Section 3.3 Objectives

- Convert among fractions, decimals, and percent.
- Rewrite percent problems as algebraic equations.
- Solve percent problems for the unknown whole, part, or percent.
- Solve application word problems involving percent.



INTRODUCTION

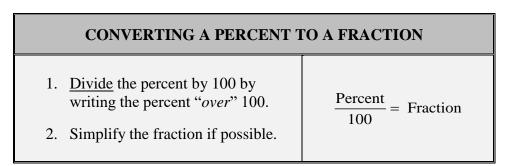
Percentages are widely used in daily life. Some examples are shopping, sports, weather, food, and finance. Because there are so many practical applications, it is important to understand the meaning of percentages and to learn how to calculate percentages. In this section we begin with the definition of percent. Then we will review how to convert among percent, fractions, and decimals. After that, the main focus will be on calculating percent and solving application problems involving percent.

Percent is derived from the Latin phrase "per centum" which means "by the hundred." A *percent* is the number of parts out of a hundred.

For example, a 70% on a test means that you earned 70 points out of 100 points on the test.

CONVERTING A PERCENT TO A FRACTION

Remember, a percent is "*out of*" one hundred. This implies division by 100. So, to convert a percent to a fraction, you divide the percent by 100. This can be written as the percent "*over*" 100.



EXAMPLES: Write each percent as a fraction.

1. Write 78% as a fraction.

$78\% = \frac{78}{100}$	Divide the percent by 100 by writing the percent "over" 100.
$=\frac{.78^{-39}}{.10050}$	Simplify the fraction by dividing out a common factor of 2.
$=\frac{39}{50}$	This is the answer.

2. Write 13% as a fraction.

$$13\% = \frac{13}{100}$$
 Divide the percent by 100 by writing the percent "over" 100
The fraction cannot be simplified, so this is the answer.

3. Write 125% as a fraction.

125% =
$$\frac{125}{100}$$
 Divide the percent by 100 by writing the percent "over" 100.
= $\frac{125}{1004}$ Simplify the fraction by dividing out a common factor of 25.
= $\frac{5}{4}$ This is the answer.

<u>REVIEW</u>: CONVERTING A PERCENT TO A FRACTION

PRACTICE: Write each percent as a fraction.

1.	65%	5.	21%
2.	17%	6.	25%
3.	20%	7.	145%
4.	130%	8.	24%

Answers:

1.	$\frac{13}{20}$		5.	$\frac{21}{100}$
	$\frac{17}{100}$		6.	$\frac{1}{4}$
3.	$\frac{1}{5}$		7.	$\frac{29}{20}$
4.	$\frac{13}{10}$	88 ►	8.	$\frac{6}{25}$

CONVERTING A FRACTION TO A PERCENT

You just learned that, to convert a percent to a fraction, you <u>divide</u> by 100. Now we will convert the opposite way, from a fraction to a percent. It makes sense that we will use the inverse operation for this and <u>multiply</u> by 100%.

CONVERTING A FRACTION TO A	PERCENT
 <u>Multiply</u> the fraction by 100%. Simplify by dividing out common factors if possible. 	Fraction $\times 100\%$ = Percent

1.

EXAMPLES: Write each fraction as a percent.

Write
$$\frac{3}{4}$$
 as a percent.
 $\frac{3}{4} = \frac{3}{4} \cdot \frac{100}{1}\%$ Multiply the fraction by 100%.
 $= \frac{3}{1 \cancel{4}} \cdot \frac{\cancel{100}}{\cancel{1}}\%$ Simplify by dividing out a common factor of 4.
 $= 75\%$ This is the answer.

2. Write
$$\frac{5}{8}$$
 as a percent.
 $\frac{5}{8} = \frac{5}{8} \cdot \frac{100}{1}\%$ Multiply the
 $= \frac{5}{2\%} \cdot \frac{100^{25}}{1}\%$ Simplify by
 $= \frac{125}{2}\%$
 $= 62.5\%$ This is the a

e fraction by 100%.

dividing out a common factor of 4.

answer.

3. Write
$$1\frac{1}{4}$$
 as a percent.
 $1\frac{1}{4} = \frac{5}{4}$ Write the mixed number as an improper fraction.
 $= \frac{5}{4} \cdot \frac{100}{1}\%$ Multiply the improper fraction by 100%.
 $= \frac{5}{14} \cdot \frac{100^{25}}{1}\%$ Simplify by dividing out a common factor of 4.
 $= 125\%$ This is the answer.

PRACTICE: Write each fraction as a percent.

1. $\frac{4}{5}$			4.	$\frac{7}{20}$
2. $\frac{3}{5}$			5.	$\frac{1}{8}$
3. $1\frac{3}{5}$			6.	$1\frac{7}{10}$
<u>Answers</u>	:			
1. 80	%		4.	35%
2. 60	%		5.	12.5%
3. 16	0%		6.	170%

CONVERTING A PERCENT TO A DECIMAL

When you converted a percent to a fraction, you divided by 100. You will use this same process now to convert a percent to a decimal.

CONVERTING A PERCENT TO	A DECIMAL
<u>Divide</u> the percent by 100. (<i>Or move the decimal point two places to the left</i>)	Percent $\div 100 = Decimal$

EXAMPLES: Write each percent as a decimal.

1. Write 82% as a decimal.

82%	=	82÷100	Divide the percent by 100.
	=	0.82	This is the answer.

2. Write 9% as a decimal.

9%	$= 9 \div 100$	Divide the percent by 100.
	= 0.09	This is the answer.

3. Write 235% as a decimal.

 $235\% = 235 \div 100$ Divide the percent by 100. = 2.35 This is the answer.

REVIEW: CONVERTING A PERCENT TO A DECIMAL



PRACTICE: Write each percent as a decimal.

1.	68%	5.	70%
2.	4%	6.	7%
3.	126%	7.	110%

4. 32%

1.	0.68	88 ►	5.	0.7
2.	0.04	88 ►	6.	0.07
3.	1.26	88 ►	7.	1.1
4.	0.32	⊛⊛		

CONVERTING A DECIMAL TO A PERCENT

When you converted a fraction to a percent, you multiplied by 100%. You will use this same process now to convert a decimal to a percent.

