

### Section 4.3 Objectives

- Write the equation of a line given its graph.
- Write the equation of a line given its slope and y-intercept.
- Write the equation of a line given its slope and one point on the line.
- Write the equation of a line given two points on the line.
- Write a linear equation to represent a real world application involving rate of change.

## SECTION 4.3

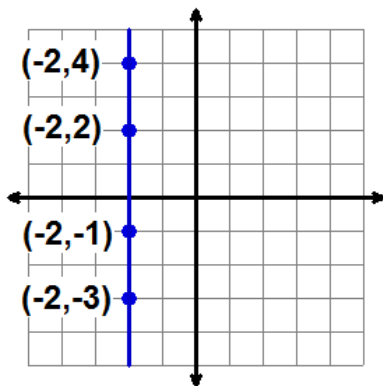
## Equation of a Line

## INTRODUCTION

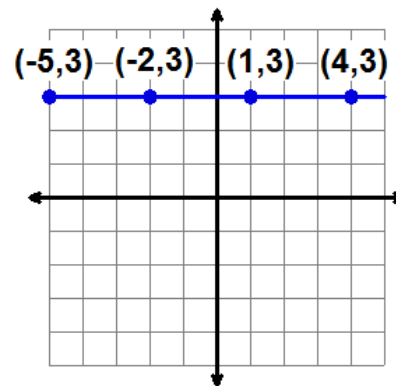
You have already studied the slope of a line as a numerical measurement of the steepness of a line. You learned how to determine slope for all types of lines, including horizontal and vertical lines. This involved the study of positive slopes, negative slopes, zero slopes, and undefined slopes. In this section, you will do more than just compute the slope of a given line. You will write an *equation* that describes all of the points on the line. Remember, a point is on a line if it is a solution of the equation that represents the line. Once again, you will examine all types of lines. We begin with horizontal and vertical lines.

## EQUATIONS OF HORIZONTAL AND VERTICAL LINES

There is an easy way to write equations that represent horizontal and vertical lines. Determining the equation for a line is finding a rule (equation or formula) that all points on the line satisfy. Look at the horizontal and vertical lines below.



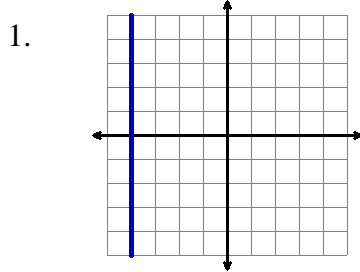
Notice that all of the points on this vertical line have  **$x$ -coordinate** =  $-2$ .  
The  $y$ -coordinate can be anything, but we know for sure that the  $x$ -coordinate is  $-2$ .  
So, the equation of the line is  $x = -2$ .



Notice that all of the points on this horizontal line have  **$y$ -coordinate** =  $3$ .  
The  $x$ -coordinate can be anything, but we know for sure that the  $y$ -coordinate is  $3$ .  
So, the equation of the line is  $y = 3$ .

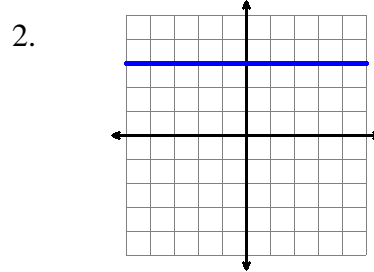
| VERTICAL LINE                             |  | HORIZONTAL LINE                           |  |
|---|--|---|--|
| All points have the same $x$ -coordinate. |  | All points have the same $y$ -coordinate. |  |
| Equation of Line:<br>$x = a$              |  | Equation of Line:<br>$y = b$              |  |

**EXAMPLES:** Write the equation of each line.



All points on the vertical line have  $x$ -coordinate  $= -4$ .

**Answer:**  $x = -4$



All points on the horizontal line have  $y$ -coordinate  $= 3$ .

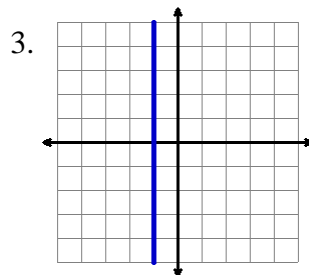
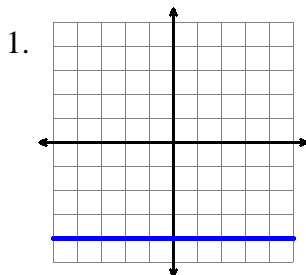
**Answer:**  $y = 3$

If you are not shown the graph of a vertical or horizontal line, you can still write the equation of the line if you know at least two points on the line.

**EXAMPLES:** Write the equation of the line that passes through the two given points.

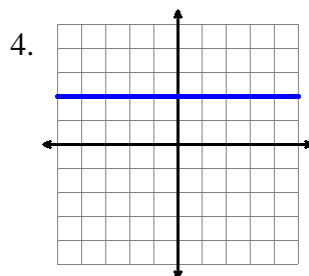
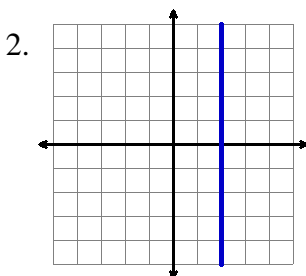
- $(3, -6)$  and  $(-2, -6)$  Since both  $y$ -coordinates are  $-6$ , the equation is  $y = -6$ .
- $(4, 3)$  and  $(4, 8)$  Since both  $x$ -coordinates are  $4$ , the equation is  $x = 4$ .
- $(-5, 0)$  and  $(-1, 0)$  Since both  $y$ -coordinates are  $0$ , the equation is  $y = 0$ .

**PRACTICE:** Write the equation of each line.



5. The line that passes through  $(5, 1)$  and  $(5, -4)$ .

6. The line that passes through  $(-3, -3)$  and  $(8, -3)$ .



7. The line that passes through  $(5, 1)$  and  $(-1, 1)$ .

8. The line that passes through  $(-2, -7)$  and  $(-2, 7)$ .

**ANSWERS:**

1.  $y = -4$



2.  $x = 2$



3.  $x = -1$

4.  $y = 2$

5.  $x = 5$



6.  $y = -3$



7.  $y = 1$

8.  $x = -2$

## EQUATIONS OF LINES THAT ARE NOT HORIZONTAL OR VERTICAL

You just learned how to determine the equation of a horizontal line. Recall that a horizontal line has a slope of 0. You also learned how to determine the equation of a vertical line. Recall that a vertical line has an undefined slope.

