MATH 082

SECTION 3.2

Negative Exponents

"The harder you work for something,
the greater you'll feel when you finally achieve it."

AUTHOR UNKNOWN

DIRECTIONS

As you work through this Learning Guide, you should:

- read carefully
- take notes
- do the CheckPoint problems on your own
- check your answers to the CheckPoint problems
- watch the videos by clicking on the 📽 icons

IMPORTANT: Get help if you don't understand a topic.

INTRODUCTION

In the last section, you focused on simplifying expressions with positive exponents.

In this section, you will learn to work with exponents that are NOT positive.

You will learn two new exponent properties/rules,

one that deals with **negative exponents**

and another that deals with **exponents of 0**.

ZERO EXPONENT RULE

To describe this rule, we will use the sample problem $\frac{a^3}{a^3}$, and we will simplify the expression two different ways.

(1) First Method

$\frac{a^3}{a^3} = a^{3-3}$ The problem is a division problem with like bases. So, we subtract the exponents.

$= a^0$ This is the simplified expression.

(2) Second Method

$\frac{a^3}{a^3} = \frac{\cancel{a^3}}{\cancel{a^3}}$ Anything divided by itself equals 1.

$= 1$ This is the simplified expression.

When we simplified using the first method, we got a^0. When we simplified using the second method, we got 1.

Therefore, $a^0 = 1$. This final result is the Zero Exponent Rule.

It says that any number *(except 0)*, variable, or expression raised to the 0 power equals 1.

ZERO EXPONENT RULE	
$x^0 = 1$	Any non-zero base raised to the 0 power equals 1.

EXAMPLES: Simplify each expression.

Problem	Solution	
1. n^0	$n^0 = 1$	The base n raised to the 0 power is 1.
2. 6^0	$6^0 = 1$	The base 6 raised to the 0 power is 1.
3. $(-6)^0$	$(-6)^0 = 1$	The parentheses indicate that the base is -6. The base -6 raised to the 0 power is 1.
4. -6^0	$-6^0 = -1$	Since there are no parentheses, the base is just 6. The base 6 raised to the 0 power is 1. Keep the negative sign in front.
5. $(5x)^0$	$(5x)^0 = 1$	The parentheses indicate that the base is $5x$. The base $(5x)$ raised to the 0 power is 1.
6. $5x^0$	$5x^0 = 5 \cdot 1 = 5$	Since there are no parentheses, the base is just x, not $5x$. The base x to the 0 power is 1. Then multiply 5 by 1.

CHECKPOINT: Simplify each expression.

1. a^0

2. 4^0

3. $(3xy)^0$

4. $(-7)^0$

5. -7^0

6. $9m^0$

7. $(4p)^0$

Go to the next slide to check your answers.

CheckPoint Answers:

Check your answers with the ones below.

Watch the videos to see step-by-step solutions and hear the explanations.

1. 1

2. 1

3. 1

4. 1

5. –1

6. 9

7. 1

NEGATIVE EXPONENT RULE

To describe this rule, we will use the sample problem $\frac{a^2}{a^6}$, and we will simplify the expression two different ways.

(1) First Method

$\frac{a^2}{a^6} = a^{2-6}$ The problem is a division problem with like bases. So, we subtract the exponents.

$= a^{-4}$ This is the simplified expression.

(2) Second Method

$\frac{a^2}{a^6} = \frac{a \cdot a}{a \cdot a \cdot a \cdot a \cdot a \cdot a}$ Rewrite the numerator and denominator as repeated multiplications.

$= \frac{\cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a \cdot a}$ Divide out common factors in the numerator and denominator.

$= \frac{1}{a \cdot a \cdot a \cdot a}$ Write the denominator using exponential notation.

$= \frac{1}{a^4}$ This is the simplified expression.

When we simplified using the first method, we got a^{-4}. When we simplified using the second method, we got $\frac{1}{a^4}$.

Therefore, $a^{-4} = \frac{1}{a^4}$. This final result illustrates the Negative Exponent Rule.

It says that a base raised to a **negative** exponent is equal to the **reciprocal** of that base raised to the **positive** exponent.

NEGATIVE EXPONENT RULE (Continued)

NEGATIVE EXPONENT RULE	
$x^{-n} = \frac{1}{x^n}$ Note: $x \neq 0$	A base raised to a negative exponent equals the reciprocal of that base raised to the positive exponent.

If an expression has a negative exponent:

1. Rewrite the Expression as a Fraction *(if it isn't already)*

   HOW? Place a denominator of 1 under the expression. For instance, n^{-2} can be written as the fraction $\frac{n^{-2}}{1}$.

