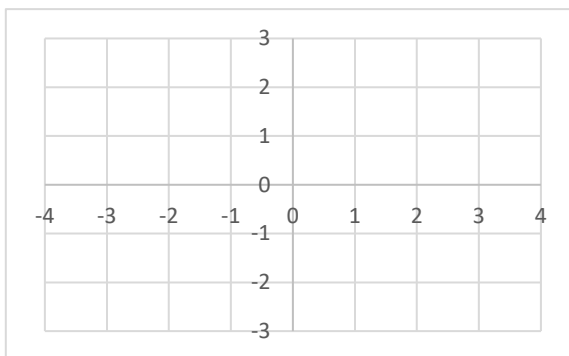


Section 1.2: Points and Lines

Objective: Graph points and lines using x and y coordinates.

Often, to get an idea of the behavior of an equation we will make a picture that represents the solutions to the equations. A **graph** is simply a picture of the solutions to an equation. Before we spend much time on making a visual representation of an equation, we first have to understand the basics of graphing. The following is an example of what is called the coordinate plane.



The plane is divided into four sections by a horizontal number line (x -axis) and a vertical number line (y -axis). Where the two lines meet in the center is called the origin. This center origin is where $x = 0$ and $y = 0$. As we move to the right the numbers count up from zero, representing $x = 1, 2, 3, \dots$

To the left the numbers count down from zero, representing $x = -1, -2, -3, \dots$. Similarly, as we move up the number count up from zero, $y = 1, 2, 3, \dots$, and as we move down count down from zero, $y = -1, -2, -3, \dots$. We can put dots on the graph, which we will call points. Each point has an “address” that defines its location.

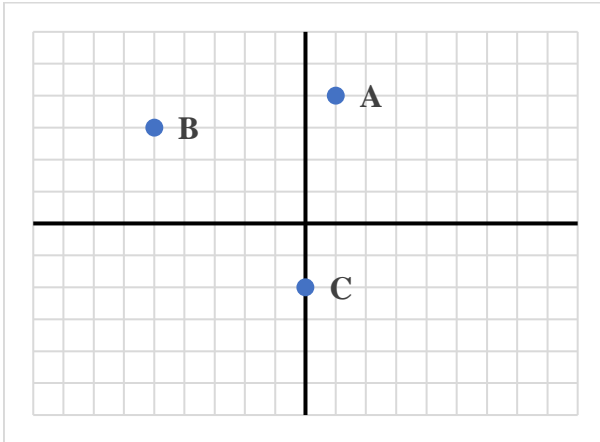
The first number will be the value on the x axis or horizontal number line. This is the distance the point moves left/right from the origin. The second number will represent the value on the y axis or vertical number line. This is the distance the point moves up/down from the origin. The points are given as an ordered pair: (x, y) .

World View Note: Locations on the globe are given in the same manner. The longitude gives the distance east or west from a central point and is like the x value. The latitude gives the distance north or south of that central point and is like the y value. The central point is just off of the western coast of Africa where the equator and prime meridian meet.

The following example finds the address, or coordinate pair, for each of several points on the coordinate plane.

Example 1.

Give the coordinates of each point.



Tracing from the origin, point A is right 1, up 4. This becomes $A(1, 4)$.

Point B is left 5, up 3. Left is backwards or negative, so we have $B(-5, 3)$.

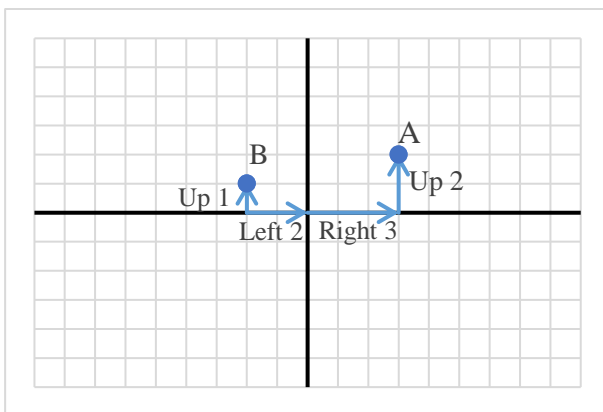
C is straight down 2 units. There is no left or right. This means we go right zero so the point is $C(0, -2)$

$A(1, 4)$, $B(-5, 3)$, $C(0, -2)$ Our Solution

Just as we can state the coordinates for a set of points, we can take a set of points and plot them on the plane.

Example 2.

Graph the points $A(3, 2)$, $B(-2, 1)$, $C(3, -4)$, $D(-2, -3)$, $E(-3, 0)$, $F(0, 2)$, $G(0, 0)$.

