Section 1.4: Slope-Intercept Form

Objective: Give the equation of a line with a known slope and *y*-intercept.

When graphing a line we found one method we could use is to make a table of values. However, if we can identify some properties of the line, we may be able to make a graph much quicker and easier. One such method is finding the slope and the $y-$intercept of the equation. The slope can be represented by m. The $y-$intercept, where the line crosses the $y-$axis can be represented by $(0,b)$ where b is the value where the graph crosses the vertical $y-$axis (thus, the x-coordinate is zero). Any other point on the line can be represented by (x, y). Using this information we will look at the slope formula and solve the formula for y.

Example 1.

$m, (0,b), (x, y)$	Using the slope formula gives:
$\frac{y-b}{x-0} = m$	Simplify
$\frac{y-b}{x} = m$	Multiply both sides by x
$y-b = mx$	Add b to both sides
$\underline{\quad +b \quad +b}$	
$y = mx+b$	Our Solution

This equation, $y = mx+b$ can be thought of as the equation of any line that has a slope of m and a $y-$intercept of b. This formula is known as the slope-intercept equation.

Slope-Intercept Equation: $\boldsymbol{y = mx+b}$

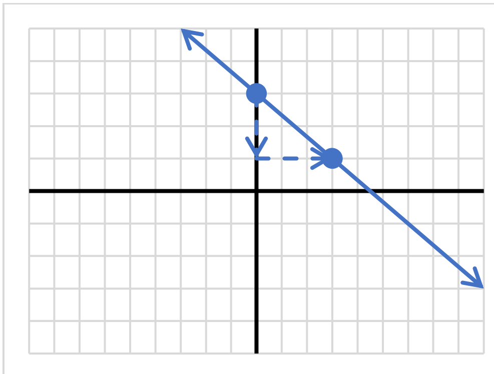
If we know the slope and the $y-$intercept we can easily find the equation that represents the line.

Example 2.

$\text{slope} = \frac{3}{4}, y-\text{intercept at } (0,-3)$	Use the slope – intercept equation
$y = mx+b$	m is the slope, b is the y-intercept
$y = \frac{3}{4}x-3$	Our Solution

We can also find the equation by looking at a graph and finding the slope and $y-$intercept.

Example 3.



Identify the point where the graph crosses the $y-$ axis $(0,3)$. This means the $y-$intercept is 3.
Draw a slope triangle to identify another point. The slope is $-\frac{2}{3}$

$y = mx + b$ Slope – intercept equation

$y = -\frac{2}{3}x + 3$ Our Solution

We can also move the opposite direction, using the equation to identify the slope and $y-$intercept and graph the equation from this information. However, it will be important for the equation to first be in slope intercept form. If it is not, we will have to solve it for y so we can identify the slope and the y-intercept.

Example 4.

Write in slope – intercept form:

$2x - 4y = 6$ Solve for y

$-2x \quad -2x$ Subtract $2x$ from both sides

Put x term first

$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{6}{-4}$ Divide each term by –4

$y = \frac{1}{2}x - \frac{3}{2}$ Our Solution

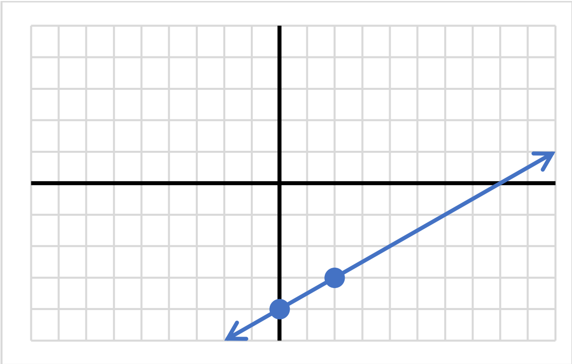
Once we have an equation in slope-intercept form we can graph it by first plotting the $y-$intercept, then using the slope, find a second point and connecting the dots.