CONVERTING A DECIMAL TO A P	PERCENT
<u>Multiply</u> the decimal by 100%. (Or move the decimal point two places to the right)	Decimal×100% = Percent

EXAMPLES: Write each decimal as a percent.

1. Write 0.29 as a percent.

$0.29 = 0.29 \times 100\%$	Multiply the decimal by 100%.
= 29%	This is the answer.

2. Write 0.9 as a percent.

 $0.9 = 0.9 \times 100\%$ Multiply the decimal by 100%. = 90% This is the answer.

3. Write 0.058 as a percent.

 $0.058 = 0.058 \times 100\%$ Multiply the decimal by 100% = 5.8% This is the answer.

REVIEW: CONVERTING A DECIMAL TO A PERCENT

PRACTICE: Write each decimal as a percent.

1.	0.49	5.	0.18
2.	0.3	6.	0.6
3.	0.025	7.	0.07
4.	5	8.	3.5

1.	49%	88 ►	5.	18%
2.	30%	88 -	6.	60%
3.	2.5%	8 ₩ ▶	7.	7%
4.	500%	88 	8.	350%

Before moving on to the next topic, we present a summary of the four conversions that you just learned. Following the summary, there is a set of mixed review problems so that you can practice all four types of conversions.

CONVERSION SUMMARY

Convert FROM a Percent: DIVIDE BY 100	
Percent \rightarrow Fraction	$\frac{P}{100}$
Percent \rightarrow Decimal	P÷100

Convert TO a Percent: MULTIPLY BY 100%		
Fraction \rightarrow Percent	$\frac{n}{d} \times 100\%$	
Decimal \rightarrow Percent	Decimal × 100%	

NOTE: Remember to reduce all fractional answers.

PRACTICE: Convert.

- 1. Write 35% as a decimal.
- 2. Write $\frac{17}{20}$ as a percent.
- 3. Write 9% as a fraction.
- 4. Write 0.41 as a percent.

- 5. Write $1\frac{1}{2}$ as a percent.
- 6. Write 6% as a decimal.
- 7. Write 0.08 as a percent.
- 8. Write 28% as a fraction.

Answers:

 1. 0.35 5. 150%

 2. 85% 6. 0.06

 3. $\frac{9}{100}$ 7. 8%

 4. 41% 8. $\frac{7}{25}$

SOLVING PERCENT PROBLEMS USING A FORMULA

Now we will focus on solving problems involving percent. You will learn two methods that can be used to solve basic percent problems. One method uses a formula and the other method uses proportions. We begin with the formula method.

<u>Percent Formula</u>: **PERCENT × WHOLE = PART** <u>Note</u>: also written **PART = PERCENT × WHOLE**

The formula relates three quantities:

- *Percent* is a number in front of a % symbol and is used to represent an amount out of 100.
- *Whole* is a total amount.
- *Part* is a portion of the total amount.

The procedure for solving a percent problem involves translating the problem into math and substituting in the given values to set up the formula. To help translate the problem, look for these <u>Key Words</u>:

- *"of"* means multiply
- "is" means equal
- "what" refers to an unknown quantity that we represent with a variable

Example: If we say, "50% of 80 is 40," we mean exactly what the formula states, that $50\% \times 80 = 40$.

Only two values will be given in percent problems. You will substitute the two values into the formula in place of the two quantities they represent (*Percent*, *Whole*, or *Part*). You will use a <u>variable</u> to represent the quantity whose value is <u>not given</u>.

Once the equation is set up, you will solve for the variable using the algebra methods you have already learned. There are a couple details to keep in mind when you solve percent problems.

- If there is a percent value in an equation, you will first need to change the percent to its decimal form by dividing by 100 (*or moving the decimal point two places to the left*).
- If the percent is the unknown quantity in an equation, the solution to the equation will be in decimal form and you will need to change the decimal to a percent by multiplying by 100% (*or moving the decimal point two places to the right*).

SOLVING A PERCENT PROBLEM USING A FORMULA

1. Translate the problem into an algebraic equation set up like the percent formula shown below and substitute the given values into the equation.

PERCENI	X	WHOLE	=	PARI
Number with %,	<i>"of</i> "	Total	" <i>is</i> "	Portion
Means out of 100	-]	Amount		of Whole

Note: The word "what" in the original problem refers to the variable (the unknown quantity).

- 2. Solve the equation for the variable using algebra.
 - If there is a percent in the equation, change it to a decimal by dividing by 100 (or moving the decimal point two places to the left).
 - If the percent is unknown, change the answer from a decimal to a percent by multiplying by 100% (*or moving the decimal point two places to the right*).

EXAMPLES: Solve each percent problem. (In these problems, the unknown value is the *part*.)

1. What is 20% of 35?

Translate the problem into an algebraic equation.

Problem:	What	is	20%	of	35?
Equation:	n	=	20%	×	35

Solve the equation for the variable.

$n = 20\% \times 35$	Divide the percent by 100 to change it to a decimal.
$n = 0.20 \times 35$	Multiply.
<i>n</i> = 7	This is the answer.

2. 18% of 220 is what number?

Translate the problem into an algebraic equation.

Problem:	18%	of	220	is	what number?
Equation:	18%	×	220	=	n

Solve the equation for the variable.

$18\% \times 220 = n$	Divide the percent by 100 to change it to a decimal.
$0.18 \times 220 = n$	Multiply.
39.6 = n	This is the answer.

3. What is 5.5% of 248?

Translate the problem into an algebraic equation.

Problem:	What	is	5.5%	of	248?
Equation:	n	=	5.5%	×	248

Solve the equation for the variable.

$n = 5.5\% \times 248$	Divide the percent by 100 to change it to a decimal.
$n = 0.055 \times 248$	Multiply.
<i>n</i> =13.64	This is the answer.

PRACTICE: Solve each percent problem.

- 1. What is 30% of 90?
- 2. What is 145% of 60?
- 3. 15% of 60 is what number?
- 4. What is 5% of 820?

Answers:

1.	27	***	5.
2.	87		6.
3.	9		7.
4.	41		8.

- 5. What is 120% of 45?
- 6. What is 2.5% of 76?