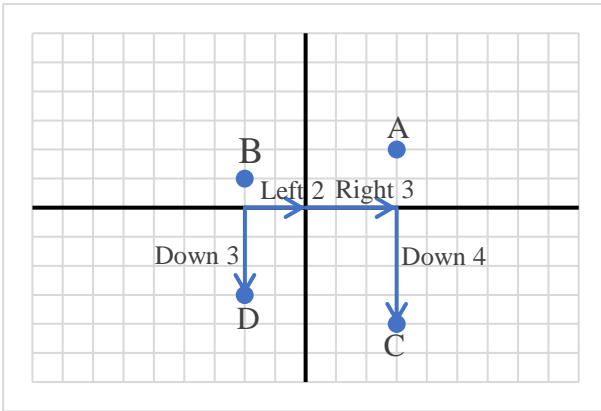


The first point, A is at $(3, 2)$ this means $x = 3$ (right 3) and $y = 2$ (up 2).

Following these instructions, starting from the origin, we get our point.

The second point, $B(-2, 1)$, is left 2 (negative moves backwards), up 1.

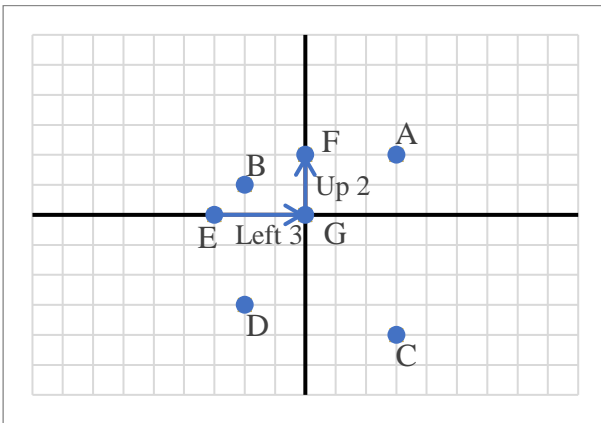
This is also illustrated on the graph.



The third point, $C(3, -4)$ is right 3, down 4 (negative moves backwards).

The fourth point, $D(-2, -3)$ is left 2, down 3 (both negative, both move backwards)

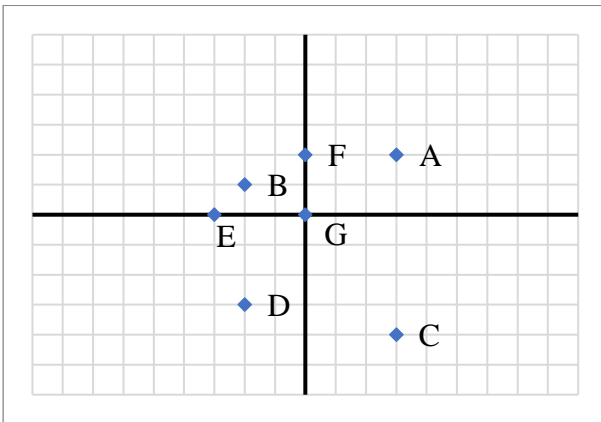
The last three points have zeros in them. We still treat these points just like the other points. If a coordinate is zero, then there is just no movement.



Next is $E(-3, 0)$. This is left 3 (negative is backwards), and up zero, which is on the x -axis.

Then is $F(0, -2)$. This is right zero, and up two, which is on the y -axis.

Finally is $G(0, 0)$. This point has no movement. Thus, the point is exactly where the two axes meet, which is known as the origin.



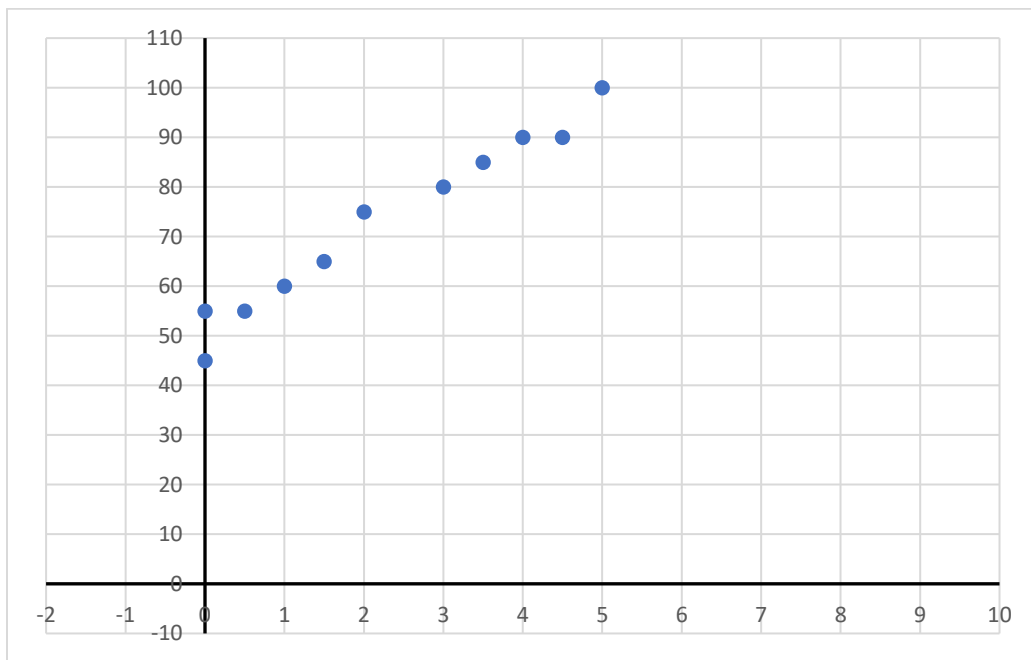
Our Solution

Example 3.