Example 5.

Graph $y = \frac{1}{2}x - 4$ Recall the slope – intercept formula

$y = mx + b$ Identify the slope, m, and the y-intercept, b

$m = \frac{1}{2}, b = -4$ Make the graph

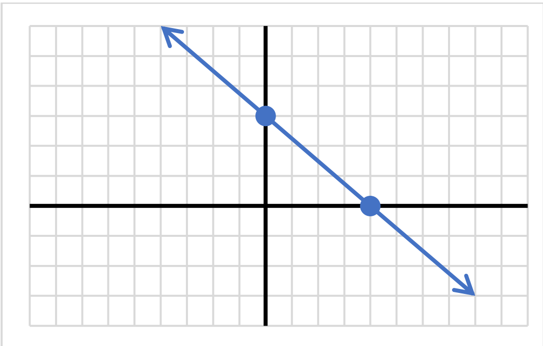


Start with a point at the $y-$ intercept of -4.
Then use the slope $\frac{\text{rise}}{\text{run}}$, so we will rise 1 unit and run 2 units to find the next point.
Once we have both points, connect the dots to get our graph.

World View Note: Before our current system of graphing, in 1323 French mathematician Nicole Oresme, suggested graphing lines that would look more like a bar graph with a constant slope!

Example 6.

Graph $3x+4y=12$	Not in slope intercept form
$-3x \qquad -3x$	Subtract $3x$ from both sides
$4y=-3x+12$	Put the x term first
$\frac{4y}{4}=\frac{-3x}{4}+\frac{12}{4}$	Divide each term by 4
$y=-\frac{3}{4}x+3$	Recall slope – intercept equation
$y=mx+b$	Identify m and b
$m=-\frac{3}{4}, b=3$	Make the graph



Start with a point at the $y-$ intercept of 3.
Then use the slope $\frac{\text{rise}}{\text{run}}$, but it's negative so it will go downhill, so we will drop 3 units and run 4 units to find the next point.
Once we have both points, connect the dots to get our graph.

We want to be very careful not to confuse using slope to find the next point with using a coordinate such as $(4,-2)$ to find an individual point. Coordinates such as $(4,-2)$ start from the origin and move horizontally first, and vertically second.

Slope starts from a point on the line that could be anywhere on the graph. The numerator is the vertical change and the denominator is the horizontal change.

Example 7.

A driving service charges an initial service fee of $6 and an additional $3 per mile traveled. Construct an equation that expresses the total cost, y, for traveling x miles. Identify the slope and $y-$intercept, and their meaning in context to this problem.

It may be helpful to calculate the total cost for several cases. The $6 service fee is constant; every traveler will pay at least $6. Added to this service fee is $3 for every mile. The total costs for three cases follow.

Mileage	Total cost
7	$6+3(7)=\$27$
8	$6+3(8)=\$30$
9	$6+3(9)=\$33$

If y represents the total charge and x represents the mileage, this can be generalized as

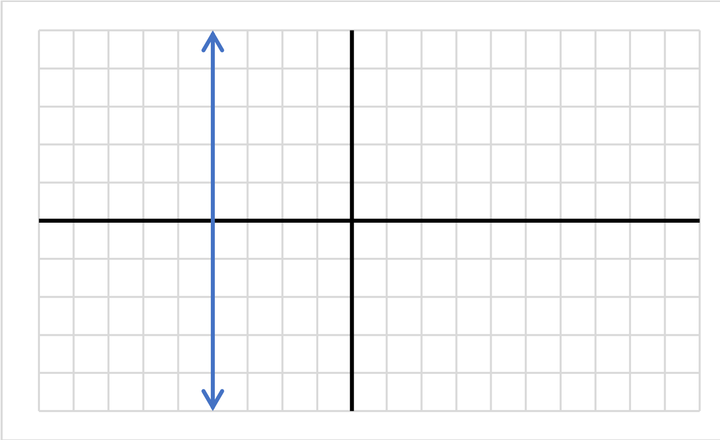
$$y=6+3x$$

Therefore, the slope is 3 and $y-$intercept is $(0,6)$. The slope represents the average rate of change. As the mileage increases by 1 mile, the total cost increases by $3: This can be seen in the examples given when traveling 7, 8, or 9 miles: The $y-$intercept represents the case where x, or mileage, is 0. When miles traveled is 0, meaning you just entered the vehicle, your cost is $6.