54 1.9 28.7 10.4

- 7. 35% of 82 is what number?
- 8. 3.2% of 325 is what number?

EXAMPLES: Solve each percent problem. (In these problems, the unknown value is the *whole*.)

1. 30% of what number is 45?

Translate the problem into an algebraic equation.

Problem:	30%	of	what number	is	45?
Equation:	30%	×	n	=	45

Solve the equation for the variable.

$30\% \times n = 45$	Divide the percent by 100 to change it to a decimal.
$0.3 \times n = 45$	Rewrite $0.3 \times n$ as $0.3n$.
$\frac{0.3n}{0.3} = \frac{45}{0.3}$	Divide both sides by 0.3.
n = 150	This is the answer.

2. 42 is 15% of what number?

Translate the problem into an algebraic equation.

Problem:	42	is	15%	of	what number?
Equation:	42	=	15%	×	n

Solve the equation for the variable.

$42 = 15\% \times n$	Divide the percent by 100 to change it to a decimal.
$42 = 0.15 \times n$	Rewrite $0.15 \times n$ as $0.15n$.
$\frac{42}{0.15} = \frac{0.15n}{0.15}$	Divide both sides by 0.15.
280 = n	This is the answer.

3. 88 is $2\frac{1}{2}$ % of what number?

Translate the problem into an algebraic equation.

88 is $2\frac{1}{2}$ % of what number? **Problem:** Equation: $88 = 2\frac{1}{2}\% \times$ n

Solve the equation for the variable.

$88 = 2\frac{1}{2}\% \times n$	First, change the mixed number to an improper fraction.
$88 = \frac{5}{2}\% \times n$	Next, change the fraction to a decimal.
$88 = 2.5\% \times n$	Now, divide the percent by 100 to change it to a decimal.
$88 = 0.025 \times n$	Rewrite $0.025 \times n$ as $0.025n$.
$\frac{88}{0.025} = \frac{0.025n}{0.025}$	Divide both sides by 0.025.
3520 = n	This is the answer.

18 is 45% of what number?

PRACTICE: Solve each percent problem.

1.	60 is 5% of what number?	4.

- 5. 60 is $3\frac{1}{5}$ % of what number? 6. 160% of what number is 48? 2. 32 is 20% of what number?
- 3. 200% of what number is 90?

1.	1200	**	2	1.	40
2.	160	**	5	5.	1875
3.	45	**	6	5.	30

EXAMPLES: Solve each percent problem. (In these problems, the unknown value is the *percent*.)

1. What percent of 20 is 5?

Translate the problem into an algebraic equation.

Problem:	What %	of	20	is	5?
Equation:	n	×	20	=	5

Solve the equation for the variable.

$n \times 20 = 5$	Rewrite $n \times 20$ as $20n$.
$\frac{20n}{20} = \frac{5}{20}$	Divide both sides by 20.
n = 0.25	This is the decimal form of the answer.
$n = 0.25 \times 100\%$	Change to a percent by multiplying by 100%.
<i>n</i> = 25%	This is the final answer.

2. What percent of 320 is 64?

Translate the problem into an algebraic equation.

Problem:	What %	of	320	is	64?
Equation:	n	×	320	=	64

Solve the equation for the variable.

Rewrite $n \times 320$ as $320n$.
Divide both sides by 320.
This is the decimal form of the answer.
Change to a percent by multiplying by 100%.
This is the final answer.

3. What percent of 32 is 20?

Translate the problem into an algebraic equation.

Problem:	What %	of	32	is	20?
Equation:	n	×	32	=	20

Solve the equation for the variable.

$n \times 32 = 20$	Rewrite $n \times 32$ as $32n$.
$\frac{32n}{32} = \frac{20}{32}$	Divide both sides by 32.
n = 0.625	This is the decimal form of the answer.
$n = 0.625 \times 100\%$	Change to a percent by multiplying by 100%.
n = 62.5%	This is the final answer.

4. 120 is what percent of 80?

Translate the problem into an algebraic equation.

Problem:	120	is	what %	of	80?
Equation:	120	=	n	×	80

Solve the equation for the variable.

$120 = n \times 80$	Rewrite $n \times 80$ as $80n$.
$\frac{120}{80} = \frac{80n}{80}$	Divide both sides by 80.
1.5 = n	This is the decimal form of the answer.
$1.5 \times 100\% = n$	Change to a percent by multiplying by 100%.
150% = n	This is the final answer.

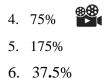
PRACTICE: Solve each percent problem.

- 1. What percent of 72 is 36?
- 2. What percent of 200 is 50?
- 3. 42 is what percent of 168?

Answers:

1.	50%	\$ ₽
2.	25%	♦€
3.	25%	⊛⊛

- 4. What percent of 120 is 90?
- 5. 133 is what percent of 76?
- 6. What percent of 120 is 45?



Before we discuss the other method for solving percent problems, we give you a chance to review how to solve using a formula. There is a set of mixed review problems below so that you can practice all types of problems. In these problems, you will determine which quantity is the unknown (percent, whole, or part) and solve for it.



<u>REVIEW</u>: SOLVING PERCENT PROBLEMS USING A FORMULA

PRACTICE: Solve each percent problem.

- 1. What is 75% of 164?
- 2. 60% of what number is 168?
- 3. 78 is what percent of 60?

- 1. 123
- 2. 280
- 3. 130%

- 4. 546 is 10.5% of what number?
- 5. What percent of 325 is 45.5?
- 6. 3% of 95 is what number?
- 4. 5200
- 5. 14%
- 6. 2.85

SOLVING PERCENT PROBLEMS USING PROPORTIONS

Now we will present the other method that can be used to solve percent problems. This method uses a special proportion that relates the Percent, Whole, and Part. The proportion is $\frac{\text{PERCENT}}{100} = \frac{\text{PART}}{\text{WHOLE}}$. Every percent problem is set up based on this proportion. Once the two values given in the problem are substituted in the proportion, you will solve for the unknown quantity by setting the cross products equal to each other and then isolating the variable.

