

Section 4.2 Objectives

- Determine whether the slope of a graphed line is positive, negative, 0, or undefined.
- Determine the slope of a line given its graph.
- Calculate the slope of a line given the ordered pairs of two points on the line.

SECTION 4.2 Slope of a Line

INTRODUCTION

The steepness of a mountain is sometimes measured by a grade. If you see a sign that says “Mountain Grade 4%,” it means that for every 100 feet traveled horizontally, you ascend or descend 4 feet. The steepness of a mountain can also be measured by the angle at which it goes up or down. A particular mountain might have an incline of 15° . Think of the angle at which you would have to tilt your head looking up from ground level to the top of a mountain. In math, we also need to measure steepness. In this section, we measure the steepness of a line. That measurement is called the slope of the line.

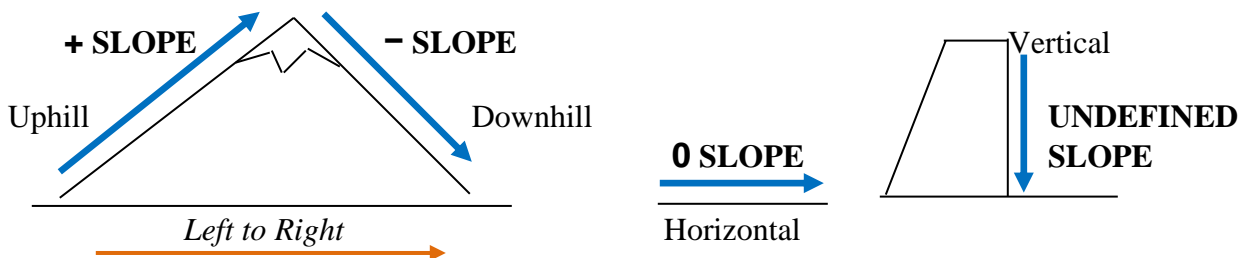
SLOPE

One of the most important properties of a line is its slope. The *slope* of a line is a number that describes how steep the line is. Later in this section you will learn how to determine the number representing the slope of a line.

SLOPE
A number that describes the <i>steepness</i> (or <i>slant</i>) of a line.

The *slope* of a line also describes the general direction of a line. If the *slope* of a line is a positive number (+), then the line looks like the uphill part of a mountain. If the *slope* of a line is a negative number (–), then the line looks like the downhill part of a mountain. When we measure *slope*, it is important to always read from left to right, just like we read a sentence in English. So, always place your finger on the left of the line and trace the line from left to right. If the line slants up, it has a positive *slope*. If the line slants down, it has a negative *slope*.

There are two other types of lines to consider: horizontal lines and vertical lines. The *slope* of a horizontal line is 0 and the *slope* of a vertical line is undefined.

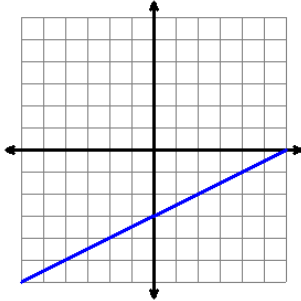


POSITIVE SLOPE	NEGATIVE SLOPE
Uphill	Downhill

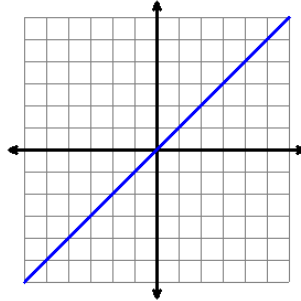
0 SLOPE	UNDEFINED SLOPE
Horizontal	Vertical

EXAMPLES:**1. Slopes of Lines that Slant Up**

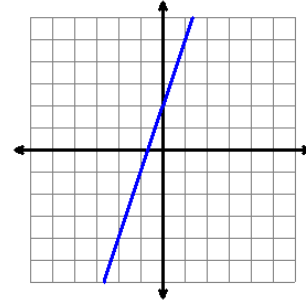
If you place your finger on the left of each line below and trace the line from left to right, you notice that all the lines slant up. Therefore, all the lines have a positive slope.



$$\text{Slope} = \frac{1}{2}$$



$$\text{Slope} = 1$$

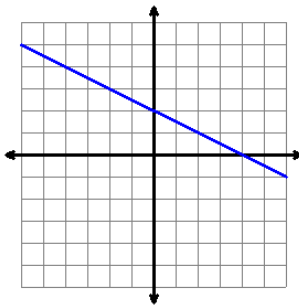


$$\text{Slope} = 3$$

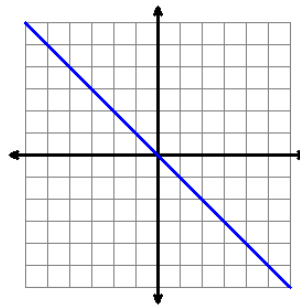
Right now, don't worry about how we determined the numbers for the slopes. You will learn that soon.

2. Slopes of Lines that Slant Down

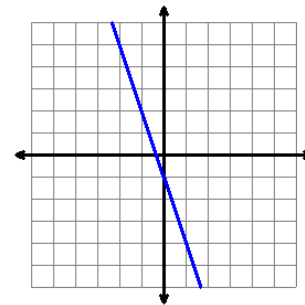
If you place your finger on the left of each line below and trace the line from left to right, you notice that all the lines slant down. Therefore, all the lines have a negative slope.



$$\text{Slope} = -\frac{1}{2}$$

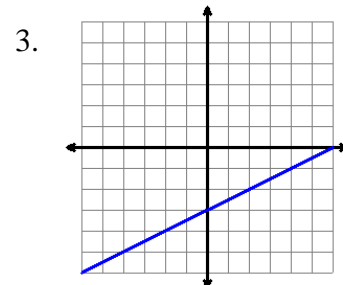
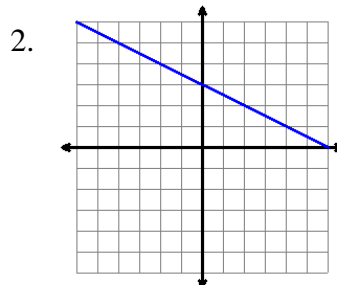
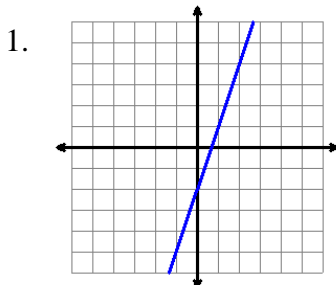


$$\text{Slope} = -1$$



$$\text{Slope} = -3$$

PRACTICE: Determine whether the slope of each of the lines below is positive or negative.