2. Apply the Negative Exponent Rule

   HOW? **Move the base** with the negative exponent to the opposite position in the fraction, and **change its exponent to positive**.

   numerator OR *denominator*

EXAMPLES: Simplify and write each expression with positive exponents only.

1) $x^{-9} = \frac{x^{-9}}{1} = \frac{1}{x^9}$

Rewrite the expression as a fraction.

Move the base x to the denominator and change its exponent to **positive** 9.

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2) $3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2} = \frac{1}{9}$

Rewrite the expression as a fraction.

Move the base 3 to the denominator and change its exponent to **positive** 2.

Simplify 3^2.

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3) $-3^{-2} = -\frac{3^{-2}}{1} = -\frac{1}{3^2} = -\frac{1}{9}$

Rewrite the expression as a fraction. Keep the negative sign in front of the fraction.

Move the base 3 to the denominator and change its exponent to **positive** 2.

Simplify 3^2.

4) $5m^{-4} = \dfrac{5m^{-4}}{1} = \dfrac{5^{1}m^{-4}}{1} = \dfrac{5}{m^{4}}$

Rewrite the expression as a fraction.

No exponent is shown on 5, so the exponent is 1.

Move the base m to the denominator and change its exponent to **positive** 4.

NOTE: 5^{1} does not move because the exponent is not negative.

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5) $\dfrac{a^{2}}{b^{-3}} = \dfrac{a^{2}b^{3}}{1} = a^{2}b^{3}$

Already a fraction.

Move the base b to the numerator and change its exponent to **positive** 3.

NOTE: a^{2} does not move because the exponent is not negative.

Simplify.

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6) $\dfrac{m^{-6}}{2n^{-5}} = \dfrac{m^{-6}}{2^{1}n^{-5}} = \dfrac{n^{5}}{2m^{6}}$

Already a fraction.

No exponent is shown on 2, so the exponent is 1.

Move the base m to the denominator and change its exponent to **positive** 6.

Move the base n to the numerator and change its exponent to **positive** 5.

NOTE: 2^{1} does not move because the exponent is not negative.

CHECKPOINT: Simplify and write each expression with positive exponents only.

1. x^{-3}

2. $\dfrac{1}{x^{-3}}$

3. 2^{-3}

4. -6^{-2}

5. $(-8)^{-2}$

6. $\dfrac{a^{-3}}{b^{-4}}$

7. $\dfrac{1}{5p^{-7}}$

8. $2x^{-3}$

9. $\dfrac{2}{m^{-9}}$

10. $2x^{-3}y^{4}$

11. $\dfrac{2}{3}a^{-5}b^{-3}c^{2}$

12. $\dfrac{x^{4}w^{-2}}{4y^{-6}z^{8}}$

Go to the next slide to check your answers.

CheckPoint Answers:

Check your answers with the ones below.

Watch the videos to see step-by-step solutions and hear the explanations.

1. $\frac{1}{x^3}$ 

2. x^3 

3. $\frac{1}{8}$ 

4. $-\frac{1}{36}$ 

5. $\frac{1}{64}$ 

6. $\frac{b^4}{a^3}$ 

7. $\frac{p^7}{5}$ 

8. $\frac{2}{x^3}$ 

9. $2m^9$ 

10. $\frac{2y^4}{x^3}$ 

11. $\frac{2c^2}{3a^5b^3}$ 

12. $\frac{x^4y^6}{4w^2z^8}$ 

ALL PROPERTIES OF EXPONENTS

You have now completed your study of all the Properties of Exponents:

- Product Rule, Quotient Rule, and Power Rules *(studied in the last section)*
- Zero Exponent Rule and Negative Exponent Rule *(studied in this section)*

You may need to use more than one rule to simplify some exponential expressions.

A good practice is to apply the Product, Quotient, or Power Rule first.
Then, simplify further using the Zero Exponent Rule or Negative Exponent Rule.

The final answer should contain only positive exponents.

SUMMARY: SIMPLIFYING EXPRESSIONS CONTAINING EXPONENTS

1) **Multiplication Problem**

   Add the exponents on like bases.

2) **Division Problem**

   Subtract the exponents on like bases.

3) **Parentheses / Powers to Powers Problem**

   Multiply every exponent inside the parentheses with the exponent outside the parentheses.

4) **Zero Exponent**

   A base raised to the 0 power equals **1**.

5) **Negative Exponent**

   If a base has a negative exponent,

   - switch its **position** in the fraction *(numerator* or *denominator)* and
   - change the exponent to **positive**.

Multiplication Problems

EXAMPLES: Simplify each expression.

