9}{5}}$

Set the cross products equal to each other by cross multiplying.

$\frac{1}{2}\cdot\frac{9}{5}=\frac{3}{5}\cdot n$

Perform the multiplication on both sides of the equation.

$\frac{9}{10}=\frac{3}{5}n$

To isolate the variable, divide by $\frac{3}{5}$ on both sides of the equation.

$\frac{\frac{9}{10}}{\frac{3}{5}}=\frac{\frac{3}{5}n}{\frac{3}{5}}$

Rewrite the left side of the equation as a division of two fractions.

$\frac{9}{10}\div\frac{3}{5}=n$

Multiply the first fraction by the reciprocal of the second fraction.

$\frac{9}{10}\cdot\frac{5}{3}=n$

$\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}}\cdot\frac{\cancel{5}^{1}}{\cancel{3}_{1}}=n$

Divide out common factors.

$\frac{3}{2}=n$

$n=\frac{3}{2}$ OR $n=1.5$

This is the answer.

PRACTICE: Solve each proportion.

1. Solve for *n*: $\frac{3}{5}=\frac{9}{n}$

2. Solve for *x:* $\frac{x}{6.5}=\frac{3.5}{0.25}$

3. Solve for *x:* $\frac{7}{x}=\frac{-2}{9}$

4. Solve for *n*: $\frac{\frac{1}{3}}{\frac{3}{4}}=\frac{n}{\frac{15}{4}}$

5. Solve for *n*: $\frac{1\frac{1}{2}}{\frac{1}{2}}=\frac{6}{n}$

6. Solve for *n*: $\frac{n}{\frac{5}{8}}=\frac{2\frac{4}{5}}{4}$

Answers:

1. $n=15$ 

2. $x=91$ 

3. $x=-31.5$ 

4. $n=\frac{5}{3}$ 

5. $n=2$

6. $n=\frac{7}{16}$

APPLICATIONS OF PROPORTIONS

You are in the grocery store because you need to purchase eggs. You are going to make cupcakes for a family reunion. Your recipe makes 18 cupcakes and requires 3 eggs. But you want to make 4 dozen cupcakes – this is going to be a large reunion. How many eggs will you need to buy? Looks like you have a proportion problem that needs to be solved!

In this type of application, we are dealing with quantities that are proportional. The number of cupcakes you bake is proportional to the number of eggs you use. To make 18 cupcakes you need 3 eggs. If you wanted to double the number of cupcakes and make 36, then you would have to double the number of eggs and use 6. If you wanted to triple the number of cupcakes and make 54, then you would have to triple the number of eggs and use 9. But you have decided to make 4 dozen cupcakes which is 48 cupcakes. The number of eggs is not so easy to calculate for 48 cupcakes. So, we will use algebra. We will set up a proportion to solve the problem. Study the steps of the procedure and the solution that follows.

USING PROPORTIONS TO SOLVE APPLICATION PROBLEMS

1. Variable: Assign a variable to the unknown quantity.
2. Ratio in Words: Set up a fraction using words to identify the two quantities being compared.
3. Known Ratio: Write a fraction using the two given values that relate the two quantities. Make sure the fraction is set up like Step 2.
4. Unknown Ratio: Write another fraction involving the variable. Make sure the fraction is set up like Step 2.
5. Proportion: Write a proportion by setting the ratios (fractions) equal to each other.
6. Solve: Solve the proportion using cross products and inverse operations.

***EXAMPLES*:** Solve each problem by using a proportion.

1. A recipe that makes 18 cupcakes requires 3 eggs. How many eggs are needed to make 4 dozen, or 48, cupcakes?

STEP 1: Variable – assign variable to unknown quantity

n = number of eggs to make 4 dozen cupcakes

STEP 2: Ratio in Words – fraction with quantities being compared

$$\frac{\text{\# of Cupcakes}}{\text{\# of Eggs}}$$

STEP 3: Known Ratio – fraction with given values, set up like step 2

$$\frac{18 \text{ Cupcakes}}{3 \text{ Eggs}}$$

STEP 4: Unknown Ratio – fraction involving the variable, set up like step 2

$$\frac{48 \text{ Cupcakes}}{n \text{ Eggs}}$$

NOTE: *In all three ratios above, cupcakes were in the numerator of the fraction and eggs in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal

$$\frac{18}{3} = \frac{48}{n}$$

STEP 6: Solve – set cross products equal

$$18n = 144$$

– divide to isolate the variable

$$\frac{18n}{18} = \frac{144}{18}$$

Answer: You will need **8 eggs** to bake 48 cupcakes.

$$n = 8$$

2. Water is being pumped out of a basement at a rate of 140 gallons per hour. How many hours will it take to pump 2030 gallons of water out of the basement?