Now you will learn how to determine the equations of lines that are neither horizontal nor vertical. These lines will have either a positive slope or a negative slope. The equations of these lines can be written in two ways. One way is the Standard form  $Ax + By = C$  which you saw in a previous section. Here in this section you will be writing equations of lines in another form called Slope-Intercept form.

### SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

The equation of a line can be written in the form  $y = mx + b$ . This form of a linear equation is appropriately called *slope-intercept form* since  $m$  represents the *slope* of the line, and  $b$  represents the *y-intercept* of the line. We should point out that horizontal lines, but *not* vertical lines, can also be written in this form.

| SLOPE-INTERCEPT FORM OF A LINEAR EQUATION |  |
|---|--|
| $y = mx + b$                              | $m =$ slope of line<br>$b =$ y-intercept of line |

We can write the *slope-intercept form* of the equation of a line if we are shown the graph of the line. We can even write the *slope-intercept form* of the equation of a line without seeing the graph! Either way, we just need to identify two values: the *slope* of the line and the *y-intercept* of the line. We begin by showing how to find these two values if we are given the graph of the line. Later we will show how to find the values if we are not given the graph.

### IDENTIFYING THE SLOPE AND Y-INTERCEPT FROM A GRAPH

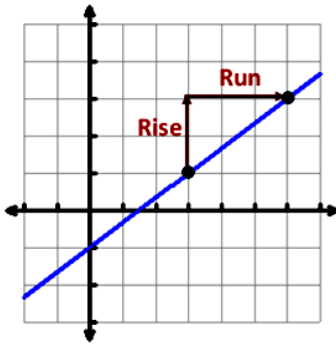
To write the equation of a line in *slope-intercept form*, we need the *slope* and the *y-intercept*. These two values can be determined easily if the graph of the line is given. Previously, you learned how to find the slope of a line from a graph. We will present a brief review of the procedure here. As for the y-intercept, that is fully explained below. Once you determine the slope and the y-intercept, writing the equation of the line is very straightforward.

### Slope of a Line

Review how to find the slope of a line given its graph. Study the definition and example below.

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{\text{Vertical Distance (Up or Down)}}{\text{Horizontal Distance (Right or Left)}}$$

$$\text{RISE} \begin{cases} \text{Up: } + \text{ Number} \\ \text{Down: } - \text{ Number} \end{cases} \quad \text{RUN} \begin{cases} \text{Right: } + \text{ Number} \\ \text{Left: } - \text{ Number} \end{cases}$$

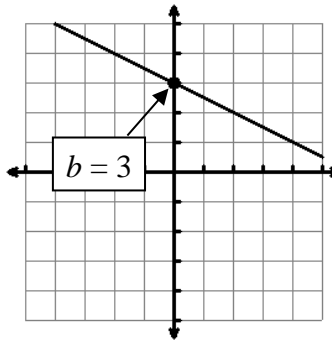


$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Up } 2}{\text{Right } 3} = \frac{2}{3}$$

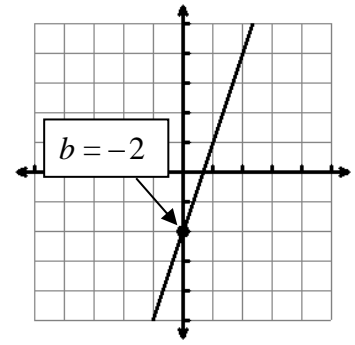
### y-Intercept of a Line

The y-intercept is the y-coordinate of the point where the line intersects (crosses) the y-axis. It is often easy to identify the y-intercept on the graph of a line if the y-intercept value is an integer. Study the examples below.

*NOTE: The y-intercept has nothing to do with the slope.*



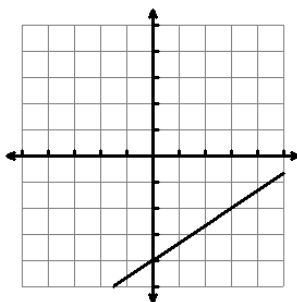
The y-intercept is  $b = 3$   
because the line intersects the y-axis at 3.



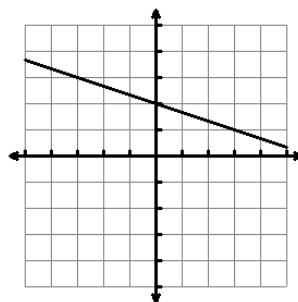
The y-intercept is  $b = -2$   
because the line intersects the y-axis at  $-2$ .

**PRACTICE:** Determine the y-intercept for each line.

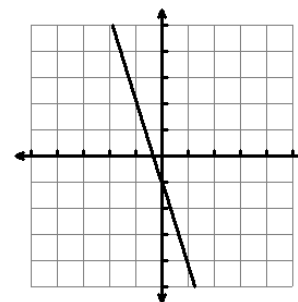
1.



2.



3.



**ANSWERS:**

1.  $b = -4$

2.  $b = 2$

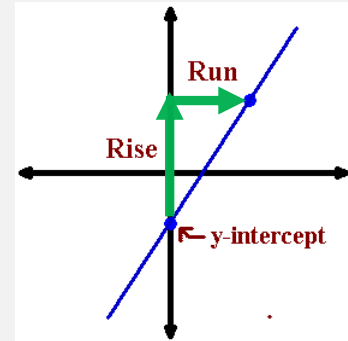
3.  $b = -1$

## WRITING THE EQUATION OF A LINE USING ITS GRAPH

Once the slope and the  $y$ -intercept of a line are determined from the graph, writing the equation of the line in slope-intercept form is very straightforward. We write the values for  $m$  and  $b$  in the equation  $y = mx + b$ . The entire process is described below and in the examples that follow.

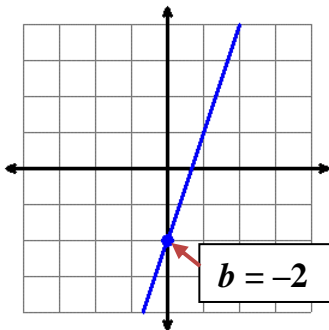
### WRITING THE SLOPE-INTERCEPT EQUATION OF A LINE USING ITS GRAPH

1. Determine the  $y$ -intercept:
  - a. Identify where the line crosses the  $y$ -axis.
  - b. This is the  $b$  value.
2. Determine the slope:
  - a. Locate another point on the line.
  - b. Count the *rise* and *run* from the  $y$ -intercept to the other point.
  - c. Express the slope as the fraction  $\frac{\text{rise}}{\text{run}}$  in simplest form.
  - d. This is the  $m$  value.
3. Substitute the values for  $m$  and  $b$  into the equation  $y = mx + b$ .