An instructor distributes the results of a midterm examination and then surveys the students to determine how many hours each student spent preparing for the exam. The results are summarized in the following table.

hours of preparation	exam score
0	45
0	55
0.5	55
1	60
1.5	65
2	75
3	80
3.5	85
4	90
4.5	90
5	100

We are given two variables: hours and scores. To visualize the impact the hours of preparation has on respective test scores, we can construct a graph. Each student can be represented by a point in the form (hours, score).



The graph illustrates the relationship between preparation hours and exam scores.

The main purpose of graphs is not to plot random points, but rather to give a picture of the solutions to an equation. Consider $y = 2x - 3$. We may be interested in all of the possible solutions to this equation which involves a combination of an x and y value that make the equation true. Graphing can help visualize these solutions. We will do this using a table of values.

Example 4.

Graph $y = 2x - 3$. We make a table of values in which the first column is for the x values and the second column is for the y values.

x	y
-1	
0	
1	

We will test three values for x . Any three can be used

x	y
-1	-5
0	-3
1	-1

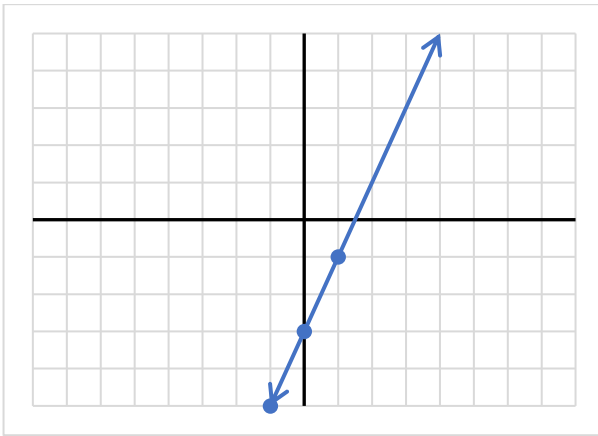
Evaluate each by replacing x with the given value

$$x = -1; y = 2(-1) - 3 = -2 - 3 = -5$$

$$x = 0; y = 2(0) - 3 = 0 - 3 = -3$$

$$x = 1; y = 2(1) - 3 = 2 - 3 = -1$$

$(-1, -5), (0, -3), (1, -1)$ These then become the points to graph on our equation



Plot each point.

Once the points are on the graph, connect the dots to make a line.

The graph is our solution

What this line tells us is that any point on the line will work in the equation $y = 2x - 3$. For example, notice the graph also goes through the point $(2, 1)$.

If we use $x = 2$, we should get $y = 1$. Sure enough, $y = 2(2) - 3 = 4 - 3 = 1$, just as the graph suggests. Thus, we have the line is a picture of all the solutions for $y = 2x - 3$. We can use this table of values method to draw a graph of any linear equation.

Example 5.

Graph $2x - 3y = 6$. We will use a table of values.

x	y
-3	
0	
3	

We will test three values for x . Any three can be used

$$2(-3) - 3y = 6 \quad \text{Substitute each value in for } x \text{ and solve for } y$$

$$-6 - 3y = 6 \quad \text{Start with } x = -3, \text{ multiply first}$$

$$\begin{array}{r} +6 \quad +6 \\ \hline \end{array} \quad \text{Add 6 to both sides}$$

$$-3y = 12$$

$$\frac{-3y}{-3} = \frac{12}{-3} \quad \text{Divide both sides by } -3$$

$$y = -4 \quad \text{Solution for } y \text{ when } x = -3, \text{ add this to table}$$

$$2(0) - 3y = 6 \quad \text{Next } x = 0$$

$$-3y = 6 \quad \text{Multiplying clears the constant term}$$

$$\frac{-3y}{-3} = \frac{6}{-3} \quad \text{Divide each side by } -3$$

$$y = -2 \quad \text{Solution for } y \text{ when } x = 0, \text{ add this to table}$$

$$2(3) - 3y = 6 \quad \text{Next } x = 3$$

$$6 - 3y = 6 \quad \text{Multiply}$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array} \quad \text{Subtract } -6 \text{ from both sides}$$