When you set up the proportion, it is very important to place each given value in the correct position. To help identify each value as the percent, whole, or part, think of the formula $PERCENT \times WHOLE = PART$. In particular, remember the translation $PERCENT \circ f$ WHOLE is PART. The WHOLE is the total amount and follows the word "of." The PART is the amount compared to the total and appears with the word "is." The PERCENT value is easy to identify since it contains the % symbol.

SOLVING A PERCENT PROBLEM USING A PROPORTION

- 1. Use the percent formula **PERCENT** "of" **WHOLE** "is" **PART** to identify whether the values given in the problem represent the percent, whole, or part. Use these hints:
 - **PERCENT** contains the % symbol
 - WHOLE the total following the key word "of"
 - **PART** the amount appearing on the left or right with the word "is"
- 2. Substitute the two given values in the percent proportion below. Be sure to place each value in the correct position of the proportion.

$$\frac{\text{PERCENT}}{100} = \frac{\text{PART}}{\text{WHOLE}}$$

3. Solve the proportion by setting the cross products equal and then isolating the variable.

EXAMPLES: Solve each percent problem.

1. What percent of 20 is 5?

Identify the given values and the unknown.

Percent = n	There is no number with a % symbol. So, the percent is unknown.
<i>Whole</i> = 20	The number that follows the word "of" is 20. So, the whole is 20.
<i>Part</i> = 5	The number with the word "is" is 5. So, the part is 5.

Set up and solve the proportion.

$\frac{Percent}{100} = \frac{Part}{Whole}$	Substitute the given values and the variable in the proportion.
$\frac{n}{100} = \frac{5}{20}$	Set the cross products equal.
20n = 500	
$\frac{20n}{20} = \frac{500}{20}$	Divide by 20.
<i>n</i> = 25%	This is the answer. 25% of 20 is 5.

2. What is 18% of 220?

Identify the given values and the unknown.

<i>Percent</i> = 18%	The number with the % symbol is 18. So, the percent is 18.
<i>Whole</i> = 220	The number that follows the word "of" is 220. So, the whole is 220.
<i>Part</i> = <i>n</i>	The part is the unknown.

Set up and solve the proportion.

$\frac{Percent}{100} = \frac{Part}{Whole}$	Substitute the given values and the variable in the proportion.
$\frac{18}{100} = \frac{n}{220}$	Set the cross products equal.
$\frac{3960}{100} = \frac{100n}{100}$	Divide by 100.
39.6 = n	This is the answer. 18% of 220 is 39.6 .

3. 30 is 15% of what number?

Identify the given values and the unknown.

<i>Percent</i> = 15%	The number with the % symbol is 15. So, the percent is 15.
Whole = n	There is no number that follows the word "of." So, the whole is the unknown.
<i>Part</i> = 30	The number with the word "is" is 30. So, the part is 30.

Set up and solve the proportion.

$\frac{Percent}{100} = \frac{Part}{Whole}$	Substitute the given values and the variable in the proportion.
$\frac{15}{100} = \frac{30}{n}$	Set the cross products equal.
$\frac{15n = 3000}{\frac{15n}{15}} = \frac{3000}{15}$	Divide by 15.
n = 200	This is the answer. 30 is 15% of 200 .

REVIEW: SOLVING PERCENT PROBLEMS USING A PROPORTION

PRACTICE: Solve each percent problem.

- 1. What percent of 72 is 36?
- 2. 15% of 60 is what number?
- 3. 60 is 5% of what number?

1.	50%	8 € ►	4.	35%
2.	9	8 € ►	5.	43.2
3.	1200		6.	450

- 4. What percent of 60 is 21?
- 5. What is 90% of 48?
- 6. 81 is 18% of what number?

APPLICATIONS OF PERCENT PROBLEMS

The percent equations that we solved in the previous examples were the most basic type. Now we will solve problems that deal with real world applications of percent. We will continue to use the percent formula **PERCENT** of **WHOLE** is **PART** to solve the problems. You may find the formula more useful when it is written as **PART** is **PERCENT** of **WHOLE**.

The application problems that follow provide a context and meaning to the values in the formula. For example, sales tax, commission rates, and tip amounts are all examples of real world applications of percent. While the meaning may vary from problem to problem, in each case the problem can be simplified to the basic percent problems covered so far. In the application problems, there will still be three quantities involved (percent, whole, and part) where two are given and one needs to be found. Rather than using the variable n to represent the unknown, it may help to use a variable that reminds you of what it is representing. For instance, you may choose to use t for tax.

EXAMPLES: Solve each application problem.

1. Julia is buying a shirt for \$12.00. How much tax will she pay if the sales tax rate is 6%?

Translate the problem into an algebraic equation.

Formula:	Part	is	Percent	of	Whole
Problem:	Tax	is	%	of	Shirt Price
Equation:	t	=	6%	×	\$12

Solve the equation for the variable.

$t = 6\% \times 12$	Divide the percent by 100 to change it to a decimal.
$t = 0.06 \times 12$	Multiply.
t = 0.72	This is the answer.

The tax amount on the shirt is **\$0.72**.

2. The restaurant bill was \$68.75. Ingrid paid the bill and left a tip of \$11.00 for the waiter. What percent of the bill was the tip that Ingrid left?

Translate the problem into an algebraic equation.

Formula:	Part	is	Percent	of	Whole
Problem:	Tip	is	%	of	Bill
Equation:	\$11	=	p	×	\$68.75

Solve the equation for the variable.

 $11 = p \times 68.75$ Rewrite the right side.11 = 68.75 pDivide to isolate the variable. $\frac{11}{68.75} = \frac{68.75 p}{68.75}$ Divide to isolate the variable.0.16 = pMultiply the decimal by 100% to change it to a percent.16% = pThis is the answer.

Ingrid left 16% of the bill for the tip.

3. Seeley is a salesman and earns a commission of 2% on his sales. He received a commission check of \$7,000 for the month of March. What was Seeley's sales amount in March?

Translate the problem into an algebraic equation.

Formula:	Part	is	Percent	of	Whole
Problem:	Commission	is	%	of	Sales
Equation:	\$7000	=	2%	×	S

Solve the equation for the variable.