**EXAMPLE 1:** Write the equation of the line shown below.

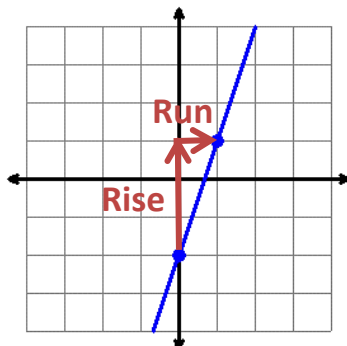
**y-intercept**



We see that the line intersects the  $y$ -axis at  $-2$ .

Therefore,  $b = -2$ .

**Slope**



Locate another point on the line. (*Use a point at the corner of a square on the grid.*)

Now that we have two points, we determine the rise and run.

Start at the  $y$ -intercept.

Count 3 units up to get the rise.

Count 1 unit right to get the run.

The slope is:  $m = \frac{\text{Rise}}{\text{Run}} = \frac{3}{1} = 3$

Therefore,  $m = 3$ .

**Equation of Line**

$$y = mx + b$$

$$\downarrow \quad \downarrow$$

$$y = 3x + -2$$

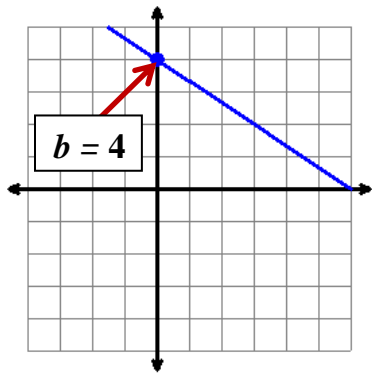
$$y = 3x - 2$$

Substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

**Answer:** The equation of the line is  $y = 3x - 2$

**EXAMPLE 2:** Write the equation of the line shown below.

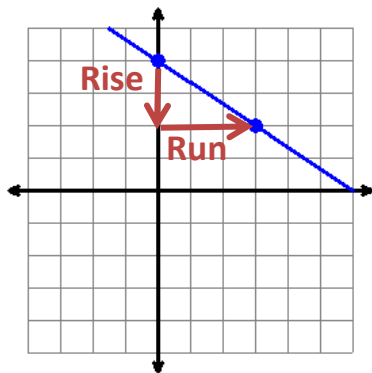
**y-intercept**



We see that the line intersects the y-axis at 4.

Therefore,  $b = 4$ .

**Slope**



Locate another point on the line. (Use a point at the corner of a square on the grid.)

Now that we have two points on the line, we determine the rise and run.

Start at the y-intercept.

Count 2 units down to get the rise. Express this as  $-2$ .

Count 3 units right to get the run.

$$\text{The slope is: } m = \frac{\text{Rise}}{\text{Run}} = \frac{-2}{3} = -\frac{2}{3}$$

Therefore,  $m = -\frac{2}{3}$ .

**Equation of Line**

$$y = mx + b$$

$$\downarrow \quad \downarrow$$

$$y = -\frac{2}{3}x + 4$$

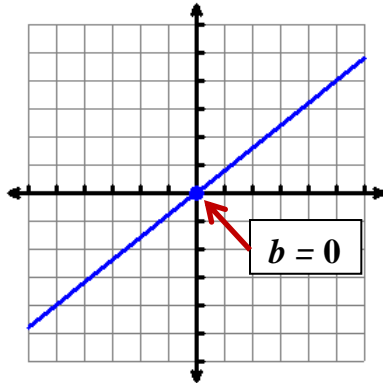
$$y = -\frac{2}{3}x + 4$$

Substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

**Answer:** The equation of the line is  $y = -\frac{2}{3}x + 4$

**EXAMPLE 3:** Write the equation of the line shown below.

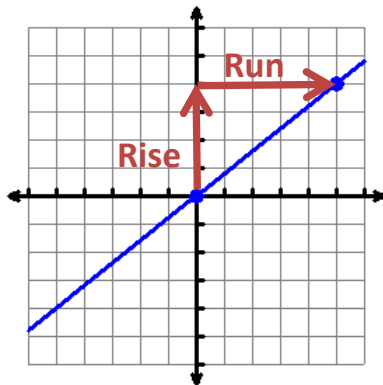
**y-intercept**



The line intersects the y-axis at 0.

Therefore,  $b = 0$ .

**Slope**



Locate another point on the line.

Determine the rise and the run.

Start at the y-intercept.

Count 4 units up to get the rise.

Count 5 units right to get the run.

The slope is:  $m = \frac{\text{Rise}}{\text{Run}} = \frac{4}{5}$

Therefore,  $m = \frac{4}{5}$ .

**Equation of Line**

$$y = m x + b$$

$$y = \frac{4}{5} x + 0$$

$$y = \frac{4}{5} x$$

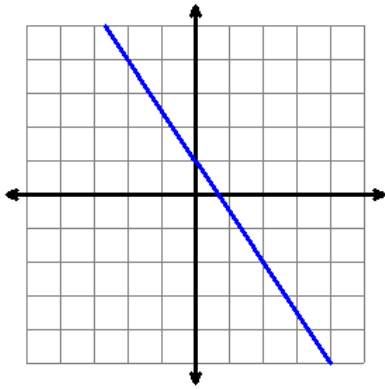
Substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

**Answer:** The equation of the line is  $y = \frac{4}{5} x$

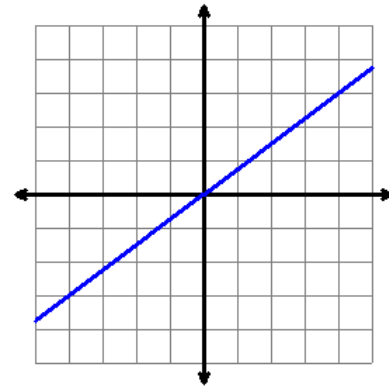


**PRACTICE:** Write the equation of each line graphed below.

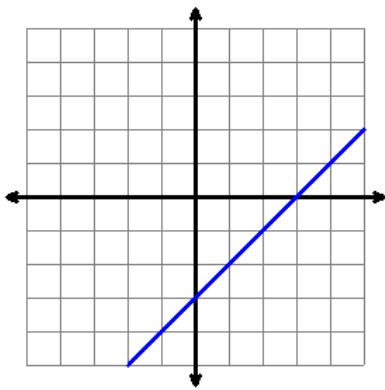
1.