Recall that a horizontal line has slope equal to 0. Replacing *m* with 0 in the slope-intercept equation gives the equation $y=0x+b$ or just $y=b$. So, the equation of any horizontal line is of the form $y=b$, where *b* is the y-intercept of the line.

Recall that a vertical line has undefined slope; so, we cannot use the slope-intercept form of the equation at all. The equation of any vertical line is $x=a$, where *a* is the x-intercept of the line.

Example 8.



Give the equation of the line in the graph.

Because we have a vertical line, the slope is undefined; therefore, we cannot use the slope – intercept equation. Instead, we use the $x-$intercept of -4 to write the equation of the line.

$x=-4$ Our Solution

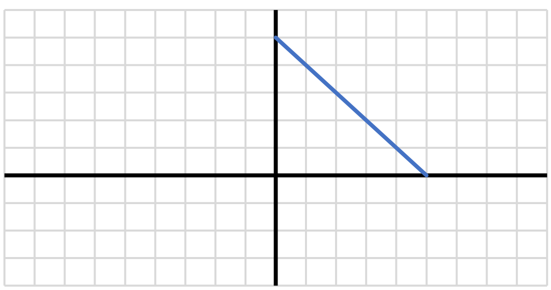
1.4 Practice

Write the slope-intercept form of the equation for each line when given the slope and the *y*-intercept.

1) $\text{Slope} = 2,\ y - \text{intercept} = 5$
2) $\text{Slope} = -6,\ y - \text{intercept} = 4$
3) $\text{Slope} = 1,\ y - \text{intercept} = -4$
4) $\text{Slope} = -1,\ y - \text{intercept} = -2$
5) $\text{Slope} = -\frac{3}{4},\ y - \text{intercept} = -1$
6) $\text{Slope} = -\frac{1}{4},\ y - \text{intercept} = 3$
7) $\text{Slope} = \frac{1}{3},\ y - \text{intercept} = 1$
8) $\text{Slope} = \frac{2}{5},\ y - \text{intercept} = 5$

Write the slope-intercept form of the equation of each line graphed below.

9)



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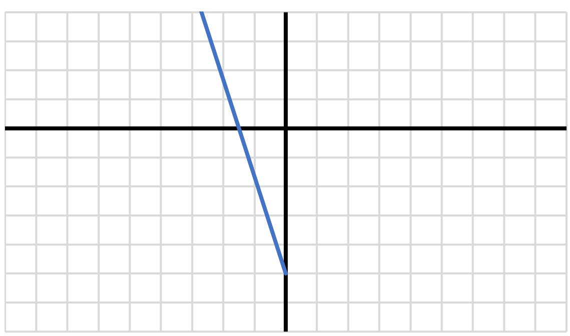

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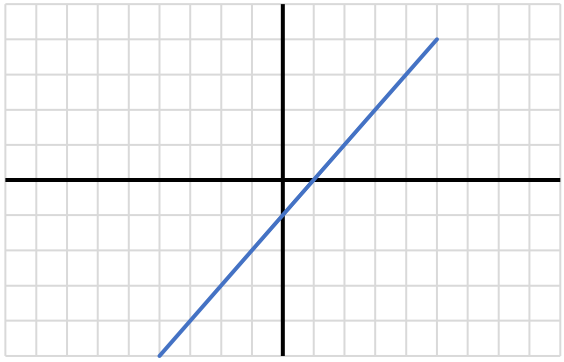