$$-3y = 0$$

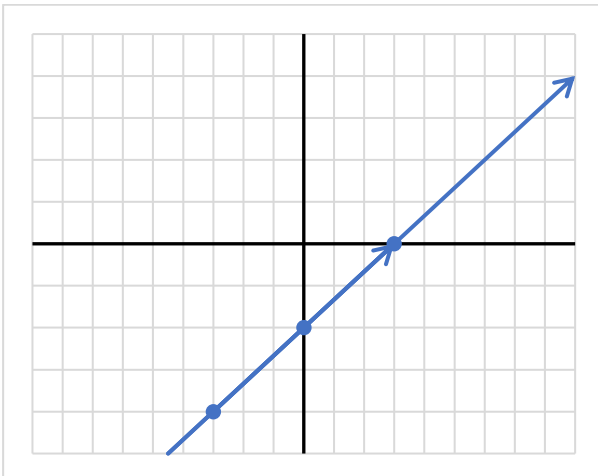
$$\frac{-3y}{-3} = \frac{0}{-3} \quad \text{Divide each side by } -3$$

$$y = 0 \quad \text{Solution for } y \text{ when } x = 3, \text{ add this to table}$$

x	y
-3	-4
0	-2
3	0

Out completed table

$(-3, -4), (0, -2), (3, 0)$ Our completed table represents ordered pairs to plot.



Graph points and connect the dots

Our Solution

Example 6.

Graph $y = \frac{2}{3}x - 1$. We will use a table of values.

x	y
0	
3	
6	

We will test three values for x . Note that in this case, choosing multiples of 3 will eliminate the fraction.

$$y = \frac{2}{3}(0) - 1 \quad \text{Substitute each value in for } x \text{ and solve for } y$$

$$y = 0 - 1 \quad \text{Multiply first, now subtract.}$$

$$y = -1 \quad \text{Solution for } y \text{ when } x = 0, \text{ add this to table}$$

$$y = \frac{2}{3}(3) - 1 \quad \text{Next, } x = 3.$$

$$y = \frac{6}{3} - 1 \quad \text{Multiply first, now divide}$$

$$y = 2 - 1 \quad \text{Subtract}$$

$$y = 1 \quad \text{Solution for } y \text{ when } x = 3, \text{ add this to table}$$

$$y = \frac{2}{3}(6) - 1 \quad \text{Next, } x = 6$$

$$y = \frac{12}{3} - 1 \quad \text{Multiply first, now divide}$$

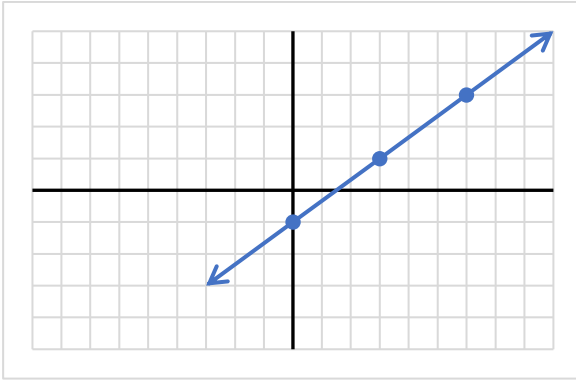
$$y = 4 - 1 \quad \text{Subtract}$$

$$y = 3 \quad \text{Solution for } y \text{ when } x = 6, \text{ add this to table}$$

x	y
0	-1
3	1
6	3

Our completed table.

$(0, -1), (3, 1), (6, 3)$ Our table represents ordered pairs to plot:



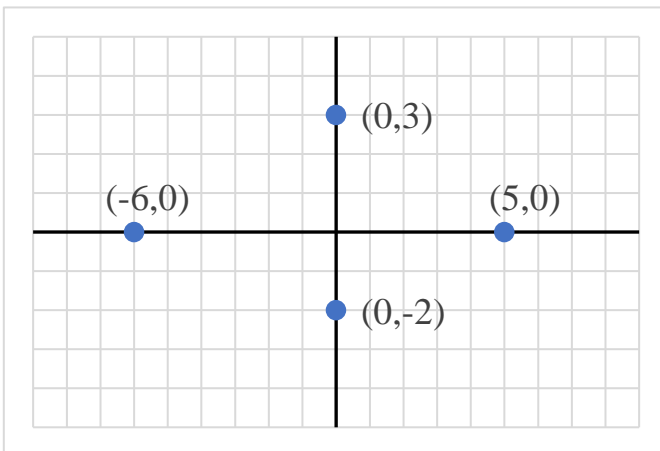
Graph points and connect dots

Our Solution

Objective: Graph lines using intercepts.

In the previous examples we constructed graphs of lines by plotting randomly selected points. Now we will find specific points on the graph of a line: x - and y -intercepts. The **x -intercept** of the graph of a line is the point where the line intersects the x axis (horizontal axis). The **y -intercept** of the graph of a line is the point where the line intersects the y axis (vertical axis).