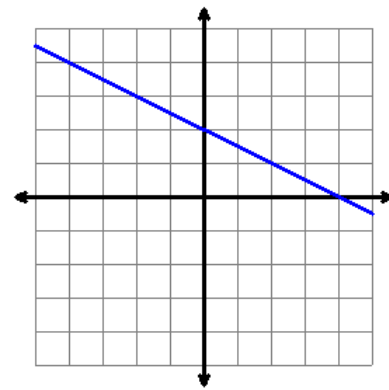
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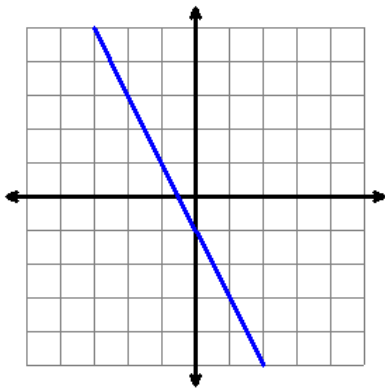
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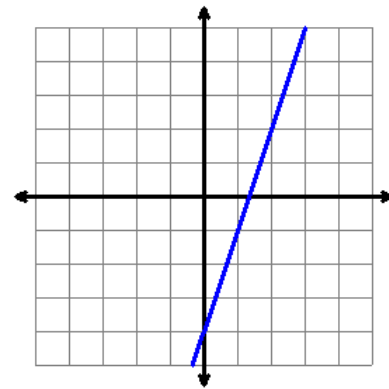
5.



3.



6.



**ANSWERS:**

1.  $y = -\frac{3}{2}x + 1$



4.  $y = \frac{3}{4}x$

2.  $y = x - 3$



5.  $y = -\frac{1}{2}x + 2$

3.  $y = -2x - 1$

6.  $y = 3x - 4$

## WRITING THE EQUATION OF A LINE WITHOUT THE GRAPH

Sometimes we need to write the equation of a line without seeing its graph. We could use the information that we are given to draw the graph, but then our answer might be wrong if our drawing is not precise – especially if the  $y$ -intercept is not an integer. However, there is an algebraic way to determine the equation of a line. Just as before, the method relies on getting the slope and the  $y$ -intercept of the line and then substituting those values in the equation  $y = mx + b$ .

### WRITING THE SLOPE-INTERCEPT EQUATION OF A LINE WITHOUT THE GRAPH

1. If the slope is not given, calculate it using the slope formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .
2. If the  $y$ -intercept is not given:
  - a. Substitute the following values into  $y = mx + b$ :
    - the slope value  $m$
    - the values of  $x$  and  $y$  from a given point
  - b. Then solve the equation for  $b$ .
3. Rewrite the equation  $y = mx + b$ , substituting in just the values for  $m$  and  $b$ . The result is the equation of the line.

**EXAMPLE 1:** Write the equation of the line with slope  $\frac{2}{5}$  and  $y$ -intercept  $-7$ .

**Slope:**

$$m = \frac{2}{5}$$

It is given that the slope is  $\frac{2}{5}$ .

**$y$ -intercept:**

$$b = -7$$

It is also given that the  $y$ -intercept is  $-7$ .

**Equation of Line:**

$$y = mx + b$$

$$\quad \downarrow \quad \downarrow$$

$$y = \frac{2}{5}x + -7$$

Substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

$$y = \frac{2}{5}x - 7$$

**Answer:** The equation of the line is  $y = \frac{2}{5}x - 7$ .

**EXAMPLE 2:** Write the equation of the line with slope  $-4$  and that passes through the point  $(3, -2)$ .

**Slope:**

$$m = -4$$

It is given that the slope is  $-4$ .

**y-intercept:**

$$b = ?$$

The y-intercept is not given, but we can calculate it using the information we know.

$$(3, -2)$$

$$\begin{array}{cc} \uparrow & \uparrow \\ x & y \end{array}$$

In addition to the slope, we know that one point on the line is  $(3, -2)$ . Recall that an ordered pair gives an  $x$ -coordinate and a  $y$ -coordinate.

$$y = mx + b$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \end{array}$$

$$-2 = -4(3) + b$$

$$-2 = -12 + b$$

$$\begin{array}{cc} +12 & +12 \\ \hline \end{array}$$

$$10 = b$$

Substitute the values for  $x$ ,  $y$ , and  $m$  into the slope-intercept equation. Since the only unknown value left in the equation is  $b$ , we can solve for its value.

Multiply  $-4$  and  $3$  to get  $-12$ .

Add  $12$  to both sides.

Now we have the value of the y-intercept.

**Equation of Line:**

$$y = mx + b$$

$$\begin{array}{cc} \downarrow & \downarrow \end{array}$$

$$y = -4x + 10$$

Last, we substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

**Answer:** The equation of the line is  $y = -4x + 10$ .

**Check:**

$$y = -4x + 10$$

$$\begin{array}{cc} \downarrow & \downarrow \end{array}$$

$$-2 = -4(3) + 10$$

$$-2 = -12 + 10$$

$$-2 = -2 \quad \checkmark$$

Check the answer by substituting the  $x$  and  $y$  coordinates of the given point into the linear equation we just found.

Simplify.

Since the result is a true statement, the equation of the line is correct.

**EXAMPLE 3:** Write the equation of the line that passes through the points  $(1, 3)$  and  $(4, 9)$ .

**Slope:**

$$m = ?$$

The slope is not given, but we can calculate it using the other information in the problem.

$$\begin{array}{c} (1, 3) \\ \uparrow \uparrow \\ (x_1, y_1) \end{array} \text{ and } \begin{array}{c} (4, 9) \\ \uparrow \uparrow \\ (x_2, y_2) \end{array}$$

The problem gives two points on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Therefore, we can use the slope formula to calculate the slope.

$$m = \frac{9 - 3}{4 - 1} = \frac{6}{3} = 2$$

Substitute the values from the ordered pairs into the slope formula, then simplify.

$$\boxed{m = 2}$$

Now we have the value of the slope.