**ANSWERS:**

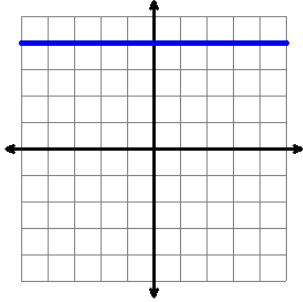
1. positive slope

2. negative slope

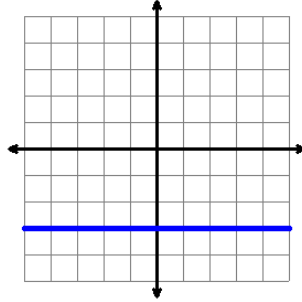
3. positive slope

EXAMPLES:**1. Slopes of Horizontal Lines**

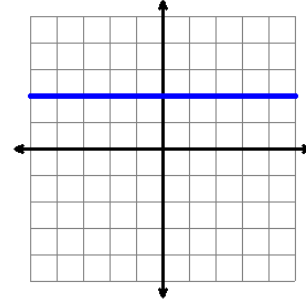
All horizontal lines have a slope of zero. (slope = 0)



Slope = 0



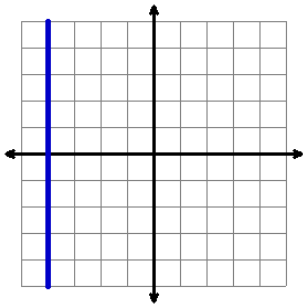
Slope = 0



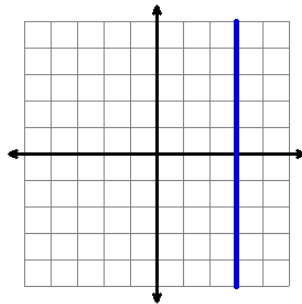
Slope = 0

2. Slopes of Vertical Lines

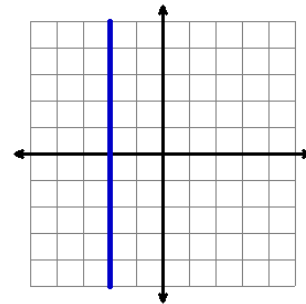
All vertical lines have an undefined slope.



Undefined Slope



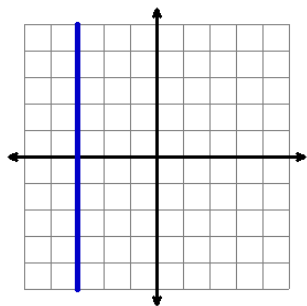
Undefined Slope



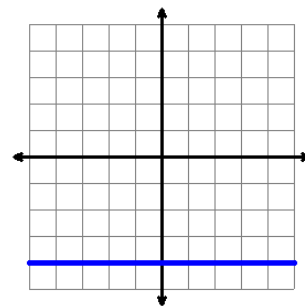
Undefined Slope

PRACTICE: Determine whether the slope of each of the lines below is zero or undefined.

1.





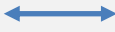

2.

**ANSWERS:**

1. undefined slope

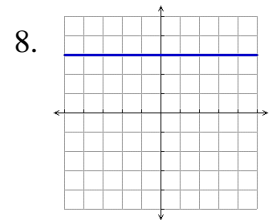
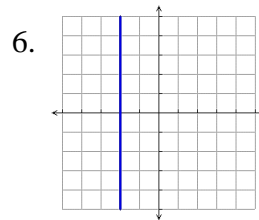
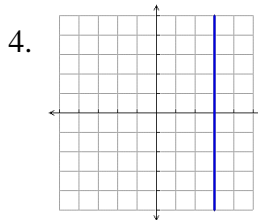
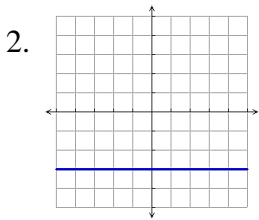
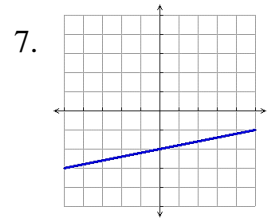
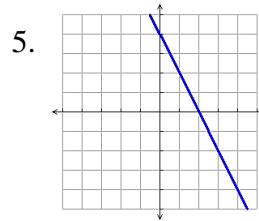
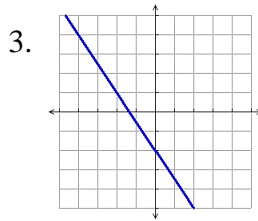
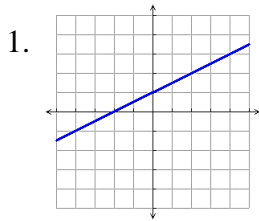
2. slope = 0

The chart and review video below summarize how the slope of a line relates to the direction of the line.

SLOPE OF LINE	DIRECTION OF LINE <i>(from Left to Right)</i>
Positive Number	Slants Up 
Negative Number	Slants Down 
0	Horizontal 
Undefined	Vertical 

REVIEW: SLOPE AND DIRECTION OF A LINE 

PRACTICE: Determine whether the slope of each line is positive, negative, 0, or undefined.



ANSWERS:

- 1. Positive
- 2. 0
- 3. Negative
- 4. Undefined



- 5. Negative
- 6. Undefined
- 7. Positive
- 8. 0

DETERMINING SLOPE

In many of the problems presented so far, the value of the slope of the line was given. For example, $Slope = 3$ was written below the graph of the line. But we told you not to worry about how that number was determined and that you would learn the procedure later. Well, now it is time for you to study this topic. You will learn to determine the number that gives the slope of a line. Two methods will be shown. The first method involves determining the slope using the graph of the line, and the other method involves determining the slope without the graph.

DETERMINING THE SLOPE OF A LINE FROM ITS GRAPH

The slope of a line is simply the ratio of the *rise* to the *run*. The *rise* is the vertical distance traveled and the *run* is the horizontal distance traveled as you move from one point on the line to another point on the line.

To determine the slope of any line from its graph, you first need to identify two points on the line. Any two points you choose will result in the same answer. To find the slope, it is recommended to start at the leftmost point and then determine the *rise* and *run* needed to move to the other point. The *rise* is the number of units you must move up or down, and the *run* is the number of units you must move left or right. Remember that if you move up, the *rise* is positive. And if you move down, the *rise* is negative. The *run* will always be positive as long as you start at the leftmost point.