Before we determine the location of these intercepts, given a particular equation, examine the following graph. What do you notice?



Points along the x -axis have $y = 0$, and points along the y -axis have $x = 0$.

Therefore, to determine the intercepts, we will substitute values of 0 into the given equation one variable at a time.

Example 7.

Determine the x - and y -intercepts and sketch the graph of $9x - 6y = 18$.

First, we determine the x -intercept.

$$9x - 6y = 18$$

$$9x - 6(0) = 18$$

$$9x = 18$$

$$\frac{9x}{9} = \frac{18}{9}$$

$$x = 2$$

(2, 0)

Our original equation.

Let $y = 0$.

Divide each side by 9.

Write as an ordered pair.

Second, we determine the y -intercept.

$$9x - 6y = 18$$

$$9(0) - 6y = 18$$

$$-6y = 18$$

$$\frac{-6y}{-6} = \frac{18}{-6}$$

$$y = -3$$

(0, -3)

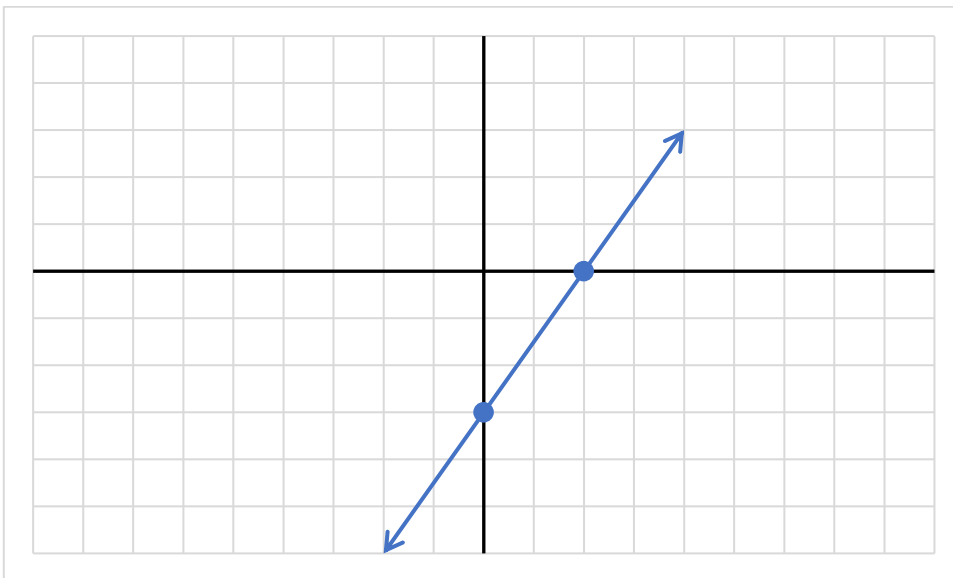
Our original equation.

Let $x = 0$.

Divide each side by -6 .

Write as an ordered pair.

Now we can sketch the graph of the line by plotting the intercepts.



Example 8.

Determine the x - and y -intercepts and sketch the graph of $y = \frac{3}{5}x - 2$.

First, we determine the x -intercept.

$$y = \frac{3}{5}x - 2$$

Our original equation.

$$0 = \frac{3}{5}x - 2$$

Let $y = 0$.

$$0 = \frac{3}{5}x - 2$$

Solve for x . Add 2 to both sides.

$$0 = \frac{3}{5}x - 2$$

$$\begin{array}{r} +2 \quad \quad +2 \\ \hline \end{array}$$

$$2 = \frac{3}{5}x$$

Multiply both sides by $\frac{5}{3}$.

$$\frac{5}{3} \cdot 2 = \frac{5}{3} \cdot \frac{3}{5}x$$

$$\frac{10}{3} = x$$

$$\left(\frac{10}{3}, 0\right) \text{ or } \left(3\frac{1}{3}, 0\right)$$

Write as an ordered pair.

Second, we determine the y -intercept.

$$y = \frac{3}{5}x - 2$$

Our original equation.

$$y = \frac{3}{5}(0) - 2$$

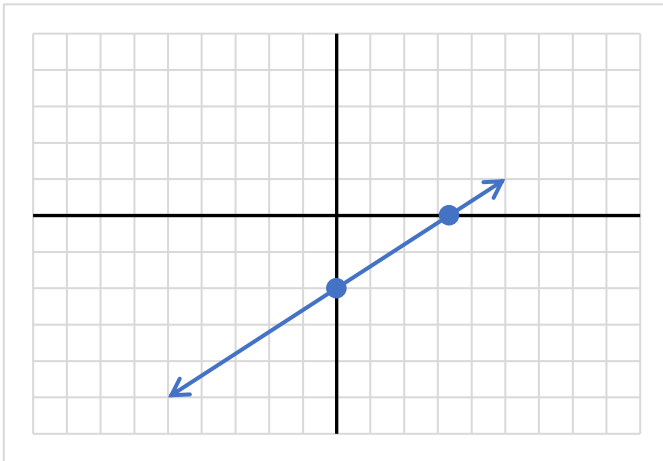
Let $x = 0$

$$y = -2$$

$$(0, -2)$$

Write as an ordered pair

Now we can sketch the graph of the line by plotting the intercepts.