**y-intercept:**

$$b = ?$$

The y-intercept is not given, but we can calculate it using the information we know.

$$m = 2 \quad \begin{array}{c} (1, 3) \\ \uparrow \uparrow \\ x \quad y \end{array}$$

We will use the slope that we just found and **either** of the two points that were given in the problem. We choose to use the first point.

$$\begin{array}{l} y = m x + b \\ \downarrow \downarrow \downarrow \\ 3 = 2(1) + b \\ 3 = 2 + b \\ \underline{-2 \quad -2} \end{array}$$

Substitute the values for  $x$ ,  $y$ , and  $m$  into the slope-intercept equation. Since the only unknown value left in the equation is  $b$ , we can solve for its value.

$$\boxed{1 = b}$$

Now we have the value of the y-intercept.

**Equation of Line:**

$$\begin{array}{l} y = m x + b \\ \downarrow \quad \downarrow \\ y = 2x + 1 \end{array}$$

Last, we substitute the values for  $m$  and  $b$  in the slope-intercept form for the equation of a line.

**Answer:** The equation of the line is  $\boxed{y = 2x + 1}$ .

**Check:**

$$\begin{array}{cc} \underline{(1, 3)} & \underline{(4, 9)} \\ y = 2x + 1 & y = 2x + 1 \\ \downarrow \quad \downarrow & \downarrow \quad \downarrow \\ 3 = 2(1) + 1 & 9 = 2(4) + 1 \\ 3 = 2 + 1 & 9 = 8 + 1 \\ 3 = 3 \checkmark & 9 = 9 \checkmark \end{array}$$

Substitute each of the given points into the equation of the line that you just found.

Simplify.

Since both points are solutions of the equation, the equation of the line is correct!

**EXAMPLE 4:** Write the equation of the line that passes through the points  $(-4, 3)$  and  $(-2, -4)$ .

**Slope:**

$$m = ?$$

The slope is not given, but we can calculate it.

$$\begin{array}{ccc} (-4, 3) & & (-2, -4) \\ \uparrow \uparrow & \text{and} & \uparrow \uparrow \\ (x_1, y_1) & & (x_2, y_2) \end{array}$$

The problem gives two points on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, we can use the slope formula to calculate the slope.

$$m = \frac{-4-3}{-2-(-4)} = \frac{-4-3}{-2+4} = \frac{-7}{2}$$

Substitute the values from the ordered pairs into the slope formula, then simplify.

$$\boxed{m = -\frac{7}{2}}$$

Now we have the value of the slope.

**y-intercept:**

$$b = ?$$

The y-intercept is not given, but we can calculate it using the information we know.

$$m = -\frac{7}{2} \quad \begin{array}{c} (-4, 3) \\ \uparrow \uparrow \\ x \quad y \end{array}$$

We will use the slope that we just found and **either** of the two points that were given in the problem. We choose to use the first point.

$$\begin{array}{r} y = m x + b \\ \downarrow \quad \downarrow \quad \downarrow \\ 3 = \left(-\frac{7}{2}\right)(-4) + b \end{array}$$

Substitute the values for  $x$ ,  $y$ , and  $m$  into the slope-intercept equation. Since the only unknown value left in the equation is  $b$ , we can solve for its value.

$$3 = \left(-\frac{7}{2}\right)\left(-\frac{4}{1}\right) + b$$

$$\begin{array}{r} 3 = 14 + b \\ -14 \quad -14 \\ \hline \end{array}$$

$$\boxed{-11 = b}$$

Now we have the value of the y-intercept.

**Equation of Line:**

$$\begin{array}{l} y = m x + b \\ \quad \downarrow \quad \downarrow \\ y = -\frac{7}{2} x + (-11) \\ y = -\frac{7}{2} x - 11 \end{array}$$

Last, we substitute the values for  $m$  and  $b$  in the slope-intercept form of the equation of a line.

**Answer:** The equation of the line is  $\boxed{y = -\frac{7}{2}x - 11}$ .

**PRACTICE:** Write the equation of each line described below.

1. Write the equation of the line with slope  $\frac{2}{3}$  and y-intercept  $-5$ .
2. Write the equation of the line that has slope  $-2$  and that passes through the point  $(4, -3)$ .
3. Write the equation of the line that has slope  $\frac{1}{2}$  and that passes through the point  $(8, -1)$ .
4. Write the equation of the line that has slope  $4$  and that passes through the point  $(-5, -6)$ .
5. Write the equation of the line that has slope  $-\frac{2}{3}$  and that passes through the point  $(-9, 2)$ .
6. Write the equation of the line that passes through the points  $(0, 4)$  and  $(6, 1)$ .
7. Write the equation of the line that passes through the points  $(-1, -3)$  and  $(-2, -6)$ .
8. Write the equation of the line that passes through the points  $(7, 2)$  and  $(10, 8)$ .
9. Write the equation of the line that passes through the points  $(-3, -5)$  and  $(-1, 3)$ .
10. Write the equation of the line that passes through the points  $(-2, 6)$  and  $(-8, 9)$ .

**ANSWERS:**

- |                            |   |                             |   |
|----------------------------|---|-----------------------------|---|
| 1. $y = \frac{2}{3}x - 5$  |  | 6. $y = -\frac{1}{2}x + 4$  |  |
| 2. $y = -2x + 5$           |  | 7. $y = 3x$                 |  |
| 3. $y = \frac{1}{2}x - 5$  |  | 8. $y = 2x - 12$            |   |
| 4. $y = 4x + 14$           |   | 9. $y = 4x + 7$             |   |
| 5. $y = -\frac{2}{3}x - 4$ |   | 10. $y = -\frac{1}{2}x + 5$ |   |

## APPLICATIONS OF LINEAR EQUATIONS

In this section, you studied the slope-intercept form of the equation of a line:  $y = mx + b$ . Now we will consider some practical uses of linear equations. We will show how the slope and y-intercept are meaningful in real life applications.

The slope of a line is a measure of how much the y-values change (the rise) divided by how much the x-values change (the run) for any two points on the line. In application problems, slope refers to how much one quantity changes in relation to how much another quantity changes. When *slope* is used to describe changes in real, measureable quantities (such as time, money, population, etc.), it is called the *rate of change*.

The y-intercept of a line is the value of y where the graph crosses the y-axis. In other words, it is the value of y when  $x = 0$ . In application problems, the *y-intercept* can be interpreted as the *initial (starting) value*.

| APPLICATIONS OF LINEAR EQUATIONS |   |
|----------------------------------|---|
| $y = mx + b$                     | $m = \text{slope} = \text{rate of change}$<br>$b = \text{y-intercept} = \text{initial value}$ |

**EXAMPLES:** Complete each application problem.

- Janine decides to start giving an allowance to her son Adam. She starts by giving him \$10 now and then she will give him \$25 every two weeks. Write an equation that shows the relationship between how many weeks have passed and how much money Adam has.