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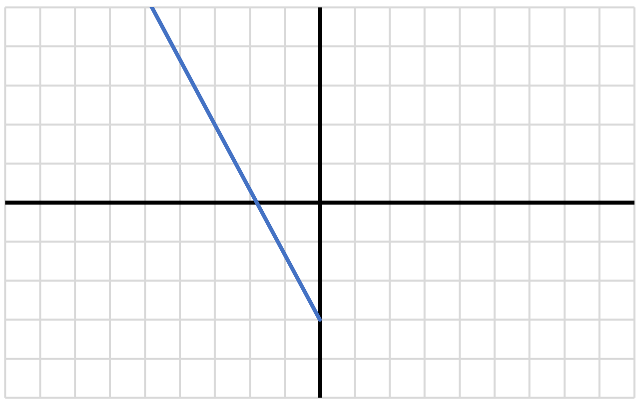

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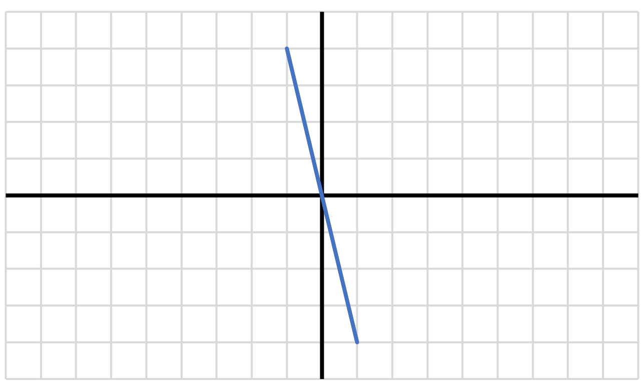

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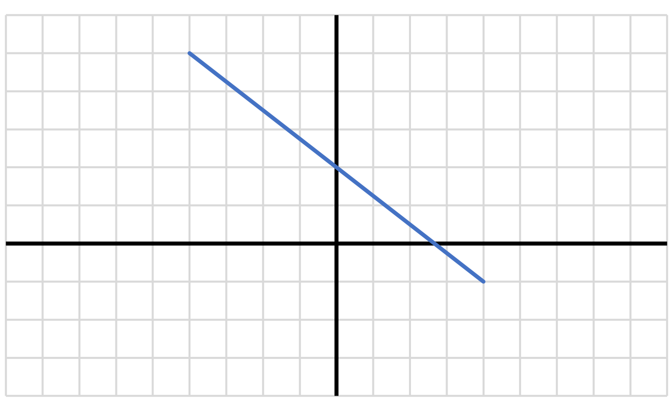


Write the equation of each line using the slope-intercept form.

15) $x + 10y = -37$
16) $x - 10y = 3$
17) $2x + y = -1$
18) $6x - 11y = -70$
19) $7x - 3y = 24$
20) $4x + 7y = 28$
21) $x - 7y = -42$
22) $y - 4 = -(x + 5)$
23) $y - 5 = \frac{5}{2}(x - 2)$
24) $y - 4 = 4(x - 1)$
25) $y - 3 = -\frac{2}{3}(x + 3)$
26) $y + 5 = -4(x - 2)$
27) $y + 1 = -\frac{1}{2}(x - 4)$
28) $y + 2 = \frac{6}{5}(x + 5)$

Sketch the graph of each line.

29) $y = \frac{1}{3}x + 4$
30) $y = -\frac{1}{5}x - 4$
31) $y = \frac{6}{5}x - 5$
32) $y = -\frac{3}{2}x - 1$
33) $y = \frac{3}{2}x$
34) $y = -\frac{3}{4}x + 1$
35) $x - y + 3 = 0$
36) $4x + 5 = 5y$
37) $-y - 4 + 3x = 0$
38) $-8 = 6x - 2y$
39) $-3y = -5x + 9$
40) $-3y = 3 - \frac{3}{2}x$

Consider each scenario and develop an applicable model.