STEP 1: Variable – assign variable to unknown quantity

n = number of hours to pump 2030 gal of water out

STEP 2: Ratio in Words – fraction with quantities being compared

$$\frac{\text{gallons of Water}}{\text{Number of hours}}$$

STEP 3: Known Ratio – fraction with given values, set up like step 2

$$\frac{140 \text{ gallons}}{1 \text{ hour}}$$

STEP 4: Unknown Ratio – fraction involving the variable, set up like step 2

$$\frac{2030 \text{ gallons}}{n \text{ hours}}$$

NOTE: *In all three ratios above, gallons were in the numerator of the fraction and hours in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal

$$\frac{140}{1} = \frac{2030}{n}$$

STEP 6: Solve – set cross products equal and isolate the variable

$$140n = 2030$$

$$\frac{140n}{140} = \frac{2030}{140}$$

$$n = 14.5$$

Answer: It will take **14.5 hours** to pump 2030 gallons of water out.

3. You need to combine 98 grams of sulfuric acid and 70 grams of sodium hydroxide to produce sodium sulfate (a kind of chemical salt). How many grams of sulfuric acid would need to combine with 20 grams of sodium hydroxide to produce sodium sulfate?

STEP 1: Variable – assign variable to unknown quantity

n = grams of sulfuric acid to combine with 20 grams of sodium hydroxide

STEP 2: Ratio in Words – fraction with quantities being compared

$$\frac{\text{grams of Sulfuric Acid}}{\text{grams of Sodium Hydroxide}}$$

STEP 3: Known Ratio – fraction with given values

$$\frac{98 \text{ grams of SA}}{70 \text{ grams of SH}}$$

STEP 4: Unknown Ratio – fraction involving the variable

$$\frac{n \text{ grams of SA}}{20 \text{ grams of SH}}$$

NOTE: *In all three ratios above, Sulfuric Acid was in the numerator and Sodium Hydroxide was in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal

$$\frac{98}{70} = \frac{n}{20}$$

STEP 6: Solve – set cross products equal and isolate the variable

$$1960 = 70n$$

$$\frac{1960}{70} = \frac{70n}{70}$$

$$28 = n$$

Answer: You need **28 grams** of Sulfuric Acid to combine with 20 grams of Sodium Hydroxide.

4. You know that there are 56 milligrams of cholesterol in $3\frac{1}{2}$ ounces of trout. How much cholesterol is there in 8 ounces of trout?

STEP 1: Variable – assign variable to unknown — n = mg of cholesterol in 8 oz. of trout

STEP 2: Ratio in Words – quantities being compared — $\frac{\text{mg of Cholesterol}}{\text{ounces of Trout}}$

STEP 3: Known Ratio – fraction with given values — $\frac{56 \text{ mg of Cholesterol}}{3\frac{1}{2} \text{ ounces of Trout}}$

STEP 4: Unknown Ratio – fraction involving variable — $\frac{n \text{ mg of Cholesterol}}{8 \text{ ounces of Trout}}$

NOTE: *In all three ratios, Cholesterol was in the numerator and Trout in the denominator.*

STEP 5: Proportion – set ratios equal

$$\frac{56}{3\frac{1}{2}} = \frac{n}{8}$$

– rewrite the mixed number as an improper fraction

$$\frac{56}{\frac{7}{2}} = \frac{n}{8}$$

STEP 6: Solve – set cross products equal

$$448 = \frac{7}{2}n$$

– multiply both sides of the equation by the LCD 2

$$2 \cdot 448 = 2 \cdot \frac{7}{2}n$$

$$2 \cdot 448 = \frac{2}{1} \cdot \frac{7}{2}n$$

$$896 = 7n$$

$$\frac{896}{7} = \frac{7n}{7}$$

$$128 = n$$

Answer: There are **128 mg** of cholesterol in 8 ounces of trout.