$7000 = 2\% \times s$	Divide the percent by 100 to change it to a decimal.				
7000 = 0.02s					
$\frac{7000}{100} = \frac{0.02s}{1000}$	Divide to isolate the variable.				
0.02 0.02					
350000 = s	This is the answer.				
Seeley's sales total for March was \$350,000.					

4. Bridget registered to take classes at the local community college. Due to a program with her employer, she only had to pay 60% of the tuition. If a three-credit class cost \$1,101, how much did Bridget have to pay?

Translate the problem into an algebraic equation.

Formula:	Part	is	Percent	of	Whole
Problem:	Payment	is	%	of	Tuition
Equation:	р	=	60%	×	\$1,101

Solve the equation for the variable.

$p = 60\% \times 1101$	Divide the percent by 100 to change it to a decimal.
$p = 0.60 \times 1101$	Multiply.
p = 660.60	This is the answer.
Bridget paid \$660.60	for the class.

5. Angela sells cars for a living. In one week, she sold \$37,500 worth of cars and earned a commission of \$1,875. What is her commission rate?

Translate the problem into an algebraic equation.

Formula:	Part	is	Percent	of	Whole
Problem:	Commission	is	%	of	Sales
Equation:	\$1,875	=	p	×	\$37,500

Solve the equation for the variable.

$$1875 = p \times 37500$$
Rewrite the right side. $1875 = 37500 p$ Divide to isolate the variable. $\frac{1875}{37500} = \frac{37500 p}{37500}$ Divide to isolate the variable. $0.05 = p$ Multiply the decimal by 100% to change it to a percent. $5\% = p$ This is the answer.

Angela's commission rate is 5%.

6. Glynis earns \$43,500 this year. Due to a promotion, she will get a 5% raise next year. What is the dollar amount of her raise? What will her salary be next year?

AMOUNT OF RAISE

Formula: Problem: Equation:	Raise	is	%	of	Salary	
$r = 5\% \times 43$ $r = 0.05 \times 4$ r = 2175			ivide the pe Iultiply.	ercent	by 100 to change it to a decimal.	

Glynis' raise will be **\$2,175**.

NEW SALARY

New Salary = Current Salary + Raise New Salary = \$43,500 + \$2,175 New Salary = \$45,675

Glynis' salary next year will be \$45, 675.

7. Gretchen's favorite store is having a 20% off sale, so Gretchen decides to buy an outfit she likes. If the regular price of the outfit is \$82, how much will it cost on sale?

AMOUNT OF DISCOUNT

Formula:	Part	is	Percent	of	Whole
Problem:	Discount	is	%	of	Regular Price
Equation:	d	=	20%	×	\$82
$d = 20\% \times 8$ $d = 0.2 \times 82$ $d = 16.40$	-	Divid Multi	•	nt by	100 to change it to a decimal.

The discount on the outfit will be **\$16.40**.

SALE PRICE

Sale Price = Regular Price - Discount Sale Price = \$82.00 - \$16.40 Sale Price = \$65.60

The outfit will cost **\$65.60** on sale.

ALTERNATE METHOD: Now we show a different way to solve this same problem.

Discount Amount (amount deducted from regular price) is 20% of the regular price. **Sale Price** (remaining amount Gretchen will have to pay) **is 80% of the regular price**.

Formula: Part is Percent of Whole Problem: Sale Price is % of Regular Price *s* = Equation: 80% \$82 X $s = 80\% \times 82$ $s = 0.80 \times 82$ s = 65.6We get the same answer – the outfit will cost **\$65.60**.

8. Little Timmy has been sick all week. On Monday his fever was 102.4 degrees. By Friday it had decreased 2.7%. What was his temperature on Friday? Round your answer to the nearest tenth.

DECREASE IN TEMPERATURE - how many degrees Timmy's fever went down

Formula:	Part	is	Percent	of	Whole
Problem:	Decrease in Temp	is	%	of	Original Temp
Equation:	d	=	2.7%	×	102.4

 $d = 2.7\% \times 102.4$ $d = 0.027 \times 102.4$ d = 2.7648

From Monday to Friday, Timmy's temperature dropped 2.7648 degrees.

NEW TEMPERATURE - temperature on Friday

Friday's Temperature = Original Temperature – Decrease in Temperature f = 102.4 - 2.7648f = 99.6352

Timmy's temperature on Friday, rounded to the nearest tenth, was 99.6 degrees.

We used the formula method to solve the previous examples. We should point out that the proportion method can also be used to solve application problems.

PRACTICE: Solve each application problem.

- 1. Julio is buying a shirt for \$14.00. How much tax will he pay for the item if the tax rate is 7%?
- 2. The mixer that Patti wants to purchase costs \$229. How much sales tax will she pay if the sales tax rate is 6%? What will the final cost of the mixer be?
- 3. Catherine is buying a couch that costs \$535. After a 6% sales tax, what is the final cost?
- 4. Silas found a cell phone that he would like to buy. He has \$84 to spend. If the price is \$79 and the tax rate is 6%, does Silas have enough money to buy the phone? Explain your answer.
- 5. Sally bought a pair of earrings for \$125 and paid \$8.75 in sales tax. What is the tax rate for the state where she bought the item?
- 6. The cost of a salon visit was \$55 (not including tip). An \$11 tip was given to the stylist. What percent of the cost was given as a tip?
- 7. Bridget applied to take classes at the local community college. Due to a program with her employer, she only had to pay 75% of the tuition. If a three-credit course cost \$1,101, how much did Bridget have to pay?
- 8. Delia sold a house to one of her clients for \$125,000. Her commission rate is 5% of the selling price of the house. How much commission did she earn?
- 9. Jamie earns a commission of 4% on his sales. He received a commission check of \$1,440 for the month of April. What was the amount of Jamie's sales in April?
- 10. If Josh sold a house for \$235,000 last month and got a commission check for \$14,100, what was his commission rate?
- 11. The temperature outside was 75 degrees in the afternoon. It dropped 44% during the day. What temperature was it in the evening? Round the answer to the nearest degree.
- 12. Cassidy earns \$54,300 this year. Due to a promotion, she will get a 3% raise next year. What is the dollar amount of her raise? What will her salary be next year?
- 13. Heather's favorite store is having a 25% off sale, so Heather decides to buy an outfit she likes. If the regular cost of the outfit is \$76, how much will the outfit cost on sale?
- 14. April purchased a Raven's sweatshirt for \$45. This was 30% off the original price. What was the original price of the sweatshirt?