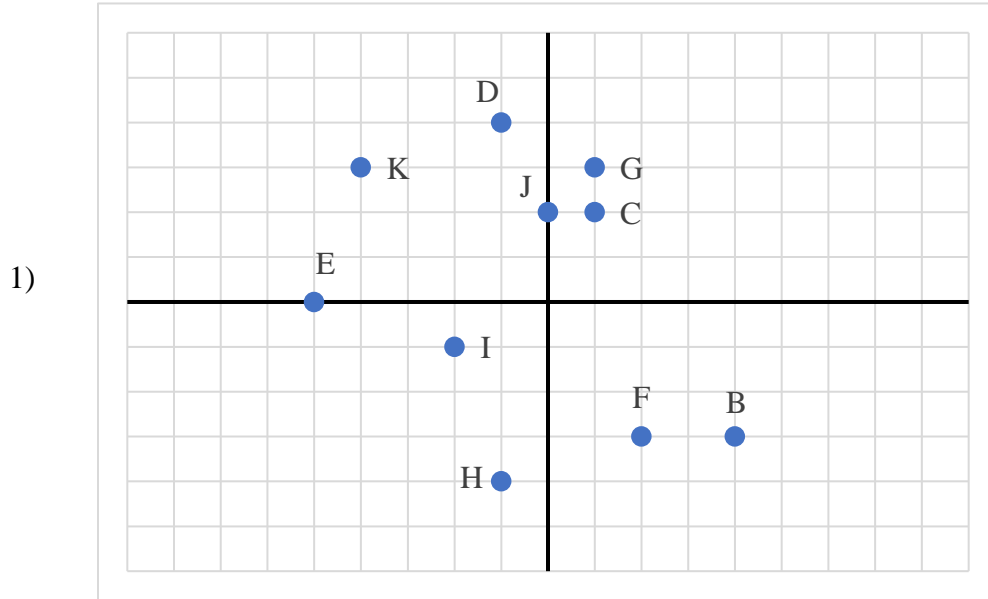


Improper fractions are sometimes better expressed as mixed numbers so they can be estimated on the graph.

The x -intercept of $\left(\frac{10}{3}, 0\right)$ is expressed as $\left(3\frac{1}{3}, 0\right)$ and its point is estimated, slightly to the right of $+3$ along the x -axis.

1.2 Practice

State the coordinates of each point in the graph.



Plot each point on a graph.

- 2) $C(0,4)$ $K(1,0)$ $J(-3,4)$ $I(-3,0)$ $H(-4,2)$ $G(4,-2)$ $F(-2,-2)$ $E(3,-2)$ $D(0,3)$
 $L(-5,5)$

Sketch the graph of each line by plotting points.

- 3) $y = -2x - 3$
- 4) $y = 5x - 4$
- 5) $y = -4x + 2$
- 6) $y = \frac{3}{2}x - 5$
- 7) $y = \frac{4}{5}x - 3$
- 8) $y = -x - 5$
- 9) $4x + y = 5$
- 10) $2x - y = 2$
- 11) $x + y = -1$
- 12) $x - y = -3$
- 13) $y = 3x + 1$

$$14) y = \frac{5}{3}x + 4$$

$$15) y = -x - 2$$

$$16) y = \frac{1}{2}x$$

$$17) 8x - y = 5$$

Determine the x – and y – intercepts, and sketch the graph of each line.