PRACTICE: Solve each problem by using a proportion.

1. To make a cup of hot cocoa, Bob mixes 3 teaspoons of cocoa powder with 2 cups of milk. How much cocoa powder would be needed to mix with 12 cups of milk?
2. Water is pumped into a pool at a rate of 120 gallons per hour. How many hours will it take to pump 3000 gallons of water into the pool?
3. You need to combine 98 grams of sulfuric acid (H2SO4) and 80 grams of sodium hydroxide (NaOH) to produce sodium sulfate (a kind of chemical salt). How many grams of sulfuric acid (H2SO4) would you need to combine with 40 grams of sodium hydroxide (NaOH) to produce sodium sulfate?
4. A single tablet of One-A-Day Vitamin for men contains 75 milligrams of Vitamin C. How many milligrams of Vitamin C are in $2\frac{1}{3}$ tablets of One-A-Day Vitamins?

Answers:

1. 18 teaspoons 
2. 25 hours 
3. 49 grams 
4. 175 milligrams 

CONVERSIONS USING PROPORTIONS

Measurement is very important to everyday life. We measure distance, time, weight, space, and many other aspects of our physical world. From one culture to another culture, or even within the same community, a particular aspect may be measured in different ways using different units of measurement. For example, the length of a full semester 3 credit course at CCBC could be given as 37.5 hours or as 2250 minutes. An important skill is learning to convert from one type of measuring unit to another. Unit conversion problems can be solved with proportions using the same procedure that we used to solve application problems. The steps are reviewed below.

USING PROPORTIONS TO PERFORM UNIT CONVERSIONS
1. Variable: Assign a variable to the unknown quantity. 2. Ratio in Words: Set up a fraction using words to identify the two units being compared. 3. Known Ratio: Write a fraction using the given conversion fact that relates the two units. 4. Unknown Ratio: Write another fraction involving the variable. 5. Proportion: Write a proportion by setting the ratios (fractions) equal to each other. 6. Solve: Solve the proportion using cross products and inverse operations.

There are two primary systems of measurement that we will study, the U.S. System of Measurement and the Metric System of Measurement. We begin with the U.S. System.

CONVERSIONS WITHIN THE U.S. SYSTEM

You are probably most familiar with the system of measurement that we use here in the United States. The units of measurement are typically categorized based on what they measure: length, time, volume, and weight. The chart below lists the basic unit conversion facts in each category.

BASIC CONVERSION FACTS – U.S. SYSTEM OF MEASUREMENT			
LENGTH	12 inches (in) = 1 foot (ft) 3 feet (ft) = 1 yard (y) 5280 feet (ft) = 1 mile (mi)	WEIGHT	16 ounces (oz) = 1 pound (lb) 2000 pounds (lbs) = 1 ton (T)
VOLUME	8 fluid ounces (fl oz) = 1 cup (c) 2 cups (c) = 1 pint (pt) 2 pints (pt) = 1 quart (qt) 4 quarts (qt) = 1 gallon (gal)	TIME	60 seconds (s) = 1 minute (min) 60 minutes (min) = 1 hour (hr) 24 hours (hrs) = 1 day 7 days = 1 week 365 days = 1 year

Now we will convert a measurement from one unit to another by setting up and solving a proportion. The given conversion fact will be used as the *known* ratio in the proportion.