1. $-2x^3y^{-8} \cdot 4x^2y^8$ — Combine coefficients. Combine like bases.

$= -2 \cdot 4 \; x^{3+2} \; y^{-8+8}$ — Multiply coefficients. Add exponents on like bases.

$= -8\,x^5\,y^0$ — y^0 equals **1**.

$= -8\,x^5 \cdot 1$ — Simplify.

$= -8x^5$

2. $a^{-4} \cdot a^{-2}$ — Combine like bases.

$= a^{-4+-2}$ — Add exponents on like bases.

$= a^{-6}$ — Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{a^{-6}}{1}$ — Apply Negative Exponents Rule:

- In fraction, switch position of a^{-6}.
- Change exponent to positive 6.

$= \dfrac{1}{a^6}$

3. $a^{-3}b^{-2} \cdot a^{-4}b^{5}$ Combine like bases.

$= a^{-3+-4}\, b^{-2+5}$ Add exponents on like bases.

$= a^{-7}b^{3}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{a^{-7}b^{3}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of a^{-7}.
- Change exponent to positive 7.

$= \dfrac{b^{3}}{a^{7}}$

4. $-4x^{-3}y^{5} \cdot 3x^{6}y^{-9}$ Combine coefficients. Combine like bases.

$= -4 \cdot 3\, x^{-3+6}\, y^{5+-9}$ Multiply coefficients. Add exponents on like bases.

$= -12x^{3}y^{-4}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{-12x^{3}y^{-4}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of y^{-4}.
- Change exponent to positive 4.

$= \dfrac{-12x^{3}}{y^{4}}$

CHECKPOINT: Simplify each expression.

1. $p^3 \cdot p^{-6}$

2. $p^{-4} \cdot p^2 \cdot p$

3. $x^3 y^{-1} \cdot x^{-5} y^4$

4. $-6x^7 y^{-4} \cdot 3x^{-7} y^6$

5. $\left(6m^{-4} n^6\right)\left(2m^{-2} n\right)$

Go to the next slide to check your answers.

CheckPoint Answers:

Check your answers with the ones below.

Watch the videos to see step-by-step solutions and hear the explanations.

1. $\frac{1}{p^3}$ 

2. $\frac{1}{p}$ 

3. $\frac{y^3}{x^2}$ 

4. $-18y^2$ 

5. $\frac{12n^7}{m^6}$ 

Division Problems

EXAMPLES: Simplify each expression.

1. $\dfrac{a^{-5}}{a^{-3}}$ Combine like bases.

$= a^{-5-(-3)}$ Subtract exponents on like bases.

$= a^{-5+3}$ Change subtraction to adding the opposite.

$= a^{-2}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{a^{-2}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of a^{-2}.
- Change exponent to positive **2**.

$= \dfrac{1}{a^{2}}$

2. $\dfrac{x^{7}y^{2}}{xy^{5}}$ Write x as x^{1}.

$= \dfrac{x^{7}y^{2}}{x^{1}y^{5}}$ Combine like bases.

$= x^{7-1}\,y^{2-5}$ Subtract exponents on like bases.

$= x^{6}y^{-3}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{x^{6}y^{-3}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of y^{-3}.
- Change exponent to positive **3**.

$= \dfrac{x^{6}}{y^{3}}$

3. $\dfrac{8x^{-7}y^{3}}{16x^{-2}y^{3}}$ Notice y^3 in the numerator and denominator.

$= \dfrac{8\,x^{-7}\,\cancel{y^{3}}}{16\,x^{-2}\,\cancel{y^{3}}}$ Anything divided by itself is 1.

$= \dfrac{\cancel{8}^{\,1}}{\cancel{16}_{\,2}}\,x^{-7-(-2)}$ Coefficients: Simplify the fraction. Subtract the exponents on like bases.

$= \dfrac{1}{2}\,x^{-7+2}$ Change subtraction to adding the opposite.

$= \dfrac{1}{2}x^{-5}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{1}{2}\cdot\dfrac{x^{-5}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of x^{-5}.
- Change exponent to positive 5.

$= \dfrac{1}{2}\cdot\dfrac{1}{x^{5}}$

$= \dfrac{1}{2x^{5}}$

4. $\dfrac{6a^{-4}b^{6}}{9a^{7}b^{-2}}$ Combine like bases.

$= \dfrac{\cancel{6}^{\,2}}{\cancel{9}_{\,3}}\,a^{-4-7}\,b^{6-(-2)}$ Coefficients: Simplify the fraction. Subtract exponents on like bases.

$= \dfrac{2}{3}\,a^{-4+-7}\,b^{6+2}$ Change subtraction to adding the opposite.