### Variables

$x$  = the number of weeks that have passed  
 $y$  = the amount of money Adam has

Define variables to represent the related quantities.

### $b$ = $y$ -Intercept = Initial $y$ Value

When  $x = 0$  (for 0 weeks),  
 $y = 10$  (for the \$10 that Adam has).

The  $y$ -intercept is the initial  $y$  value.

In this case, the  $y$ -intercept is the initial amount of money that Adam was given at week 0.

So,  $\boxed{b = 10}$

### $m$ = Slope = Rate of Change

$$\begin{aligned} m &= \frac{\text{amount } y \text{ changes}}{\text{amount } x \text{ changes}} \\ &= \frac{\text{amount } \mathbf{money} \text{ changes}}{\text{amount } \mathbf{weeks} \text{ change}} \\ &= \frac{25}{2} \end{aligned}$$

Slope is the amount  $y$  changes divided by the amount  $x$  changes.

Since  $y$  represents the money and  $x$  represents the weeks, we get the money divided by the weeks.

Adam gets \$25 every 2 weeks.  
 So, the rate of change is 25 divided by 2.

So,  $\boxed{m = \frac{25}{2}}$

### Equation of Line

$$\begin{aligned} y &= mx + b \\ &\quad \downarrow \quad \downarrow \\ y &= \frac{25}{2}x + 10 \end{aligned}$$

Substitute the  $m$  and  $b$  values in the slope-intercept form of a linear equation.

This is the equation showing the relationship between the weeks that passed and the money that Adam has.

**NOTE:** Once a linear equation is written, it can be used further. For instance, suppose we want to know how much money Adam has at week 12. We can express this as the question: when  $x = 12$ ,  $y = ?$  This can be solved by substituting in the  $x$  value and solving for  $y$ :

$$y = \frac{25}{2}x + 10 = \frac{25}{2}(12) + 10 = \frac{25}{\cancel{1} \cancel{2}} \left( \frac{\cancel{12}^6}{1} \right) + 10 = 150 + 10 = 160$$

Adam will have \$160 dollars at week 12.

2. A phone company charges \$3 for every gigabyte of data that is used and charges \$5 as a service fee. Write an equation that shows the relationship between how many gigabytes are used and how much money is charged.

### Variables

$x$  = the number of gigabytes used  
 $y$  = the amount of money charged

Define variables to represent the related quantities.

### $b$ = $y$ -Intercept = Initial $y$ Value

When  $x = 0$  (for 0 gigabytes),  
 $y = 5$  (for the \$5 service fee).

The  $y$ -intercept is the initial  $y$  value.

In this case, the  $y$ -intercept is the initial amount of money charged even if 0 gigabytes are used.

So,  $b = 5$

### $m$ = Slope = Rate of Change

$$\begin{aligned} m &= \frac{\text{amount } y \text{ changes}}{\text{amount } x \text{ changes}} \\ &= \frac{\text{amount } \mathbf{money} \text{ changes}}{\text{amount } \mathbf{gigabytes} \text{ change}} \\ &= \frac{3}{1} \end{aligned}$$

Slope is the amount  $y$  changes divided by the amount  $x$  changes.

Since  $y$  represents the money and  $x$  represents the gigabytes, we get the money divided by the gigabytes.

The phone company charges \$3 for every 1 gigabyte. So, the rate of change is 3 divided by 1.

So,  $m = 3$

### Equation of Line

$$\begin{aligned} y &= mx + b \\ &\quad \downarrow \quad \downarrow \\ y &= 3x + 5 \end{aligned}$$

Substitute the  $m$  and  $b$  values in the slope-intercept form of a linear equation.

This is the equation showing the relationship between how many gigabytes are used and the money charged.

**PRACTICE:** Complete each application problem.

1. A musician gets paid \$1000 for each concert he performs. He also gets paid \$360 for every 5 minutes he performs during a concert. Write an equation that shows the relationship between how much money  $y$  the musician gets paid for a concert and how many minutes  $x$  he performs during a concert.
2. A person has a meal of a sandwich and potato chips. Each potato chip is 20 calories and the sandwich is 250 calories. Write an equation that shows the relationship between how many calories  $y$  the person consumes and the number of potato chips  $x$  the person eats.

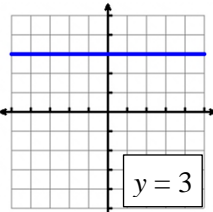
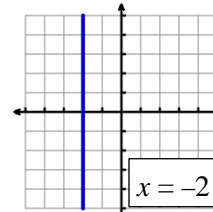
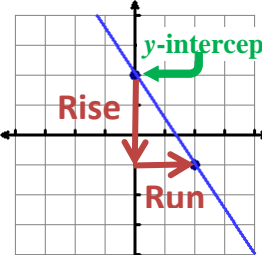
### ANSWERS:

1.  $y = 72x + 1000$
2.  $y = 20x + 250$



## SECTION 4.3 SUMMARY

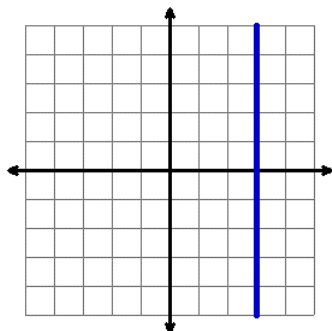
### Equation of a Line

|   |   |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
|---|---|--|----------------|--------------------|--------------|--------------|--------------------|----------------------------|----------------|--|-----------------|--|---------|--|
| <b>LINEAR EQUATION</b>  | <p><b>Slope-Intercept Form of a Linear Equation:</b> <math>y = mx + b</math></p> <p><math>m</math> = slope of the line      <math>b</math> = y-intercept of line (where the line crosses the y-axis)</p>  |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| <b>WRITING THE EQUATION OF A LINE WHEN THE GRAPH IS GIVEN</b>     | <p><b>Horizontal Lines</b></p> <p>All points on the line have the same y-coordinate.</p> <p>Equation: <math>y = b</math></p>   | <p><b>Vertical Lines</b></p> <p>All points on the line have the same x-coordinate.</p> <p>Equation: <math>x = a</math></p>    |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
|   | <p>NOTE: If you think you might get the variables reversed for the horizontal and vertical lines, then write the coordinates of a few points, and then you will see which variable remains constant.</p>  |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
|   | <p><b>Lines NOT Horizontal Or Vertical</b></p> <ol style="list-style-type: none"> <li>On the graph, identify the y-intercept (<math>b</math>).</li> <li>Use the graph to identify the slope (<math>m = \frac{\text{rise}}{\text{run}}</math>) <ul style="list-style-type: none"> <li>Count the rise and run from the y-intercept to another point on the line.</li> </ul> </li> <li>Substitute the values for <math>m</math> and <math>b</math> into the equation <math>y = mx + b</math>.</li> </ol>   | <p><i>Example:</i> Write the equation of the line below.</p>  <p><math>b = 2</math></p> $m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Down } 3}{\text{Right } 2} = \frac{-3}{2}$ $y = mx + b$ $y = -\frac{3}{2}x + 2$   |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| <b>WRITING THE EQUATION OF A LINE WHEN THE GRAPH IS NOT GIVEN</b> | <p><i>Are either the x or y coordinates repeated in the ordered pairs that are given?</i></p> <p><b>YES → Line is Horizontal or Vertical</b></p> <p>Horizontal → same y-coordinates<br/>Vertical → same x-coordinates</p> <p><b>NO → Line is NOT Horizontal or Vertical</b></p> <ol style="list-style-type: none"> <li>If the slope is not given, calculate it using the slope formula <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math>.</li> <li>If the y-intercept is not given, rewrite <math>y = mx + b</math>, substituting in the value of <math>m</math> and the values of <math>x</math> and <math>y</math> from either given point. Then solve for <math>b</math>.</li> <li>Rewrite the equation <math>y = mx + b</math> and substitute in just the values for <math>m</math> and <math>b</math>.</li> </ol> | <p><i>Examples:</i> Write the equation of the line passing through the points.</p> <p><math>(2, -5)</math> <math>(-1, -5)</math> → Equation: <math>y = -5</math></p> <p><math>(4, 7)</math> <math>(4, -9)</math> → Equation: <math>x = 4</math></p> <p><i>Example:</i> Write the equation of the line passing through the points <math>(2, -3)</math> and <math>(-3, 7)</math>.</p> $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-3)}{-3 - 2} = \frac{7 + 3}{-3 - 2} = \frac{10}{-5} = -2$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td><math>y = mx + b</math></td> <td><math>y = mx + b</math></td> </tr> <tr> <td><math>\downarrow</math></td> <td><math>\downarrow</math></td> </tr> <tr> <td><math>7 = (-2)(-3) + b</math></td> <td><math>7 = -2x + 1</math></td> </tr> <tr> <td><math>7 = 6 + b</math></td> <td></td> </tr> <tr> <td><math>\frac{-6}{-6}</math></td> <td></td> </tr> <tr> <td><math>1 = b</math></td> <td></td> </tr> </table> | $y = mx + b$   | $y = mx + b$       | $\downarrow$ | $\downarrow$ | $7 = (-2)(-3) + b$ | $7 = -2x + 1$              | $7 = 6 + b$    |  | $\frac{-6}{-6}$ |  | $1 = b$ |  |
| $y = mx + b$  | $y = mx + b$  |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| $\downarrow$  | $\downarrow$  |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| $7 = (-2)(-3) + b$  | $7 = -2x + 1$   |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| $7 = 6 + b$   |   |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| $\frac{-6}{-6}$   |   |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| $1 = b$   |   |  |                |                    |              |              |                    |                            |                |  |                 |  |         |  |
| <b>APPLICATIONS OF LINEAR EQUATIONS</b>                           | <p><math>y = mx + b</math>    <math>m</math> = slope = rate of change    <math>b</math> = y-intercept = initial value</p> <p><i>Example:</i> The technician charged \$140 for a 3.5 hour repair job. He added on a \$75 travel fee. Write an equation that shows the relationship between the total billed and the hours on the job.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td rowspan="2" style="font-size: 3em; vertical-align: middle;">}</td> <td><math>x = \text{hours}</math></td> <td><math>b = 75</math></td> <td><math>y = mx + b</math></td> </tr> <tr> <td><math>y = \text{cost}</math></td> <td><math>m = \frac{140}{3.5} = 40</math></td> <td><math>y = 40x + 75</math></td> </tr> </table>  |  | }              | $x = \text{hours}$ | $b = 75$     | $y = mx + b$ | $y = \text{cost}$  | $m = \frac{140}{3.5} = 40$ | $y = 40x + 75$ |  |                 |  |         |  |
| }   | $x = \text{hours}$  | $b = 75$   |                | $y = mx + b$       |              |              |                    |                            |                |  |                 |  |         |  |
|   | $y = \text{cost}$   | $m = \frac{140}{3.5} = 40$   | $y = 40x + 75$ |                    |              |              |                    |                            |                |  |                 |  |         |  |

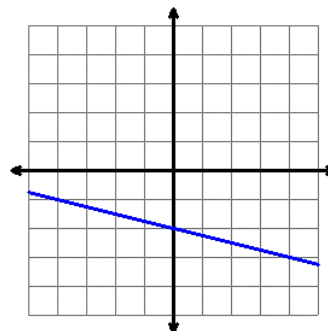
**SECTION 4.3 EXERCISES****Equation of a Line**

Write the equation of each graphed line.

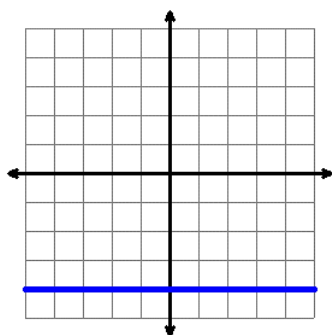
1.