***EXAMPLES*:** Perform each unit conversion by using a proportion.

1. Convert 5 feet to inches. Use the conversion fact: **12 inches (in) = 1 foot (ft)**

STEP 1: Variable – assign variable to unknown — n = inches equal to 5 feet

STEP 2: Ratio in Words – fraction using units being compared — $\frac{\text{inches}}{\text{feet}}$

STEP 3: Known Ratio – fraction using given conversion fact — $\frac{12 \text{ in}}{1 \text{ foot}}$

STEP 4: Unknown Ratio – fraction using variable — $\frac{n \text{ in}}{5 \text{ ft}}$

NOTE: *In all three ratios above, inches were in the numerator and feet in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal — $\frac{12}{1} = \frac{n}{5}$

STEP 6: Solve – set cross products equal — $60 = n$

Answer: 5 feet is equal to **60** inches. — $n = 60$

2. Convert 20 inches to feet. Use the conversion fact: **12 inches (in) = 1 foot (ft)**

STEP 1: Variable – assign variable to unknown — n = feet equal to 20 inches

STEP 2: Ratio in Words – fraction using units being compared — $\frac{\text{inches}}{\text{feet}}$

STEP 3: Known Ratio – fraction using given conversion fact — $\frac{12 \text{ in}}{1 \text{ foot}}$

STEP 4: Unknown Ratio – fraction using variable — $\frac{20 \text{ in}}{n \text{ ft}}$

NOTE: *In all three ratios above, inches were in the numerator and feet in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal — $\frac{12}{1} = \frac{20}{n}$

STEP 6: Solve – set cross products equal — $12n = 20$

– isolate the variable — $\frac{12n}{12} = \frac{20}{12}$

– simplify — $n = \frac{20}{12}$

Answer: 20 inches is equal to $\mathbf{\frac{5}{3}}$ feet. — $n = \frac{5}{3}$

3. Convert $3\frac{3}{4}$ pounds to ounces. Use the conversion fact: **16 ounces (oz) = 1 pound (lb)**

STEP 1: Variable – assign variable to unknown — n = ounces equal to $3\frac{3}{4}$ pounds

STEP 2: Ratio in Words – fraction using units being compared — $\frac{\text{ounces}}{\text{pounds}}$

STEP 3: Known Ratio – fraction using given conversion fact — $\frac{16 \text{ ounces}}{1 \text{ pound}}$

STEP 4: Unknown Ratio – fraction using variable — $\frac{n \text{ ounces}}{3\frac{3}{4} \text{ pounds}}$

NOTE: *In all ratios, ounces are in the numerator, pounds in the denominator.*

STEP 5: Proportion – set ratios equal — $\frac{16}{1} = \frac{n}{3\frac{3}{4}}$

STEP 6: Solve – set cross products equal — $16 \cdot 3\frac{3}{4} = n$

– write the mixed number as an improper fraction — $\frac{16}{1} \cdot \frac{15}{4} = n$

– divide out common factors — $\frac{\overset{4}{\cancel{16}}}{1} \cdot \frac{15}{\cancel{4}_{1}} = n$

Answer: $3\frac{3}{4}$ pounds is equal to **60** ounces. — $60 = n$

The solutions for the next three examples are presented using a shortened format.

4. Convert 2.5 hours to minutes. Use the conversion fact: **60 minutes (min) = 1 hour (hr)**

VARIABLE	UNITS	PROPORTION	ANSWER
n = minutes equal to 2.5 hours	$\frac{\text{min}}{\text{hr}} \rightarrow$	$\frac{60}{1} = \frac{n}{2.5}$	2.5 hours is equal to **150** minutes.
		$150 = n$	

5. Convert 120 fluid ounces to cups. Use the conversion fact: **8 fluid ounces (fl oz) = 1 cup (c)**

VARIABLE	UNITS	PROPORTION	ANSWER
n = cups equal to 120 fl oz	$\frac{\text{fl oz}}{\text{cups}} \rightarrow$	$\frac{8}{1} = \frac{120}{n}$	120 fl oz is equal to **15** cups.
		$8n = 120$	
		$\frac{8n}{8} = \frac{120}{8}$	
		$n = 15$	

6. Convert 30 quarts to gallons. Use the conversion fact: **4 quarts (qt) = 1 gallon (gal)**

VARIABLE	UNITS	PROPORTION	ANSWER
n = gallons equal to 30 quarts	$\frac{\text{qt}}{\text{gal}} \rightarrow$	$\frac{4}{1} = \frac{30}{n}$	30 quarts is equal to **7.5** gallons
		$4n = 30$	
		$\frac{4n}{4} = \frac{30}{4}$	
		$n = \frac{15}{2} = 7.5$	

PRACTICE: Perform each unit conversion by using a proportion.