1.	\$0.98	*** •••	8.	\$6,250	⊛ ⊛ ►
2.	Tax = \$13.74; Final Cost = \$242.74	**	9.	\$36,000	₩
3.	\$567.10		10.	6%	₩
4.	Yes.		11.	42 degrees	\$€
	Silas needs \$83.74.		12.	Raise = \$1,629	
5.	7%	8 €		New Salary = \$55,929	€€
6.	20%	*** •••	13.	\$57	*** •
7.	\$825.75	*** •••	14.	\$64.29	₩

FINANCIAL APPLICATIONS – SIMPLE INTEREST

You have already studied many financial applications of percent in the preceding problems. Another important financial application involves *interest*. There are two major financial areas in which interest is used: borrowing money and investing money. When you take out a loan to borrow money, you are charged interest. And when you deposit money into an account to save money, you earn interest. To understand financial applications dealing with interest, you should learn the definitions of the terms below.

FINANCIAL TERMS
Principal: (<i>a</i>) amount of money borrowed OR (<i>b</i>) amount of money deposited
Interest: (a) fee charged to borrow money OR (b) amount earned on money deposited
Interest Rate: (a) percent charged for money borrowed OR (b) percent earned on money deposited
Simple Interest: interest calculated on the original principal only
<u>Compound Interest</u> : interest calculated on both the original principal and any previous interest
<u>Note</u> : We will only study simple interest in this course.

There is a formula that is used to calculate simple interest. The formula is I = Prt. The variable I represents the interest, P represents the principal (original amount of money borrowed or deposited), r represents the interest rate (written as a decimal), and t represents the time (number of years the money was borrowed or deposited). The formula shows that we calculate simple interest by multiplying the three quantities: Principal, Interest Rate, and Time. Keep in mind that this formula may be used to solve for any of the four quantities (I, P, r, or t) if three of the values are given.

SIMPLE INTEREST FORMULA			
I = Prt			
<i>I</i> = Interest	P = Principal	<i>r</i> = rate	<i>t</i> = time
		<i>NOTE:</i> r is given in <u>percent</u> form and must be changed to <u>decimal</u> form.	<i>NOTE:</i> t must be expressed in <u>years</u>

EXAMPLES: Solve each percent problem.

1. What is the simple interest on a loan of \$6,000 for one year if the interest rate is 8.25%?

Determine the given and unknown values.

I = ?	Interest is unknown.
P = \$6000	Principal is the amount of money borrowed.
r = 8.25% = 0.0825	Rate is the percent charged for the loan. Divide by 100 to change it to a decimal.
t = 1 yr	Time is the length of the loan in years.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.
I = (\$6000)(0.0825)(1)	Multiply the values on the right side of the equation.
I = \$495	The interest charged on the loan is \$495.

2. If Barry borrows \$2,000 for 3 years at a 13% annual simple interest rate, how much interest will Barry pay?

Determine the given and unknown values.

I = ?	Interest is unknown.
P = \$2000	Principal is the amount of money borrowed.
r = 13% = 0.13	Rate is the percent charged for the loan. Divide by 100 to change it to a decimal.
t = 3 yrs	Time is the length of the loan in years.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.
I = (\$2000)(0.13)(3)	Multiply the values on the right side.
I = \$780	Barry will pay \$780 in interest.

3. What is the interest gained on a \$1500 investment at an 8% annual simple interest rate for 9 months?

Determine the given and unknown values.

I = ?	Interest is unknown.
P = \$1500	Principal is the amount of money invested.
r = 8% = 0.08	Rate is the percent earned on the investment. Divide by 100 to change it to a decimal.
t = 9 months = $9 \div 12$ = 0.75 yrs	Time is how long the money was invested. We need to convert months to years.Since there are 12 months in a year, divide 9 by 12.This is the time in years.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.
I = (\$1500)(0.08)(0.75)	Multiply the values on the right side.
I = \$90	The interest earned on the investment is \$90 .

4. A television costs \$900. The store gives Ray credit calculated with simple interest at 30% over 2 years. How much does Ray pay for his television?

Determine the given and unknown values.

I = ?	Interest is unknown.
P = \$900	Principal is the amount of money borrowed.
r = 30% = 0.30	Divide the rate by 100 to change the percent to a decimal.
t = 2 yrs	The time is given in years.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.
I = (\$900)(0.30)(2)	Multiply.
I = \$540	The interest charged on the loan is \$540.

Compute the total cost of the television.

Total Cost = $P + I$	Ray has to pay the principal (amount borrowed) and the interest.
Total Cost = \$900 + \$540	Ray has to pay the \$900 cost of the television plus the \$540 interest charge.
Total Cost = \$1440	The total amount that Ray pays for the television is \$1440 .

5. What annual simple interest rate is necessary for an investment to grow from \$200 to \$270 in two years?

Determine the given and unknown values.

I = \$270 - \$200 = \$70	For the investment to grow from \$200 to \$270, \$70 must be earned in interest.
P = \$200	Principal is the original amount of money invested.
r = ?	Rate is unknown.
t = 2 yrs	Time is given in years.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.		
70 = (200)(r)(2)	Multiply the integers on the right side of the equation.		
70 = 400r			
$\frac{70}{400} = \frac{400r}{400}$	Divide by 400 to isolate the variable.		
0.175 = r			
$0.175 \times 100\% = r$	Multiply by 100% to convert the decimal to a percent.		
17.5% = r	For the investment to grow to \$270, a 17.5% interest rate is needed.		

6. How much time will it take an investment of \$3,000 to double in value if the annual simple interest rate is 20%?

Determine the given and unknown values.

$I = \$6000 - \$3000 \\= \$3000$	For the investment to double in value (grow from \$3000 to \$6000), \$3000 must be earned in interest.
<i>P</i> = \$3000	Principal is the original amount of money invested.
r = 20% = 0.20	Divide the rate by 100 to change the percent to a decimal.
t = ?	Time is unknown.

Set up the formula and solve for the unknown.