$$18) 2x + 5y = 10$$

$$19) 6x + 2y = 18$$

$$20) x + 3y = 6$$

$$21) 4x - y = 4$$

$$22) x - 7y = 7$$

$$23) x + y = 6$$

$$24) x + y = 3$$

$$25) y = 5x - 10$$

$$26) y = 4x + 12$$

$$27) y = x + 8$$

$$28) y = -x - 6$$

$$29) y = \frac{1}{2}x + 3$$

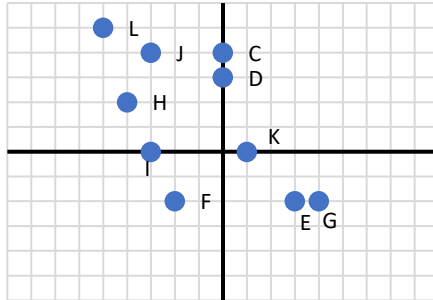
$$30) y = \frac{3}{4}x - 2$$

$$31) 3x + 4y = 16$$

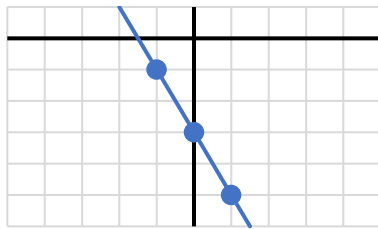
1.2 Answers

- 1) $B(4,-3)$ $C(1,2)$ $D(-1,4)$ $E(-5,0)$ $F(2,-3)$ $G(1,3)$ $H(-1,-4)$ $I(-2,-1)$
 $J(0,2)$ $K(-4,3)$

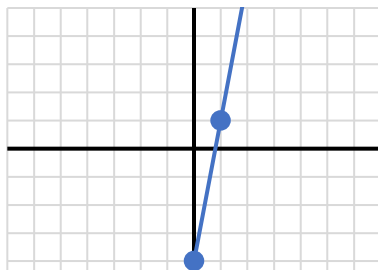
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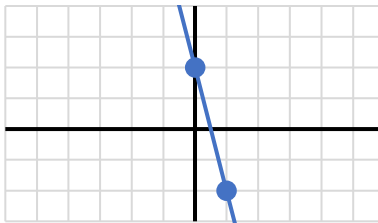
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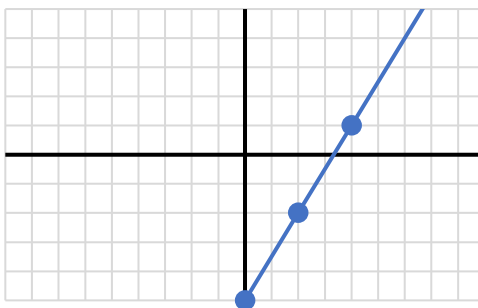
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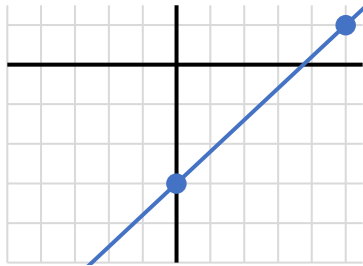
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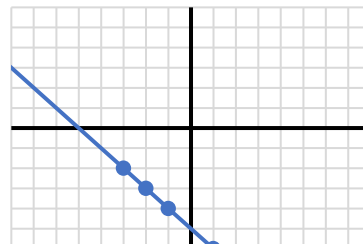
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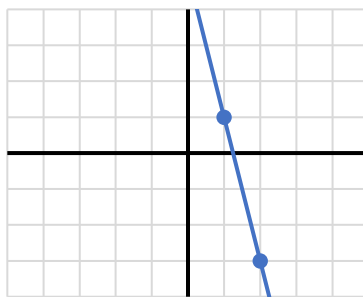
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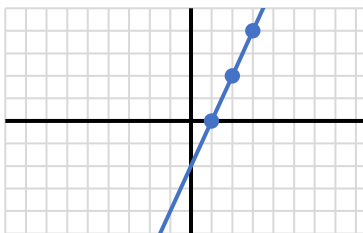
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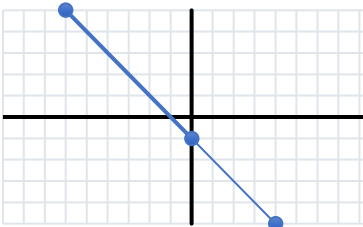
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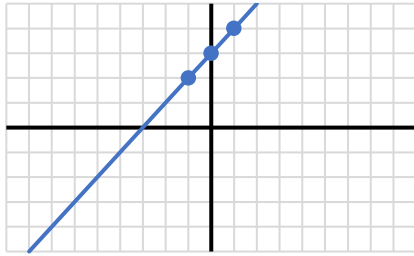
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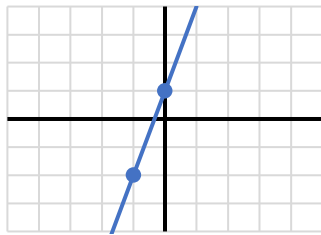
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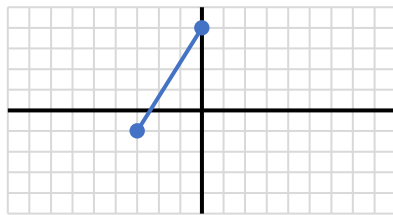
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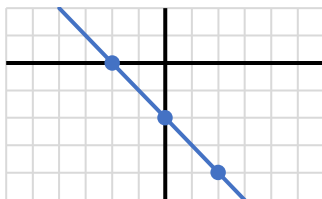
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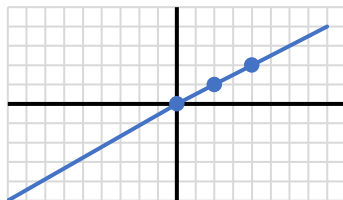
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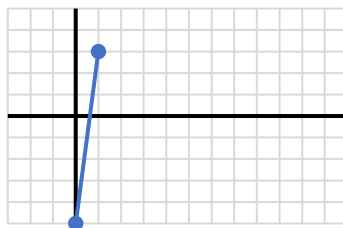
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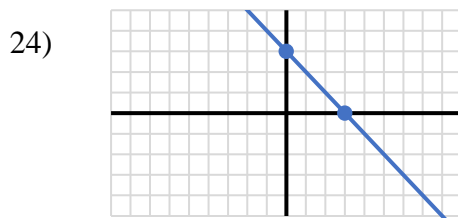
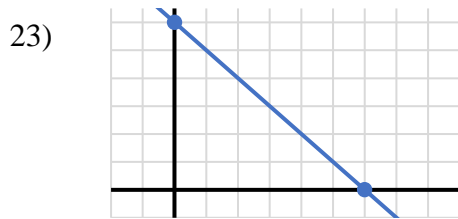
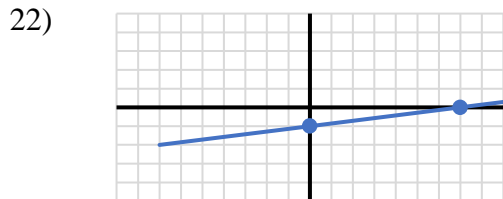
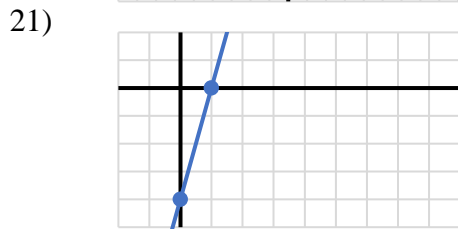
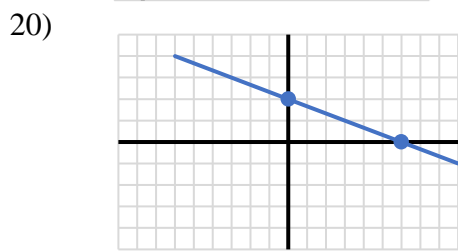
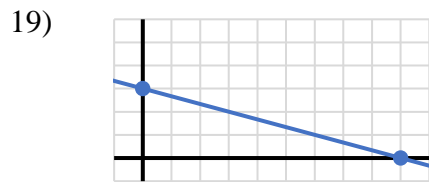
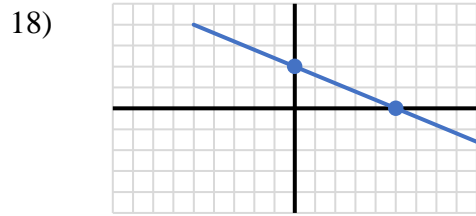