1. Convert 4 miles to feet. Use the conversion fact: **5280 feet (ft) = 1 mile (mi)**
2. Convert 81 feet to yards. Use the conversion fact: **3 feet (ft) = 1 yard (y)**
3. Convert 5 minutes to seconds. Use the conversion fact: **60 seconds (s) = 1 minute (min)**
4. Convert 96 hours to days. Use the conversion fact: **24 hours (hrs) = 1 day**
5. Convert 8.5 pints to cups. Use the conversion fact: **2 cups (c) = 1 pint (pt)**
6. Convert 11 pints to quarts. Use the conversion fact: **2 pints (pt) = 1 quart (qt)**
7. Convert 6400 pounds to tons. Use the conversion fact: **2000 pounds (lbs) = 1 ton (T)**
8. Convert $2\frac{1}{2}$ pounds to ounces. Use the conversion fact: **16 ounces (oz) = 1 pound (lb)**

Answers:

1. 21,120 ft
2. 27 yds
3. 300 sec
4. 4 days
5. 17 c
6. 5.5 qt
7. 3.2 T
8. 40 oz

CONVERSIONS WITHIN THE METRIC SYSTEM

The metric system is an international system of measurement based on powers of ten. The system has three ***basic units*** that correspond to what is being measured: length, volume, or weight. ***Prefixes*** can be attached to the basic units to form smaller or larger units of measure.

BASIC UNITS

The metric system of measurement uses the following ***basic units*** for length, volume, and weight.

Meter: The basic unit of length is the ***meter.*** The abbreviation for meter is **m**. A meter is about 3 inches longer than a yard. Michael Jordan is about 2 meters tall.

Liter: The basic unit of volume is the ***liter***. The abbreviation for liter is **L**. A liter is slightly less than a quart. It takes about 50 liters of gasoline to fill a Mini Cooper's gas tank.

Gram: The basic unit of weight is the ***gram***. The abbreviation for gram is **g**. About 30 grams equal an ounce. The average weight of a healthy newborn baby is about 3,000 grams.

PREFIXES

In the metric system, a ***prefix*** can be attached to any of the basic units to produce a new unit. The new unit is smaller or larger than the basic unit by a power of 10. The table below shows some of the common prefixes and their meanings. The table shows, for example, that a kilogram (kg) is larger than a gram because one kilogram is equal to one thousand grams. The table also shows, for example, that a centimeter (cm) is smaller than a meter because one centimeter is equal to one hundredth of a meter.

Prefix	**KILO k-**	**HECTO h-**	**DEKA da-**	**BASIC UNIT**	**DECI d-**	**CENTI c-**	**MILLI m-**
Meaning	Thousand	Hundred	Ten	**(m, L, g)**	Tenth	Hundredth	Thousandth
	1,000	100	10	**1**	0.1	0.01	0.001

Now we will express the information in the prefix table using conversion facts as we did for the U.S. System of Measurement. The conversion facts are shown using meters. However, "***meter***" (the basic unit for length) could be replaced with "***liter***" (the basic unit for volume) or with "***gram***" (the basic unit for weight).

BASIC CONVERSION FACTS – METRIC SYSTEM OF MEASUREMENT	
BASIC UNITS	PREFIXES (shown for meter)
Meter (m) – to measure length	1 **kilo**meter (**k**m) = 1000 meters (m)
Liter (L) – to measure volume	1 **hecto**meter (**h**m) = 100 meters (m)
Gram (g) – to measure weight	1 **deka**meter (**da**m) = 10 meters (m)
	10 **deci**meters (**d**m) = 1 meters (m)
	100 **centi**meters (**c**m) = 1 meter (m)
	1000 **milli**meters (**m**m) = 1 meter (m)

Now we will convert a measurement from one unit to another by setting up and solving a proportion. The given conversion fact will be used as the *known* ratio in the proportion.