I = Prt	Replace the variables with the given values.
3000 = (3000)(0.20)(t)	Multiply the integers on the right side of the equation.
3000 = 600t	
$\underline{3000}$ $\underline{-}$ $\underline{600t}$	Divide by 600 to isolate the variable.
600 600	
5 = t	It will take 5 years for the investment to double in value.

<u>REVIEW</u>: SIMPLE INTEREST FORMULA

PRACTICE: Solve each application problem.

- 1. If Sarah borrows \$1,000 for 4 years at a 13% annual simple interest rate, how much interest will Sarah pay?
- 2. What is the simple interest on a loan of \$30,000 for one year if the interest rate is 2.25%?
- 3. What is the interest earned on an account with an initial investment of \$1250 at a 4.5% annual simple interest rate for 6 months?
- 4. Sarah needs \$15,000 for college in 6 years. If she invests \$5000 now, what simple annual rate is necessary for the investment to grow to \$15,000 in 6 years?
- 5. If the annual simple interest rate at a bank is 10%, how many years does Maria need to leave her \$4000 in the bank so that she doubles the value?
- 6. Leroy buys a \$1200 computer. The store gives him credit using simple interest at 20% over 2 years. How much does Leroy pay for his computer?
- 7. Justin bought a new refrigerator for \$1060. The store gave him credit calculated with simple interest at 18% over 2½ years. What is the total amount Justin is paying for the refrigerator?
- 8. Keisha invests \$1500 at a 5% annual simple interest rate. How many years will it take for her investment to grow to \$1875?

<u>Answers</u>:

- 1. \$520

 2. \$675
- 3. \$28.13
- 4. 33.3%

5.	10 years	
6.	\$1680	⊛ € ►
7.	\$1537	
8.	5 years	

SECTION 3.3 SUMMARY Percent			
The number of parts out of 100.			
Convert FROM a Percent: DIVIDE BY 100	Convert TO a Percent: MULTIPLY BY 100%		
Percent \rightarrow Fraction $\frac{P}{100}$	Fraction \rightarrow Percent $\frac{n}{d} \times 100\%$		
<i>Example</i> : Write 8% as a fraction.	<u><i>Example</i></u> : Write $\frac{3}{4}$ as a percent.		
$8\% = \frac{8}{100} = \frac{8^2}{10025} = \frac{2}{25}$	$\frac{3}{4} \times \frac{100\%}{1} = \frac{3}{14} \times \frac{100\%^{25\%}}{1} = 75\%$		
Percent \rightarrow Decimal P \div 100	Decimal \rightarrow Percent Decimal \times 100%		
<i>Example</i> : Write 8% as a decimal.	<i>Example</i> : Write 0.75 as a percent.		
$8\% = 8 \div 100 = 0.08$	$0.75 \times 100\% = 75\%$		
FORMULA METHOD	PROPORTION METHOD		
°of" °is″ PERCENT × WHOLE = PART	$\frac{\text{PERCENT}}{100} = \frac{\text{PART}}{\text{WHOLE}}$		
<i>Example</i> : What percent of 35 is 7?	<i>Example</i> : What percent of 35 is 7?		
What percent of 35 is 7?	What percent of 35 is 7 ?		
$n \times 35 = 7$	Percent Whole Part		
35n = 7	$\frac{\text{PERCENT}}{100} = \frac{\text{PART}}{\text{WHOLE}}$		
$\frac{35n}{35} = \frac{7}{35}$	$\frac{n}{100} = \frac{7}{35}$		
n = 0.2	35n = 700		
$n = 0.2 \times 100\%$	$\frac{35n}{35} = \frac{700}{35}$		
n = 20%	<i>n</i> = 20%		
PART is PERCENT of WHOLE PART = PERCENT × WHOLE	<i>Example</i> : Jake is buying a shirt for \$17. How much sales tax will he pay if the tax rate is 6%?		
Sales Tax = Tax Rate \times Total Sales	Sales $Tax = Tax Rate \times Purchase$		
Raise = Rate Increase \times Current Salary	$T = 0.00 \times 17 T = \$1.02		
SIMPLE INTEREST FORMULA:	<i>I</i> = <i>Prt</i> <u><i>Example</i></u> : What is the simple interest on a loan of \$9000 for 4 years		
I = Interest: Money charged on a loan or ear	rned on a deposit if the interest rate is 7.5%?		
<i>P</i> = Principal: Money borrowed or deposite	1 = 1 1 1		
t = time: Length of loan or investment meas	ured in years $I = (\$9000)(0.075)(4)$ I = \$2700		
	The number of parts out of 100. Convert FROM a Percent: DIVIDE BY 100 Percent \rightarrow Fraction $\frac{P}{100}$ Example: Write 8% as a fraction. $8\% = \frac{8}{100} = \frac{8}{100}\frac{2}{25} = \frac{2}{25}$ Percent \rightarrow Decimal $P \div 100$ Example: Write 8% as a decimal. $8\% = 8 \div 100 = 0.08$ FORMULA METHOD $\frac{vof''}{vof''}$ PERCENT × WHOLE $=$ PART Example: What percent of 35 is 7? What percent of 35 is 7? What percent of 35 is 7? $\frac{35n}{n} = 7$ $\frac{35n}{35} = \frac{7}{35}$ n = 0.2 $n = 0.2 \times 100\%$ n = 20% PART is PERCENT of WHOLE PART = PERCENT × WHOLE Sales Tax = Tax Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Commission = Commission Rate × Total Sales Discount = Percent Off × Original Price Noney borrowed or deposite r = rate: Percent charged on a loan or earner % must be converted to a decimal		

SECTION 3.3 EXERCISES

Percent

Convert from a percent to a fraction.

- 1. Write 19% as a fraction.
- 2. Write 56% as a fraction.
- 3. Write 120% as a fraction.
- 4. Write 2% as a fraction.

Convert from a percent to a decimal.

- 5. Write 29% as a decimal.
- 6. Write 60% as a decimal.
- 7. Write 2% as a decimal.
- 8. Write 320% as a decimal.

Convert from a fraction to a percent.