DETERMINING SLOPE FROM A GRAPH

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{\text{Vertical Distance (Up or Down)}}{\text{Horizontal Distance (Right or Left)}}$$

1. Identify 2 points on the line. (*Points A and B*)
2. Start at the leftmost point. (*Point A*)
3. Determine the **Rise** needed to get to the other point. (*Point B*)

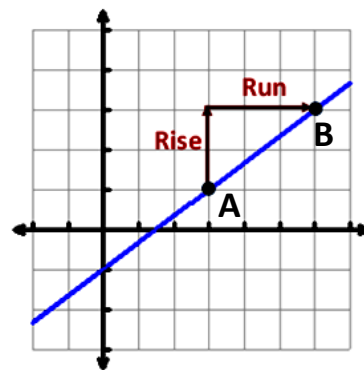
$$\text{RISE} \begin{cases} \text{Up: } + \text{ Number} \\ \text{Down: } - \text{ Number} \end{cases}$$

4. Determine the **Run** needed to get to the other point. (*Point B*)

$$\text{RUN} \begin{cases} \text{Right: } + \text{ Number} \\ \text{Left: } - \text{ Number} \end{cases}$$

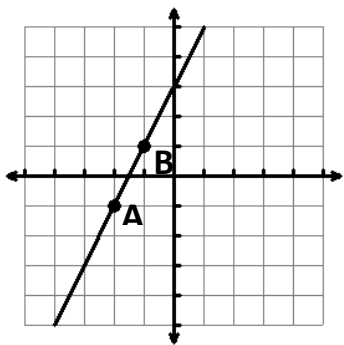
5. Express as a fraction and simplify if possible.

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}}$$



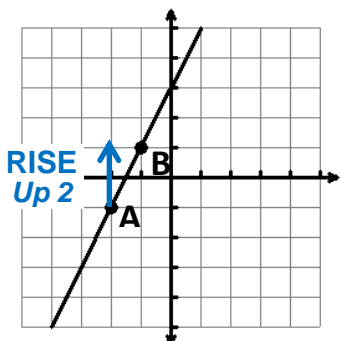
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$$

EXAMPLE 1: Determine the slope of the line.



Choose two points on the line whose coordinates are integers.

Note that points A and B are not the only two points that we could have chosen. Any two points on the line will work.

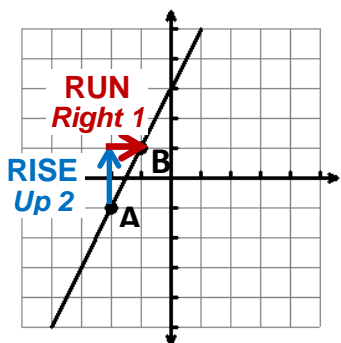


Start at the leftmost point (*Point A*).

Determine the rise by counting the number of units up until you are across from the other point (*Point B*).

We moved 2 units.
And since we moved up, the rise is positive.

Therefore, **Rise = 2**.



Start where you left off at the tip of the upward arrow.

Determine the run by counting the number of units to the right until you are on the other point (*Point B*).

We moved 1 unit.
And since we moved right, the run is positive.

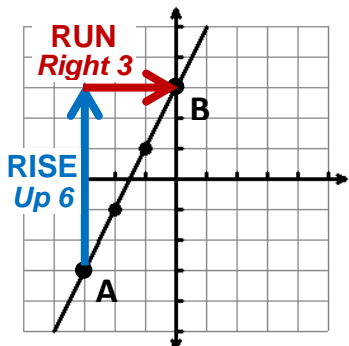
Therefore, **Run = 1**.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2}{1} = 2$$

Express the slope as a fraction and then simplify.

What if we choose different points on the same line? Will we get the same slope value? YES!

If you choose different points on the line, you will obtain a different fraction initially. But once you simplify the fraction, you will see that the answer is the same. We show an example like this below.



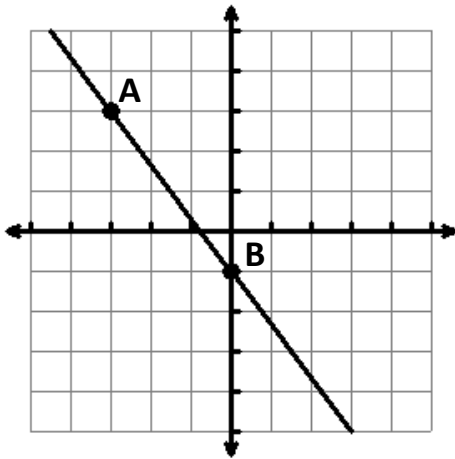
This is the same line as Example 1 above.
But we have chosen new points for A and B.

We start at point A.
We determine that the rise is 6 units up. **Rise = 6**
We determine that the run is 3 units to the right. **Run = 3**

We express this as a fraction and simplify:

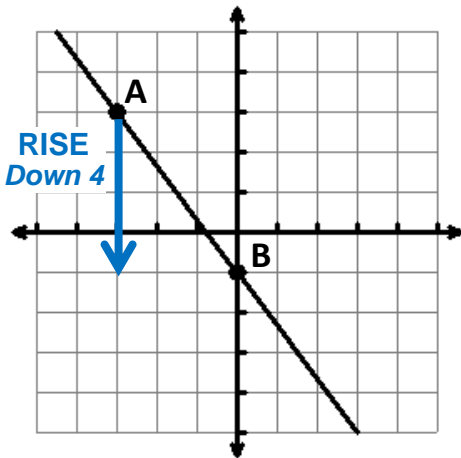
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{6}{3} = 2 \quad \leftarrow \text{The same answer!}$$

EXAMPLE 2: Determine the slope of the line.



Choose two points on the line whose coordinates are integers.

Note that points A and B are not the only two points that we could have chosen. Any two points on the line will work.



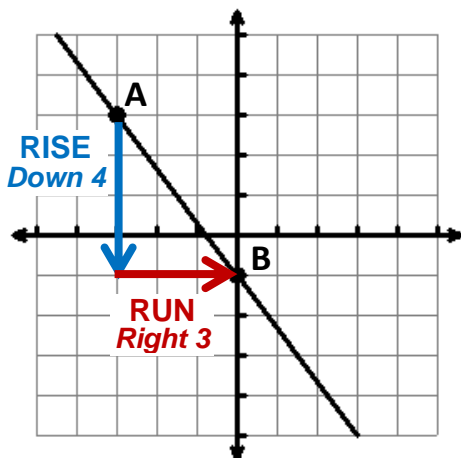
Start at the leftmost point (*Point A*).

Determine the rise by counting the number of units down until you are across from the other point (*Point B*).

We moved 4 units.

And since we moved down, the rise is negative.

Therefore, **Rise** = **-4**.