***EXAMPLES*:** Perform each unit conversion by using a proportion.

1. Convert 3 kilometers (km) to meters (m). Use the conversion fact: **1 kilometer (km) = 1000 meters (m)**

STEP 1: Variable – assign variable to unknown — n = meters equal to 3 kilometers

STEP 2: Ratio in Words – fraction using units being compared — $\frac{\text{km}}{\text{m}}$

STEP 3: Known Ratio – fraction using given conversion fact — $\frac{1\text{ km}}{1000\text{ m}}$

STEP 4: Unknown Ratio – fraction using variable — $\frac{3\text{ km}}{n\text{ m}}$

NOTE: *In all three ratios, kilometers were in the numerator and meters in the denominator. It is very important to be consistent when you set up your fractions.*

STEP 5: Proportion – set ratios equal — $\frac{1}{1000} = \frac{3}{n}$

STEP 6: Solve – set cross products equal

Answer: 3 kilometers equals **3000** meters. — $n = 3000$

2. Convert 88 centigrams (cg) to grams (g). Use the conversion fact: **100 centigrams (cg) =1 gram (g)**

VARIABLE	UNITS	PROPORTION	ANSWER
n = grams equal to 88 centigrams	$\frac{\text{cg}}{\text{g}} \rightarrow$	$\frac{100}{1} = \frac{88}{n}$	88 centigrams equals **0.88** grams.

$$100n = 88$$

$$\frac{100n}{100} = \frac{88}{100}$$

$$n = 0.88$$

3. Convert 41 liters (L) to kiloliters (kL). Use the conversion fact: **1 kiloliter (kL) = 1000 liters (L)**

VARIABLE	UNITS	PROPORTION	ANSWER
n = kiloliters equal to 41 liters	$\frac{\text{kL}}{\text{L}} \rightarrow$	$\frac{1}{1000} = \frac{n}{41}$	41 liters equals **0.041** kiloliters.

$$41 = 1000n$$

$$\frac{41}{1000} = \frac{1000n}{1000}$$

$$0.041 = n$$

PRACTICE: Perform each unit conversion by using a proportion.

1. Convert 3.52 kilograms (kg) to grams (g).
   Use the conversion fact: **1 kilogram (kg) = 1000 grams (g)**
2. Convert 5.6 centimeters (cm) to meters (m).
   Use the conversion fact: **100 centimeters (cm) = 1 meter (m)**
3. Convert 9.1 meters (m) to millimeters (mm).
   Use the conversion fact: **1000 millimeters (mm) = 1 meter (m)**
4. Convert 0.5 kiloliters (kL)to liters (L).
   Use the conversion fact: **1 kiloliter (kL) = 1000 liters (L)**
5. Convert 2.84 liters (L) to milliliters (mL).
   Use the conversion fact: **1000 milliliters (mL) = 1 liter (L)**
6. Convert 1970 milligrams (mg) to grams (g).
   Use the conversion fact: **1000 milligrams (mg) = 1 gram (g)**

Answers:

1. 3520 g
2. 0.056 m
3. 9100 mm
4. 500 L
5. 2840 mL
6. 1.97 g

SECTION 3.2 SUMMARY
Proportions

PROPORTION	A statement that two ratios (fractions) are equal.	*Example*: $\frac{4}{8}=\frac{1}{2}$
TRUE OR FALSE PROPORTION	$\frac{a}{b}=\frac{c}{d}$ only if $ad=bc$ 1. Multiply diagonally to get the cross products. 2. If the cross products are equal, the proportion is true.	*Example*: Is $\frac{3.6}{8}=\frac{4}{9.5}$ a true statement? $(3.6)(9.5)\stackrel{?}{=}(8)(4)$ $34.2\neq 32$ No, the proportion is not true.
SOLVING A PROPORTION	1. Multiply diagonally and set the cross products equal to each other. 2. Isolate the variable using inverse operations.	*Example*: $\frac{4}{15}=\frac{6}{n}$ $4\cdot n=15\cdot 6$ $4n=90$ $\frac{4n}{4}=\frac{90}{4}$ $n=22.5$
APPLICATION PROBLEMS	1. Assign Variable to Unknown 2. Write Ratio in Words 3. Write Proportion: Ratio of Given Values = Ratio with Variable 4. Solve Proportion	*Example*: On a map, 2 cm represents 9 km. How many cm would represent 36 km? n = centimeters equal to 36 km $\frac{\text{Centimeters}}{\text{Kilometers}}$ $\frac{2}{9}=\frac{n}{36}$ $2\cdot 36=9n$ $72=9n$ $\frac{72}{9}=\frac{9n}{9}$ $8=n$ So, 36 km would be represented by **8 cm**.
UNIT CONVERSIONS	1. Assign Variable to Unknown 2. Write Ratio in Words 3. Write Proportion: Ratio of Conversion Fact = Ratio with Variable 4. Solve Proportion	*Example*: Convert 3 minutes to seconds. Use the fact: 60 seconds (s) = 1 minute (min) n = seconds equal to 3 minutes $\frac{\text{Minutes}}{\text{Seconds}}$ $\frac{1}{60}=\frac{3}{n}$ $1n=60\cdot 3$ $n=180$ So, 3 minutes equals **180 seconds**.