$= \dfrac{2}{3}a^{-11}b^{8}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{2}{3}\cdot\dfrac{a^{-11}b^{8}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of a^{-11}.
- Change exponent to positive 11.

$= \dfrac{2}{3}\cdot\dfrac{b^{8}}{a^{11}}$ Simplify.

$= \dfrac{2b^{8}}{3a^{11}}$

CHECKPOINT: Simplify each expression.

1. $\dfrac{x^{-12}}{x^{-8}}$

2. $\dfrac{y^{7}}{7y^{-6}}$

3. $\dfrac{22m^{6}p^{2}}{2m^{9}p}$

4. $\dfrac{21x^{6}y^{3}}{7x^{4}y^{7}}$

5. $\dfrac{4a^{2}b}{12a^{-3}b^{5}}$

6. $\dfrac{4t^{-10}u}{6t^{-3}u^{-1}}$

Go to the next slide to check your answers.

CheckPoint Answers:

Check your answers with the ones below.

Watch the videos to see step-by-step solutions and hear the explanations.

1. $\frac{1}{x^4}$ 

2. $\frac{y^{13}}{7}$ 

3. $\frac{11p}{m^3}$ 

4. $\frac{3x^2}{y^4}$ 

5. $\frac{a^5}{3b^4}$ 

6. $\frac{2u^2}{3t^7}$ 

Powers to Powers Problems

EXAMPLES: Simplify each expression.

1. $\left(y^3\right)^{-2}$ Multiply the exponent inside the parentheses with the exponent outside the parentheses.

$= y^{3\cdot(-2)}$ Multiply the exponents.

$= y^{-6}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{y^{-6}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of y^{-6}.
- Change exponent to positive 6.

$= \dfrac{1}{y^6}$

2. $\left(a^2b^{-6}\right)^{-5}$ Multiply every exponent inside the parentheses with the exponent outside the parentheses.

$= a^{2\cdot(-5)}\, b^{-6\cdot(-5)}$ Multiply the exponents.

$= a^{-10}\, b^{30}$ Since there's a negative exponent, write the expression as a fraction.

$= \dfrac{a^{-10}\, b^{30}}{1}$ Apply Negative Exponents Rule:

- In fraction, switch position of a^{-10}.
- Change exponent to positive 10.

$= \dfrac{b^{30}}{a^{10}}$

3. $\left(\frac{x^2}{y^{-1}}\right)^4$ Multiply every exponent inside the parentheses with the exponent outside the parentheses.

$= \frac{x^{2\cdot 4}}{y^{-1\cdot 4}}$ Multiply the exponents.

$= \frac{x^8}{y^{-4}}$ Apply Negative Exponents Rule:

- In fraction, switch position of y^{-4}.
- Change exponent to positive 4.

$= \frac{x^8 y^4}{1}$ Simplify.

$= x^8 y^4$

4. $\left(4a^3b^{-5}\right)^2$ Write 4 as 4^1.

$= \left(4^1 a^3 b^{-5}\right)^2$ Multiply every exponent inside the parentheses with the exponent outside the parentheses.

$= 4^{1\cdot 2} a^{3\cdot 2} b^{-5\cdot 2}$ Multiply the exponents.

$= 4^2 a^6 b^{-10}$ Since there's a negative exponent, write the expression as a fraction.

$= \frac{4^2 a^6 b^{-10}}{1}$ Simplify 4^2.

Apply Negative Exponents Rule:

- In fraction, switch position of b^{-10}.
- Change exponent to positive 10.

$= \frac{16a^6}{b^{10}}$

CHECKPOINT: Simplify each expression.

1. $\left(x^{-2}\right)^{-4}$

2. $\left(x^{3}y^{6}\right)^{-5}$

3. $\left(x^{-4}y^{3}\right)^{-5}$

4. $\left(\frac{x^{4}}{y^{5}}\right)^{-3}$

5. $\left(\frac{x^{-4}}{y^{-3}}\right)^{2}$

6. $\left(3x^{-4}y^{7}\right)^{2}$

Go to the next slide to check your answers.

CheckPoint Answers:

Check your answers with the ones below.

Watch the videos to see step-by-step solutions and hear the explanations.

1. x^8

2. $\dfrac{1}{x^{15}y^{30}}$

3. $\dfrac{x^{20}}{y^{15}}$

4. $\dfrac{y^{15}}{x^{12}}$

5. $\dfrac{y^6}{x^8}$

6. $\dfrac{9y^{14}}{x^8}$

This is the end of the PowerPoint Learning Guide for Section 3.2.

Return to Section 3.2 of the Brightspace course to:

- study the Summary
- complete the Exercise Set