Start where you left off at the tip of the downward arrow.

Determine the run by counting the number of units to the right until you are on the other point (*Point B*).

We moved 3 units right.

And since we moved right, the run is positive.

Therefore, **Run** = **3**.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-4}{3} = -\frac{4}{3}$$

Express the slope as a fraction. It cannot be simplified, but the negative sign can be moved in front of the fraction.

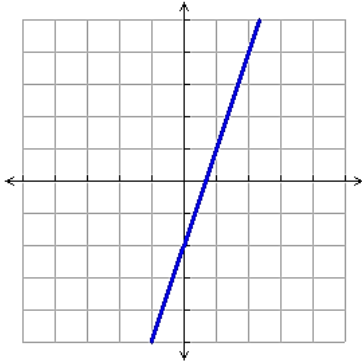
Remember that no matter which two points you choose on a line, you will always get the same slope. Also, remember to write the slope in lowest terms by simplifying the fraction if possible.

REVIEW: DETERMINING SLOPE FROM A GRAPH

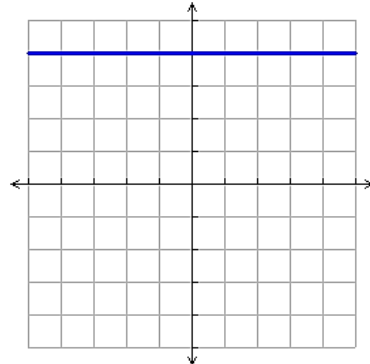
Video 1 Video 2

PRACTICE: Determine the slope of each line. Remember that there are many pairs of points to choose on a line. For a particular line, we may have used a different pair of points than the pair that you select. But as long as you simplify your fractions, you should obtain the same answers shown in the answer section.

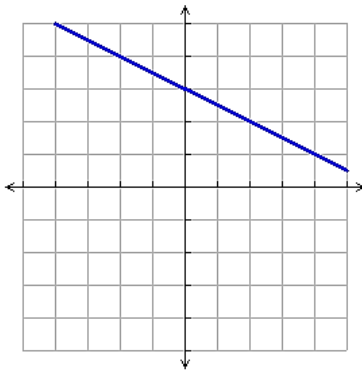
1.



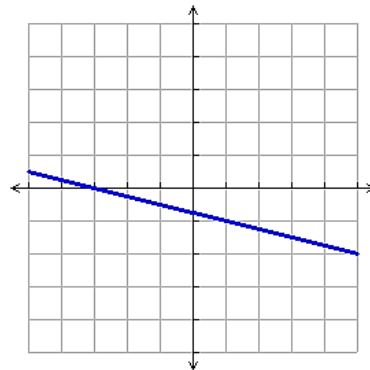
4.



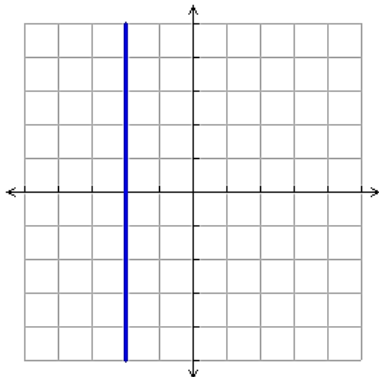
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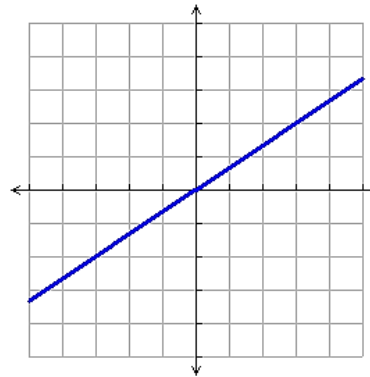
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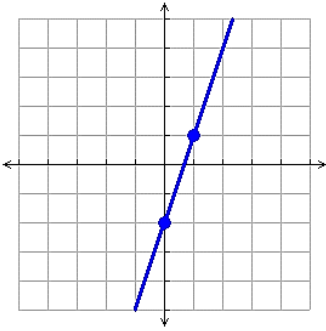


3.

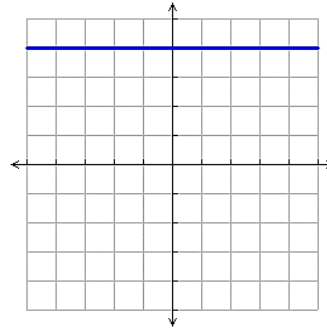


6.

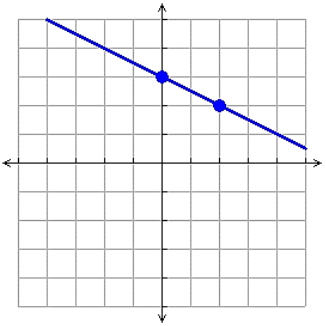


ANSWERS:1. 

$$\text{Slope} = \frac{\text{Up } 3}{\text{Right } 1} = \frac{3}{1} = 3$$

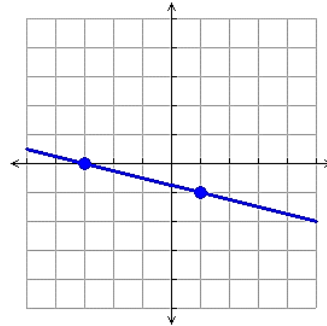
4. 

Slope = 0
because the line is Horizontal

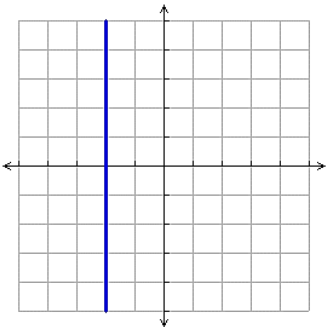
2. 

$$\text{Slope} = \frac{\text{Down } 1}{\text{Right } 2} = -\frac{1}{2}$$

5.

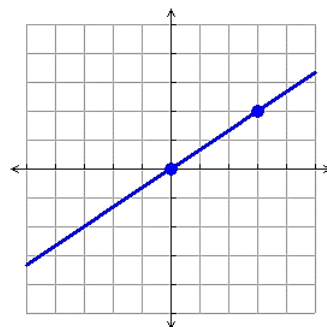


$$\text{Slope} = \frac{\text{Down } 1}{\text{Right } 4} = -\frac{1}{4}$$

3. 