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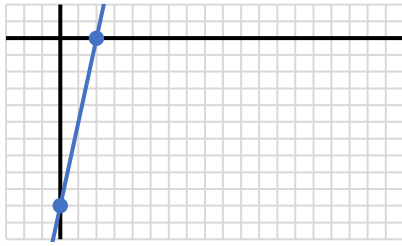


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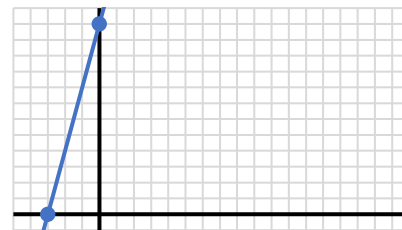




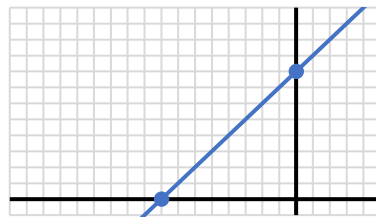
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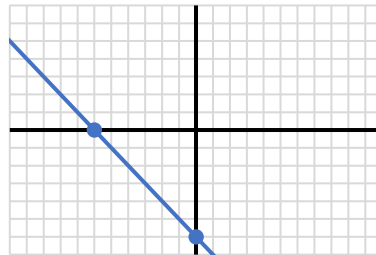
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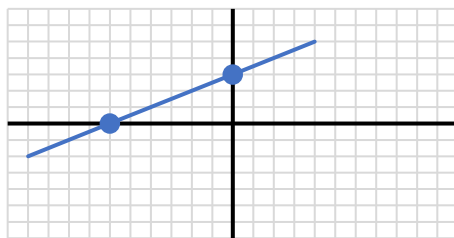
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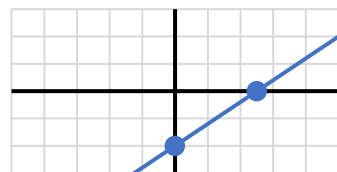
28)



29)



30)



31)