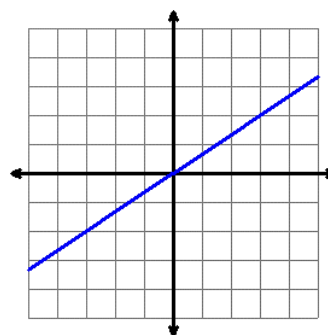
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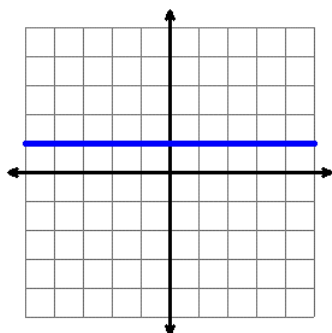
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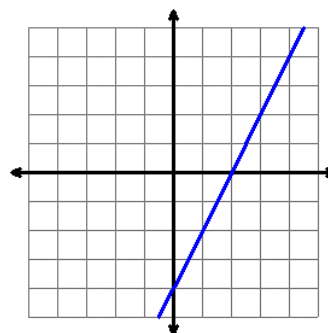
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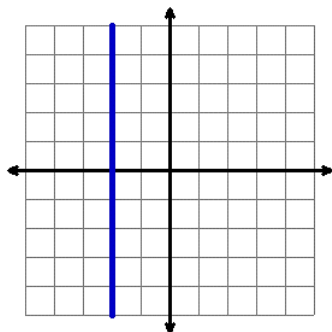
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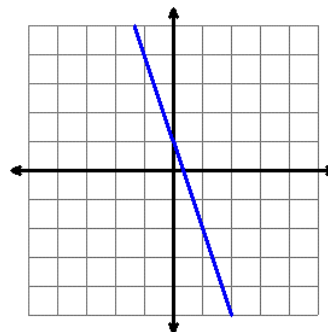
7.



4.



8.



Use the information given to write the equation of each line.

9. Write the equation of the line with slope  $-4$  and  $y$ -intercept  $8$ .
10. Write the equation of the line with slope  $\frac{3}{7}$  and  $y$ -intercept  $-10$ .
11. Write the equation of the line that has slope  $5$  and that passes through the point  $(-2, -6)$ .
12. Write the equation of the line that has slope  $-3$  and that passes through the point  $(-4, 5)$ .
13. Write the equation of the line that has slope  $-\frac{3}{4}$  and that passes through the point  $(8, -16)$ .
14. Write the equation of the line that has slope  $\frac{3}{5}$  and that passes through the point  $(-10, -6)$ .
15. Write the equation of the line that passes through the points  $(1, 5)$  and  $(1, -6)$ .
16. Write the equation of the line that passes through the points  $(7, -3)$  and  $(4, -3)$ .
17. Write the equation of the line that passes through the points  $(9, 4)$  and  $(-1, 4)$ .
18. Write the equation of the line that passes through the points  $(-5, 2)$  and  $(-5, -8)$ .
19. Write the equation of the line that passes through the points  $(6, 0)$  and  $(-2, 8)$ .
20. Write the equation of the line that passes through the points  $(-1, -5)$  and  $(2, 7)$ .
21. Write the equation of the line that passes through the points  $(-6, 1)$  and  $(2, -3)$ .
22. Write the equation of the line that passes through the points  $(-1, 3)$  and  $(0, 6)$ .
23. Write the equation of the line that passes through the points  $(4, 6)$  and  $(-4, -4)$ .

Write the equation of a line to represent each application problem.

24. A plumber charges \$100 for a service call to go to a person's house. He also charges \$105 for every 15 minutes of work. Write an equation that shows the relationship between how much money the plumber charges  $y$  and how many minutes the plumber takes to complete the job  $x$ .
25. When a tree was first planted, its diameter was 3 inches. Over the past 2 years, the diameter increased by 1 inch. Write an equation that shows the relationship between the tree's diameter  $y$  and the number of years  $x$  since the tree was planted.

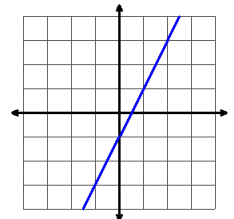
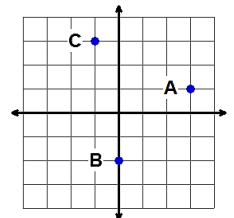
## Answers to Section 4.3 Exercises

- |                            |                              |                             |
|----------------------------|------------------------------|-----------------------------|
| 1. $x = 3$                 | 10. $y = \frac{3}{7}x - 10$  | 18. $x = -5$                |
| 2. $y = -4$                | 11. $y = 5x + 4$             | 19. $y = -x + 6$            |
| 3. $y = 1$                 | 12. $y = -3x - 7$            | 20. $y = 4x - 1$            |
| 4. $x = -2$                | 13. $y = -\frac{3}{4}x - 10$ | 21. $y = -\frac{1}{2}x - 2$ |
| 5. $y = -\frac{1}{4}x - 2$ | 14. $y = \frac{3}{5}x$       | 22. $y = 3x + 6$            |
| 6. $y = \frac{2}{3}x$      | 15. $x = 1$                  | 23. $y = \frac{5}{4}x + 1$  |
| 7. $y = 2x - 4$            | 16. $y = -3$                 | 24. $y = 7x + 100$          |
| 8. $y = -3x + 1$           | 17. $y = 4$                  | 25. $y = \frac{1}{2}x + 3$  |
| 9. $y = -4x + 8$           |                              |                             |

### Mixed Review

### Sections 1.1 – 4.3

- |  |   |
|--|---|
| <p>1. Solve <math>\frac{5}{4}x - 2 = \frac{5}{6} + \frac{1}{2}x</math></p> <p>2. Solve <math>3x - 11 \geq -\frac{2}{5}(5 - 15x)</math>, graph the solution, and write it in interval notation.</p> <p>3. Solve <math>\frac{6x - y}{2} = 10</math> for <math>x</math>.</p> <p>4. Joan found a bookcase she likes at the store. The bookcase is priced at \$230, but Joan has a coupon for 15% off any item. How much will the bookcase cost if Joan uses the coupon?</p> <p>5. 72 is what percent of 300?</p> | <p>6. Write the ordered pair <math>(x, y)</math> for each of the points shown on the graph to the right.</p> <p>7. Find the <math>x</math> and <math>y</math> intercepts of the line <math>6x - 8y = 120</math>.</p> <p>8. Determine if <math>(5, -6)</math> is a solution of the equation <math>-2x + 3y = 28</math>.</p> <p>9. Determine the slope of the line graphed to the right.</p> <p>10. Determine the slope of the line that passes through <math>(-5, 4)</math> and <math>(7, 1)</math>.</p> |
|--|---|



### Answers to Mixed Review

- |  |  |
|--|--|
| <p>1. <math>x = \frac{34}{9}</math></p> <p>2. <math>x \leq -3</math><br/><math>(-\infty, -3]</math></p> <p>3. <math>x = \frac{y+20}{6}</math></p> <p>4. \$195.50</p> <p>5. 24%</p> | <p>6. A(3,1) B(0,-2) C(-1,3)</p> <p>7. <math>x = 20</math> and <math>y = -15</math></p> <p>8. No</p> <p>9. <math>m = 2</math></p> <p>10. <math>m = -\frac{1}{4}</math></p> |
|--|--|