Slope is Undefined
because the line is Vertical

6.



$$\text{Slope} = \frac{\text{Up } 2}{\text{Right } 3} = \frac{2}{3}$$

DETERMINING THE SLOPE OF A LINE FROM TWO POINTS (WITHOUT A GRAPH)

In the last section you were shown the graph of a line on the coordinate plane. You learned to determine the slope of the line by identifying two points on the line and then counting the number of units needed to *rise* and *run* to move from one point to the other.

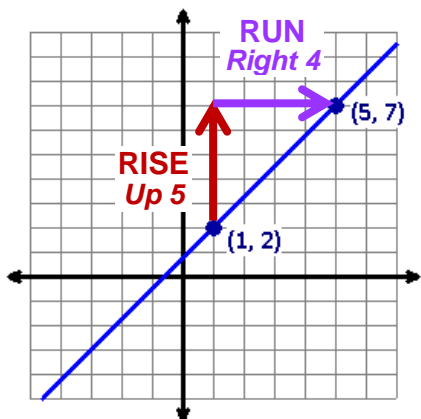
But what if you are not shown the graph of the line? Can the slope still be determined? The answer is yes. In this section we will investigate how two points can be used to determine the slope of a line even if the graph is not given.

The key to the procedure is realizing the following:

- The **rise** is the number of units that we move vertically in the y direction. This is the same as the difference between the y -coordinates of the two points.
- The **run** is the number of units that we move horizontally in the x direction. This is the same as the difference between the x -coordinates of the two points.

Let's illustrate with an example. We will count the rise and run with a graph just as we did before. Then we will calculate the rise and run without a graph and show that the results are the same.

WITH GRAPH



We choose two points on the line.

Then we determine the rise and run by counting units on the graph.

We get the following:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Up } 5}{\text{Right } 4} = \boxed{\frac{5}{4}}$$

WITHOUT GRAPH

$$\begin{array}{cc} (1, 2) & (5, 7) \\ \downarrow \downarrow & \downarrow \downarrow \\ x & y \end{array}$$

$$\text{Rise} = 7 - 2 = 5$$

$$\text{Run} = 5 - 1 = 4$$

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \boxed{\frac{5}{4}}$$

Now, suppose that we know that the two points (1, 2) and (5, 7) are on a line, but we are not given the graph of the line nor a grid on which to plot the points.

To determine the rise, we calculate the difference between the y -coordinates. The answer is 5, the same as the rise shown in the graph above.

To determine the run, we calculate the difference between the x -coordinates. The answer is 4, the same as the run shown in the graph above.

We get the slope as usual using $\frac{\text{rise}}{\text{run}}$. Notice that the result is the same as the slope we got above when the graph was given.

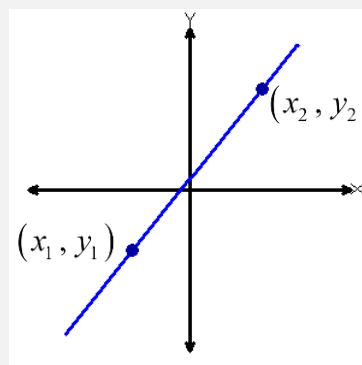
Now we will generalize the process for finding the slope of a line without a graph. As we showed, we only need two points on the line, and any two points will work. We will call the points (x_1, y_1) and (x_2, y_2) . The small numbers in the ordered pairs are called subscripts and are used to distinguish the two ordered pairs from each other. It does not matter which ordered pair is labeled (x_1, y_1) and which is labeled (x_2, y_2) . To get the **rise**, we subtract the y -coordinates. To get the **run**, we subtract the x -coordinates. The slope is the ratio $\frac{\text{rise}}{\text{run}}$. We express these math operations as a formula. The formula and the procedure for finding slope are shown in the boxes below.

SLOPE FORMULA

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

$$\text{Slope} = \frac{\text{Difference in } y\text{-coordinates}}{\text{Difference in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



*Note: It is not necessary to see the graph to compute the slope.
Only the points (ordered pairs) are needed.*

CALCULATING SLOPE FROM TWO POINTS (without a graph)

1. Label the coordinates of one point (x_1, y_1) .

Label the coordinates of the other point (x_2, y_2) .

NOTE: It does not matter which point is labeled as (x_1, y_1) and which is labeled as (x_2, y_2) .

2. Substitute the values of the coordinates into the slope formula: $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$

3. Simplify the fraction if possible.

EXAMPLES: Calculate the slope of the line that passes through each pair of points.

1. (2, 5) and (7, 9)

$$\begin{array}{cc} (2, 5) & (7, 9) \\ \uparrow \uparrow & \uparrow \uparrow \\ x_1 y_1 & x_2 y_2 \end{array}$$

We label the first point (x_1, y_1)
and the second point (x_2, y_2) .

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\begin{array}{c} y_2 \\ \downarrow \\ 9 - 5 \\ \uparrow \\ x_2 \end{array} - \begin{array}{c} y_1 \\ \downarrow \\ 5 \\ \uparrow \\ x_1 \end{array}}{\begin{array}{c} \uparrow \\ x_2 \\ \uparrow \\ x_1 \end{array}} = \frac{4}{5}$$

Substitute the values of the coordinates into the slope formula.
Then simplify.

What if we labeled the second point (x_1, y_1) and the first point (x_2, y_2) ? Let's try it!

$$\begin{array}{cc} (7, 9) & (2, 5) \\ \uparrow \uparrow & \uparrow \uparrow \\ x_1 y_1 & x_2 y_2 \end{array}$$

We reversed the way that we labeled the points.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\begin{array}{c} y_2 \\ \downarrow \\ 5 - 9 \\ \uparrow \\ x_2 \end{array} - \begin{array}{c} y_1 \\ \downarrow \\ 9 \\ \uparrow \\ x_1 \end{array}}{\begin{array}{c} \uparrow \\ x_2 \\ \uparrow \\ x_1 \end{array}} = \frac{-4}{-5} = \frac{4}{5}$$

Substitute the values of the coordinates into the slope formula.
Simplify. Remember that when we divide two negative numbers, we get a positive result.

The final answer is the same as above.

2. (4, -2) and (-5, 1)

$$\begin{array}{cc} (4, -2) & (-5, 1) \\ \uparrow \uparrow & \uparrow \uparrow \\ x_1 y_1 & x_2 y_2 \end{array}$$

We label the first point (x_1, y_1)
and the second point (x_2, y_2) .

$$\begin{aligned} \text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\begin{array}{c} y_2 \\ \downarrow \\ 1 - (-2) \\ \uparrow \\ x_2 \end{array} - \begin{array}{c} y_1 \\ \downarrow \\ -2 \\ \uparrow \\ x_1 \end{array}}{\begin{array}{c} \uparrow \\ x_2 \\ \uparrow \\ x_1 \end{array}} \\ &= \frac{1+2}{-5+4} \\ &= \frac{3}{-9} \\ &= -\frac{1}{3} \end{aligned}$$

Substitute the values of the coordinates into the slope formula.