SECTION 3.2 EXERCISES

Proportions

Determine if the following proportions are true.

1. $\frac{16}{20} = \frac{12}{15}$

2. $\frac{9.5}{14} = \frac{6.2}{10}$

Solve the proportions. For problems involving decimals, give the answer in decimal form. For problems involving fractions, give the answer in fractional form.

3. $\frac{n}{10} = \frac{7}{35}$

4. $\frac{-4}{n} = \frac{3}{12}$

5. $\frac{30}{20} = \frac{5}{n}$

6. $\frac{21}{0.3} = \frac{n}{0.07}$

7. $\frac{4}{3.2} = \frac{-6.4}{n}$

8. $\frac{\frac{2}{3}}{n} = \frac{5}{\frac{9}{8}}$

9. $\frac{3\frac{1}{2}}{\frac{1}{12}} = \frac{n}{\frac{4}{7}}$

10. $\frac{n}{6} = \frac{3\frac{1}{2}}{2\frac{2}{3}}$

Solve each problem by using a proportion.

11. Harry gets 23 miles per gallon of gasoline in his truck. How many miles can Harry drive on 4 gallons of gasoline?

12. On a map, 2 centimeters represents 3 kilometers. How many kilometers are represented by 15 centimeters?

13. A nurse has to give a patient a dose of medication. The dosage says to administer 3 ml of medication for a 150 pound person. If the patient weighs 200 pounds, how many ml of medication should the nurse give to the patient?

14. To make 4 moles of water, 2 moles of oxygen gas are needed. How many moles of oxygen gas are needed to make 42 moles of water?

15. Three ounces of a chemical are needed to treat 25 ounces of water. How many ounces of water can be treated with 12 ounces of the chemical?

16. A nurse has to give a child a dose of Tylenol. The dosage says to administer 2 teaspoons of medication for a 50 pound person. If the child weighs 75 pounds, how many teaspoons of medication should the nurse give to the child?

17. A college has a ratio of 2 male students for every 3 female students. If there are 5322 male students, how many female students attend the college?

18. An office assistant can type 525 words in 5 minutes. At this rate, how many minutes would it take the office assistant to type 2100 words?

19. There are 45 mg of cholesterol in 2 ounces of egg substitute. How many mg of cholesterol are there in 3 ounces of egg substitute?

20. A recipe calls for ¼ teaspoon of salt for every ½ cup of flour. How much salt should be used for 5 cups of flour?

Perform each unit conversion by using a proportion.

21. Convert 84 inches to feet. Use the conversion fact: **12 inches (in) = 1 foot (ft)**

22. Convert 18,480 feet to miles. Use the conversion fact: **5280 feet (ft) = 1 mile (mi)**

23. Convert 16 yards to feet. Use the conversion fact: **3 feet (ft) = 1 yard (y)**

24. Convert 600 seconds to minutes. Use the conversion fact: **60 seconds (s) = 1 minute (min)**

25. Convert $3\frac{3}{4}$ hours to minutes. Use the conversion fact: **60 minutes (min) = 1 hour (hr)**

26. Convert 5 cups to fluid ounces. Use the conversion fact: **8 fluid ounces (fl oz) = 1 cup (c)**

27. Convert 7 quarts to pints. Use the conversion fact: **2 pints (pt) = 1 quart (qt)**

28. Convert 6.5 gallons to quarts. Use the conversion fact: **4 quarts (qt) = 1 gallon (gal)**

29. Convert $2\frac{1}{2}$ tons to pounds. Use the conversion fact: **2000 pounds (lbs) = 1 ton (T)**

30. Convert 40 ounces to pounds. Use the conversion fact: **16 ounces (oz) = 1 pound (lb)**

Perform each unit conversion by using a proportion.