- 9. Write $\frac{21}{25}$ as a percent.
- 10. Write $\frac{2}{5}$ as a percent.
- 11. Write $\frac{3}{8}$ as a percent.
- 12. Write $1\frac{1}{2}$ as a percent.

Convert from a decimal to a percent.

- 13. Write 0.78 as a percent.
- 14. Write 0.5 as a percent.
- 15. Write 0.014 as a percent.
- 16. Write 1.1 as a percent.

Solve each percent problem.

- 17. What is 30% of 300?
- 18. What is 45% of 68?
- 19. What is 120% of 4000?
- 20. What is 2.25% of 120?
- 21. 12% of 75 is what number?
- 22. 4.5% of 86 is what number?

Solve each percent problem.

- 23. 32 is 80% of what number?
- 24. 15 is 25% of what number?
- 25. 1200 is 1% of what number?
- 26. 36 is $\frac{1}{2}$ % of what number?
- 27. 140% of what number is 133?

Solve each percent problem.

- 28. What percent of 60 is 24?
- 29. What percent of 260 is 65?
- 30. What percent of 78 is 117?
- 31. 3 is what percent of 120?

Solve each application problem.

- 32. Alberto wants to buy a remote control car that costs \$14.50. If the tax rate is 6%, how much will he pay in tax?
- 33. Dave sells \$56,000 worth of advertising in one month. If his commission rate is 8%, how much is his commission check?
- 34. The price tag on a dress is \$110 but Crystal has a coupon for 15% off any item. How much will Crystal save if she uses her coupon on the dress?
- 35. If you get a 3% raise next year, what is the amount of your raise and next year's salary if your salary this year is \$36,500?
- 36. Edward finds a refrigerator listed for \$1349.00 that is marked 20% off. What is the sale price of the refrigerator?
- 37. LaToya has \$95 to purchase a pair of jeans and a shirt. The jeans cost \$45 and the shirt costs \$40. If the tax rate is 7%, does LaToya have enough money to purchase the outfit?
- 38. Eliza earns a commission bonus on every house she sells. Her commission rate is 5% of the sale price of the house. How much does she have to sell a house for in order to earn a commission bonus of \$4500?
- 39. Fran sold \$4,258,000 worth of real estate last year. Her commission earnings were \$63,870. What was her commission rate?
- 40. Calvin gave the waitress a \$27 tip for a \$180 restaurant bill. What percent of the total bill did Calvin leave as the tip?

Solve each application problem.

- 41. A bank charges a 5% annual simple interest rate on loans. How much interest must you pay on a loan of \$700 for 2 years?
- 42. How much interest is earned if you invest \$25,000 for 3 years at a 5.25% annual simple interest rate?
- 43. What is the interest on a \$650 loan with a 10% annual simple interest rate for 18 months?
- 44. How much will Ed pay in total to borrow \$2000 at a 15% annual simple interest rate for 5 years?
- 45. What annual simple interest rate is needed for an initial investment of \$2000 to grow to \$5000 in 10 years?
- 46. What annual simple interest rate is needed in order to double an initial investment of \$1000 in 4 years?
- 47. How long will it take for an initial investment of \$1000 to triple in value if the annual simple interest rate is 8%?
- 48. How long will it take for an initial investment of \$800 to grow to \$1000 if it is invested at a 2.5% annual simple interest rate?
- 49. What amount is the principal in a savings account if \$3000 is earned in interest after 6 years at a 4% annual simple interest rate?
- 50. What amount is the principal in a savings account if \$420 is earned in interest after 8 years at a 3.5% annual simple interest rate?

Answers to Section 3	3.3 Exercises
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1.	$\frac{19}{100}$	25.	120,000
2		26.	7,200
2.	$\frac{14}{25}$	27.	95
3.	$\frac{6}{5}$	28.	40%
4.	$\frac{1}{50}$	29.	25%
-		30.	150%
5.	0.29	31.	2.5%
6.	0.6	32.	\$0.87
7.	0.02	33.	\$4,480
8.	3.2	34.	\$16.50
9. 10.	84% 40%	35.	Amount of raise is \$1,095. Next year's salary is \$37,595.
		36.	\$1,079.20
11.	37.5% OR $37\frac{1}{2}\%$	37.	Yes, the outfit will cost \$90.95.
12.	150%	38.	\$90,000
13.	78%	39.	1.5%
14.	50%	40.	15%
15.	1.4%	41.	\$70
16.	110%	42.	\$3,937.50
17.	90	43.	\$97.50
18.	30.6	44.	\$3,500
19.	4,800	45.	15%
20.	2.7	46.	25%
21.	9	47.	25 years
22.	3.87	48.	10 years
23.	40	49.	\$12,500
24.	60	50.	\$1,500

Sections 1.1 – 3.3

- 1. Simplify $-\frac{3}{4}(28-12x)$.
- 2. Solve 4(5-x) = -6(3x+7)-8.
- 3. Solve $\frac{5}{2}x + \frac{7}{6} = \frac{5}{3}x 3$
- 4. Solve 2x+11 > 5(4+x), graph the solution, and write the solution in interval notation.
- 5. Write an algebraic equation for the word problem. Then solve the equation to answer the question. *The school has 210 computers to place in classrooms. There are 12 middle school classrooms and 15 high school classrooms. The school wants twice as many computers in each high school classroom. How many computers should be placed in each middle school room and how many should be placed in each high school room?*
- 6. The formula to find the sum of the interior angles of a regular polygon is S = 180(n-2) where *n* is the number of sides. Find *n* if S = 900.

7. Solve
$$\frac{4A+B}{3} = C$$
 for *B*.

- 8. Solve $\frac{-14}{n} = \frac{18}{27}$
- 9. Three cups of mix make 14 pancakes. How much mix is needed to make 35 pancakes?
- 10. Convert 6.9 minutes to seconds. Use the conversion fact: 60 seconds (s) = 1 minute (min)

Ansi	wers to Mixed Review		
1.	9 <i>x</i> -21	6.	n = 7
2.	x = -5	7.	B = 3C - 4A
3.	x = -5	8.	n = -21
4.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	9.	7.5 cups
5.	12m + 15(2m) = 210 Middle Schools: 5 computers each High Schools: 10 computers each	10.	414 seconds