The formula has subtraction in it. In the numerator, the number following the subtraction sign is a negative number, so we must write both the subtraction sign and the negative sign.

Change each subtraction to addition by adding the opposite.

Simplify to reduce the fraction to lowest terms.

Do not leave the negative sign in the denominator.
Instead move it in front of the fraction.

- 3.
- $(-6, -3)$
- and
- $(-2, 0)$

$$\begin{array}{cc}
 (-6, -3) & (-2, 0) \\
 \uparrow \quad \uparrow & \uparrow \quad \uparrow \\
 x_1 \quad y_1 & x_2 \quad y_2
 \end{array}$$

$$\begin{array}{c}
 \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\begin{array}{c} y_2 \quad y_1 \\ \downarrow \quad \downarrow \\ 0 - (-3) \end{array}}{\begin{array}{c} -2 - (-6) \\ \uparrow \quad \uparrow \\ x_2 \quad x_1 \end{array}} \\
 \\
 = \frac{0+3}{-2+6} \\
 \\
 = \frac{3}{4}
 \end{array}$$

Label the first point (x_1, y_1)
and the second point (x_2, y_2) .

Substitute the values of the coordinates into the slope formula.

The slope formula has subtraction in it. Since the number following each subtraction sign is a negative number, we need to write both the subtraction sign and the negative sign.

Change each subtraction to addition by adding the opposite.

The fraction cannot be simplified.

- 4.
- $(-1, 3)$
- and
- $(8, 3)$

$$\begin{array}{cc}
 (-1, 3) & (8, 3) \\
 \uparrow \quad \uparrow & \uparrow \quad \uparrow \\
 x_1 \quad y_1 & x_2 \quad y_2
 \end{array}$$

$$\begin{array}{c}
 \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\begin{array}{c} y_2 \quad y_1 \\ \downarrow \quad \downarrow \\ 3 - 3 \end{array}}{\begin{array}{c} 8 - (-1) \\ \uparrow \quad \uparrow \\ x_2 \quad x_1 \end{array}} \\
 \\
 = \frac{3-3}{8+1} \\
 \\
 = \frac{0}{9} \\
 \\
 = 0
 \end{array}$$

Label the first point (x_1, y_1)
and the second point (x_2, y_2) .

Substitute the values of the coordinates into the slope formula.

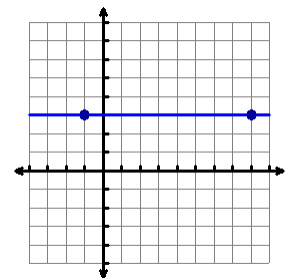
The slope formula has subtraction in it. In the denominator, the number following the subtraction sign is a negative number, so we need to write both the subtraction sign and the negative sign.

Change the subtraction to addition by adding the opposite.

Simplify.

Remember that 0 divided by any nonzero number equals 0.

Note: Whenever the y -coordinates are the same, we will get 0 in the **numerator**, and 0 divided by any nonzero number is 0. If we were to draw the line connecting the two given points, we would get the horizontal line shown to the right. As we saw earlier, the slope of a horizontal line is 0.



5. (5,7) and (5,-4)

$$\begin{matrix} (5, 7) & (5, -4) \\ \uparrow & \uparrow & \uparrow & \uparrow \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

Label the first point (x_1, y_1)
and the second point (x_2, y_2) .

$$\begin{aligned} \text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\overset{y_2}{\downarrow} - \overset{y_1}{\downarrow}}{\underset{x_2}{\uparrow} - \underset{x_1}{\uparrow}} = \frac{-4 - 7}{5 - 5} \\ &= \frac{-4 + (-7)}{5 - 5} \\ &= \frac{-11}{0} \\ &= \text{Undefined} \end{aligned}$$

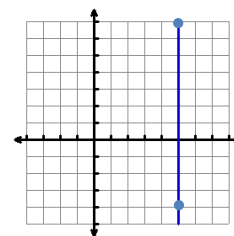
Substitute the values of the coordinates into the slope formula.

Change the subtraction to addition by adding the opposite.

Simplify.

Remember that any nonzero number divided by 0 is *undefined*.

Note: Whenever the x -coordinates are the same, we will get 0 in the **denominator**. Since division by 0 is undefined, the slope of the line is undefined. If we were to draw the line connecting the two given points, we would get the vertical line shown to the right. As we saw earlier, the slope of a vertical line is undefined.



Things to Remember When Using the Slope Formula

- Subtracting a negative number becomes adding a positive number. Example: $2 - (-3) = 2 + 3$
- 0 in the numerator \rightarrow slope = 0
- 0 in the denominator \rightarrow undefined slope

PRACTICE: Calculate the slope of the line that passes through each pair of points.

- | | |
|-----------------------|-----------------------|
| 1. (3,-2) and (-1,-7) | 5. (-3,5) and (-1,-3) |
| 2. (-1,-4) and (5,2) | 6. (-6,6) and (-1,4) |
| 3. (8,1) and (8,-3) | 7. (2,4) and (2,7) |
| 4. (1,-2) and (6,-2) | 8. (4,9) and (-5,9) |

ANSWERS:

- | | |
|---|--|
| 1. $\frac{-7 - (-2)}{-1 - 3} = \frac{-5}{-4} = \frac{5}{4}$ 🎬 | 5. $\frac{-3 - 5}{-1 - (-3)} = \frac{-3 - 5}{-1 + 3} = \frac{-8}{2} = -4$ |
| 2. $\frac{2 - (-4)}{5 - (-1)} = \frac{6}{6} = 1$ 🎬 | 6. $\frac{4 - 6}{-1 - (-6)} = \frac{4 + -6}{-1 + 6} = \frac{-2}{5} = -\frac{2}{5}$ |
| 3. $\frac{-3 - 1}{8 - 8} = \frac{-4}{0} = \text{undefined slope}$ 🎬 | 7. $\frac{7 - 4}{2 - 2} = \frac{3}{0} = \text{undefined slope}$ |
| 4. $\frac{-2 - (-2)}{6 - 1} = \frac{0}{5} = 0$ 🎬 | 8. $\frac{9 - 9}{-5 - 4} = \frac{0}{-9} = 0$ |