31. Convert 6.8 kilometers to meters. Use the conversion fact: **1 kilometer (km) = 1000 meters (m)**

32. Convert 8.75 meters to centimeters. Use the conversion fact: **100 centimeters (cm) = 1 meter (m)**

33. Convert 2600 millimeters to meters. Use the conversion fact: **1000 millimeters (mm) = 1 meter (m)**

34. Convert 800 milliliters to liters. Use the conversion fact: **1000 milliliters (mL) = 1 liter (L)**

35. Convert 950 grams to kilograms. Use the conversion fact: **1 kilogram (kg) = 1000 grams (g)**

36. Convert 0.25 grams to milligrams. Use the conversion fact: **1000 milligrams (mg) = 1 gram (g)**

Answers to Section 3.2 Exercises

1. True
2. False
3. $n = 2$
4. $n = -16$
5. $n = \frac{10}{3}$ OR $n = 3\frac{1}{3}$
6. $n = 4.9$
7. $n = -5.12$
8. $n = \frac{3}{20}$
9. $n = 24$
10. $n = \frac{63}{8}$ OR $n = 7\frac{7}{8}$
11. 92 miles
12. 22.5 kilometers
13. 4 ml
14. 21 moles of oxygen gas
15. 100 ounces of water
16. 3 teaspoons
17. 7983 females
18. 20 minutes
19. 67.5 mg
20. $2\frac{1}{2}$ teaspoons
21. 7 ft
22. $3\frac{1}{2}$ mi
23. 48 ft
24. 10 min
25. 225 min
26. 40 fl oz
27. 14 pts
28. 26 qts
29. 5000 lbs
30. 2.5 lbs
31. 6800 m
32. 875 cm
33. 2.6 m
34. 0.8 L
35. 0.95 kg
36. 250mg

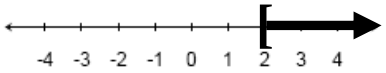
Mixed Review

Sections 1.1 – 3.2

1. Evaluate $-3b \cdot 2a + c$ if $a = \frac{1}{3}$, $b = \frac{2}{5}$, and $c = \frac{2}{3}$.
2. Simplify $4 - 3x + 8y - 6 + 7x - 2y$.
3. Simplify $-\frac{2}{3}(24x - 30)$.
4. Solve $3a - 7 = 5(a - 3) + 4$.
5. Translate the word problem into an algebraic equation. Then solve the equation.
   The difference of a number and 4 is five times the number. Determine the number.
6. Write an algebraic equation for the word problem. Then solve the equation to answer the question.
   Austin wants to buy a guitar that costs $780. He has already saved $420. If he can save $40 per month, how long will it take him to save enough to buy the guitar?
7. Solve $\frac{1}{2}x - \frac{3}{4} = \frac{3}{5}x$.
8. Solve $4(6 - 2x) \le 8$, graph the solution, and write the solution in interval notation.
9. The formula used to determine the perimeter of a rectangle is $P = 2L + 2W$ where L is the length and W is the width. Find L if $P = 15$ and $W = \frac{3}{2}$.
10. In the equation $4x + 36y = -20$, solve for x.

Answers to Mixed Review

1. $-\frac{2}{15}$
2. $4x + 6y - 2$
3. $-16x + 20$
4. $a = 2$
5. $n - 4 = 5n$
   $n = -1$
6. $40x + 420 = 780$
   9 months
7. $x = -\frac{15}{2}$
8. $x \ge 2$
   $[2, \infty)$



9. $L = 6$
10. $x = -5 - 9y$