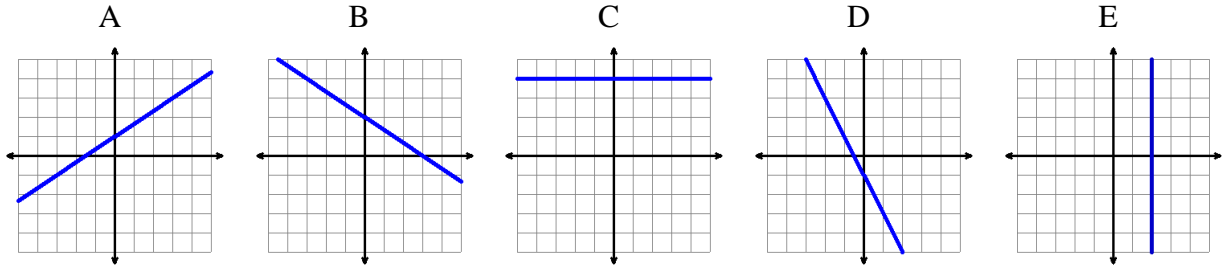
SECTION 4.2 SUMMARY

Slope of a Line

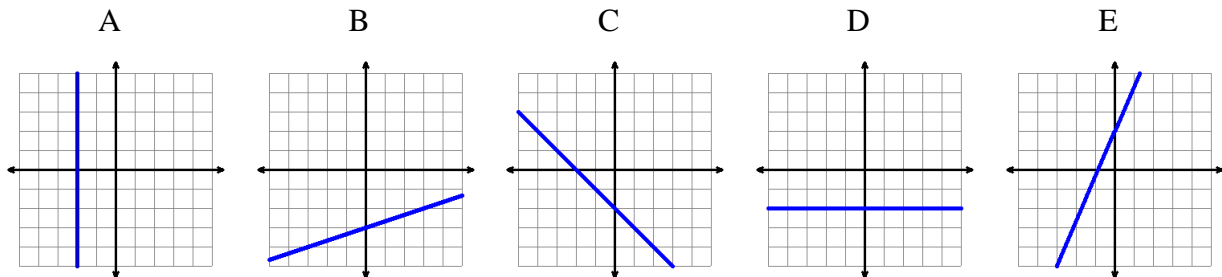
SECTION 4.2 SUMMARY											
Slope of a Line											
SLOPE	<p style="text-align: center;">Slope – a number that describes the <i>steepness</i> (or <i>slant</i>) of a line</p> <table border="1" style="width: 100%; border-collapse: collapse; margin: 10px auto;"> <thead> <tr> <th style="width: 50%; text-align: center;">Slope of Line</th> <th style="width: 50%; text-align: center;">Direction of Line (<i>from Left to Right</i>)</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Positive Number</td> <td style="text-align: center;">Slants Up </td> </tr> <tr> <td style="text-align: center;">Negative Number</td> <td style="text-align: center;">Slants Down </td> </tr> <tr> <td style="text-align: center;">0</td> <td style="text-align: center;">Horizontal </td> </tr> <tr> <td style="text-align: center;">Undefined</td> <td style="text-align: center;">Vertical </td> </tr> </tbody> </table>	Slope of Line	Direction of Line (<i>from Left to Right</i>)	Positive Number	Slants Up	Negative Number	Slants Down	0	Horizontal	Undefined	Vertical
Slope of Line	Direction of Line (<i>from Left to Right</i>)										
Positive Number	Slants Up										
Negative Number	Slants Down										
0	Horizontal										
Undefined	Vertical										
DETERMINING THE SLOPE OF A LINE FROM ITS GRAPH	<div style="text-align: center; margin-bottom: 10px;"> $\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{\text{Vertical Distance (Up or Down)}}{\text{Horizontal Distance (Right or Left)}}$ </div> <ol style="list-style-type: none"> 1. Identify 2 points on the line. (<i>Points A and B</i>) <i>Example:</i> Determine the slope of the line. 2. Start at the <u>leftmost</u> point. (<i>Point A</i>) 3. Count how many units you have to move to get to the other point (<i>Point B</i>): <ol style="list-style-type: none"> a. Count the Rise (<i>Vertical Distance</i>) <div style="margin-left: 40px;"> $\text{RISE} \begin{cases} \text{Up: } + \text{ Number} \\ \text{Down: } - \text{ Number} \end{cases}$ </div> b. Count the Run (<i>Horizontal Distance</i>) <div style="margin-left: 40px;"> $\text{RUN} \begin{cases} \text{Right: } + \text{ Number} \\ \text{Left: } - \text{ Number} \end{cases}$ </div> 4. Express the slope as a fraction $\frac{\text{RISE}}{\text{RUN}}$ and simplify if possible. <div style="text-align: right; margin-top: 10px;"> $\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Down } 2}{\text{Right } 2} = \frac{-2}{2} = -1$ </div>										
DETERMINING THE SLOPE OF A LINE FROM TWO POINTS	<ol style="list-style-type: none"> 1. Label the coordinates of one point (x_1, y_1) and the coordinates of the other point (x_2, y_2). <u>Note:</u> It does not matter which point is labeled as (x_1, y_1) and which is labeled as (x_2, y_2). 2. Substitute the values of the coordinates into the slope formula: $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$. <u>Note:</u> Remember to include any negative signs along with the subtraction sign. <i>Example:</i> $5 - (-2)$ 3. Simplify the fraction if possible. <u>Note:</u> $\frac{0}{\text{number}} = 0$ and $\frac{\text{number}}{0} = \text{undefined}$ (Do the division on your calculator if you forget these.) <p style="margin-top: 10px;"><u>Example:</u> Find the slope of the line that passes through $(3, -5)$ and $(-1, 9)$.</p> <div style="margin-top: 10px; text-align: center;"> <table style="display: inline-table; vertical-align: middle;"> <tr> <td style="text-align: center;">$(3, -5)$</td> <td style="text-align: center;">$(-1, 9)$</td> <td style="padding: 0 20px;">$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-5)}{-1 - 3} = \frac{9 + 5}{-1 + -3} = \frac{14}{-4} = -\frac{7}{2}$</td> </tr> <tr> <td style="text-align: center;">↑ ↑</td> <td style="text-align: center;">↑ ↑</td> <td></td> </tr> <tr> <td style="text-align: center;">x_1 y_1</td> <td style="text-align: center;">x_2 y_2</td> <td></td> </tr> </table> </div>	$(3, -5)$	$(-1, 9)$	$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-5)}{-1 - 3} = \frac{9 + 5}{-1 + -3} = \frac{14}{-4} = -\frac{7}{2}$	↑ ↑	↑ ↑		x_1 y_1	x_2 y_2		
$(3, -5)$	$(-1, 9)$	$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-5)}{-1 - 3} = \frac{9 + 5}{-1 + -3} = \frac{14}{-4} = -\frac{7}{2}$									
↑ ↑	↑ ↑										
x_1 y_1	x_2 y_2										

SECTION 4.2 EXERCISES

Slope of a Line



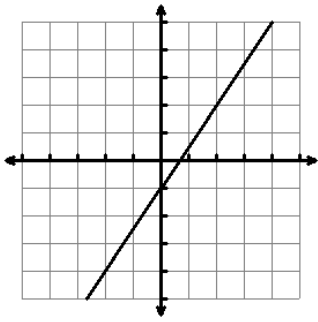
1. Which of the lines above have positive slope?
2. Which of the lines above have negative slope?
3. Which of the lines above have zero slope?
4. Which of the lines above have undefined slope?



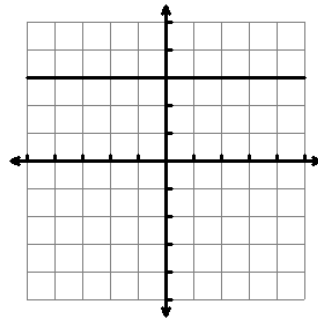
5. Which of the lines above have negative slope?
6. Which of the lines above have zero slope?
7. Which of the lines above have undefined slope?
8. Which of the lines above have positive slope?

Determine the slope of each graphed line.

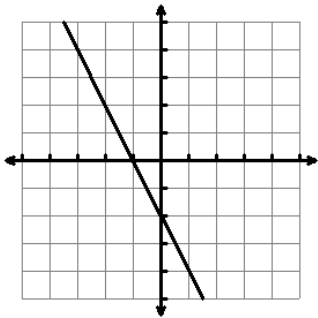
9.



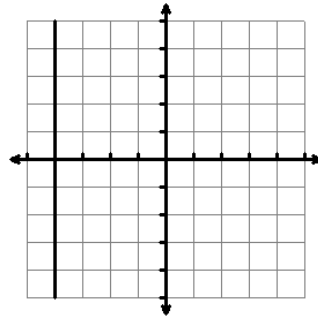
13.



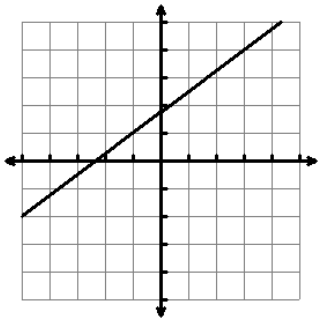
10.



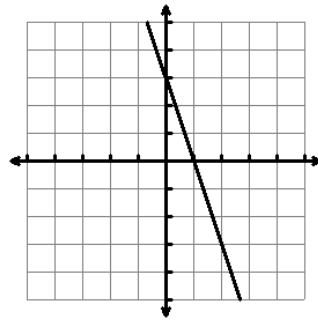
14.



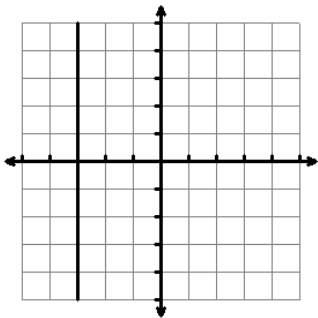
11.



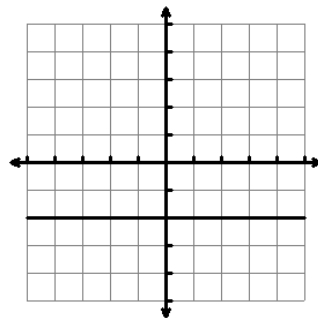
15.



12.



16.



Determine the slope of the line that passes through each pair of points.

17. $(3, 8)$ and $(9, -2)$

21. $(7, 4)$ and $(7, 6)$

18. $(1, 2)$ and $(1, 7)$

22. $(5, 8)$ and $(2, 8)$

19. $(-1, 4)$ and $(3, 4)$

23. $(-2, 6)$ and $(5, -8)$

20. $(-7, -2)$ and $(-3, 10)$

24. $(7, 3)$ and $(-4, -6)$

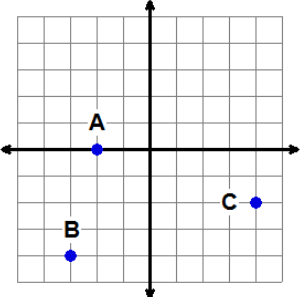
Answers to Section 4.2 Exercises

- | | |
|-------------------|--------------------|
| 1. A | 13. 0 |
| 2. B and D | 14. Undefined |
| 3. C | 15. -3 |
| 4. E | 16. 0 |
| 5. C | 17. $-\frac{5}{3}$ |
| 6. D | 18. Undefined |
| 7. A | 19. 0 |
| 8. B and E | 20. 3 |
| 9. $\frac{3}{2}$ | 21. Undefined |
| 10. -2 | 22. 0 |
| 11. $\frac{3}{4}$ | 23. -2 |
| 12. Undefined | 24. $\frac{9}{11}$ |

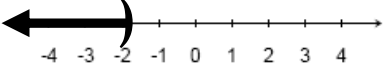
Mixed Review

Sections 1.1 – 4.2

- Solve $\frac{3}{4}(8x-4) > 10x+5$, graph the solution, and write the solution in interval notation.
- Solve $ab-c=d$ for b .
- If 15 parts are defective in a shipment of 525 parts, how many parts can be expected to be defective in a shipment of 1750 parts?
- Convert 92 grams to kilograms. Use the conversion fact: 1 kilogram (kg) = 1000 grams (g)
- What percent of 408 is 61.2?
- Sean has 4.5% deducted from each paycheck for taxes. If Sean's gross salary on his last paycheck was \$960, what was his net salary?
- Write the ordered pair for each of the points shown on the graph to the right.


- Determine if the ordered pair $(-6, -1)$ is a solution of the equation $5x-9y=-21$.
- Determine the unknown coordinate so that $(_, 7)$ is a solution of $8x+6y=10$.
- Determine the x and y intercepts of the graph of $-4x+3y=24$.

Answers to Mixed Review

- $x < -2$
 $(-\infty, -2)$

- $b = \frac{c+d}{a}$
- 50
- 0.092 kg
- 15%
- \$916.80
- A $(-2, 0)$
 B $(-3, -4)$
 C $(4, -2)$
- Yes
- $(-4, 7)$
- x -intercept: $(-6, 0)$
 y -intercept: $(0, 8)$