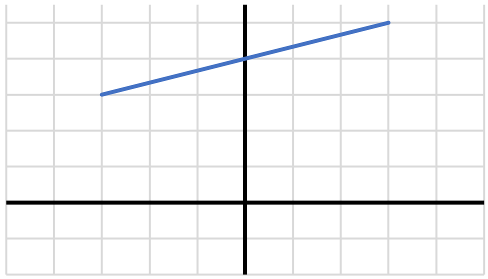
41) The initial room temperature of a beverage is 70°F. When placed in a particular refrigerator, the beverage is expected to cool (or decrease temperature) by an average of 5°F per hour. Express the temperature of the beverage, *y*, after remaining in the refrigerator for *x* hours. Identify the slope and *y*-intercept, and identify their meaning in context to this problem.

42) A reloadable banking card has an initial cost of $4.95 and a service fee of $2.95 per month. Express the total cost, *y*, of maintaining this banking card for *x* months. Identify the slope and *y*-intercept, and identify their meaning in context to this problem.

1.4 Answers

1) $y = 2x + 5$
2) $y = -6x + 4$
3) $y = x - 4$
4) $y = -x - 2$
5) $y = -\frac{3}{4}x - 1$
6) $y = -\frac{1}{4}x + 3$
7) $y = \frac{1}{3}x + 1$
8) $y = \frac{2}{5}x + 5$
9) $y = -x + 5$
10) $y = -\frac{7}{2}x - 5$
11) $y = x - 1$
12) $y = -\frac{5}{3}x - 3$
13) $y = -4x$
14) $y = -\frac{3}{4}x + 2$
15) $y = -\frac{1}{10}x - \frac{37}{10}$
16) $y = \frac{1}{10}x - \frac{3}{10}$
17) $y = -2x - 1$
18) $y = \frac{6}{11}x + \frac{70}{11}$
19) $y = \frac{7}{3}x - 8$
20) $y = -\frac{4}{7}x + 4$
21) $y = \frac{1}{7}x + 6$
22) $y = -x - 1$
23) $y = \frac{5}{2}x$
24) $y = 4x$
25) $y = -\frac{2}{3}x + 1$
26) $y = -4x + 3$
27) $y = -\frac{1}{2}x + 1$
28) $y = \frac{6}{5}x + 4$

29)



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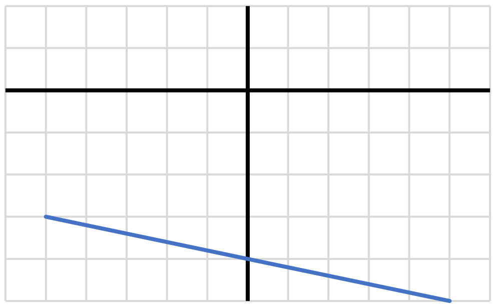

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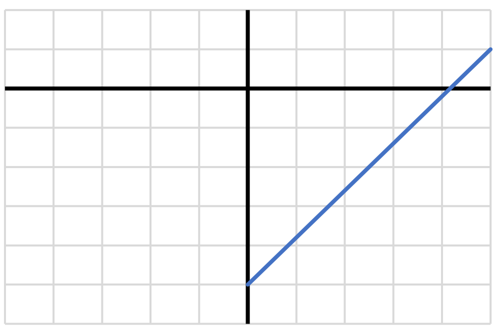

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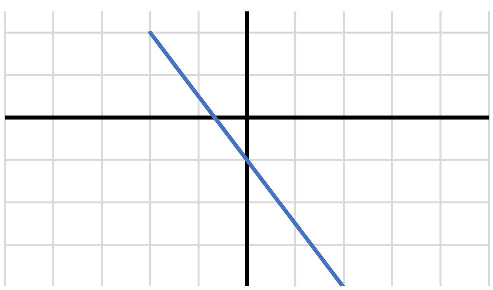

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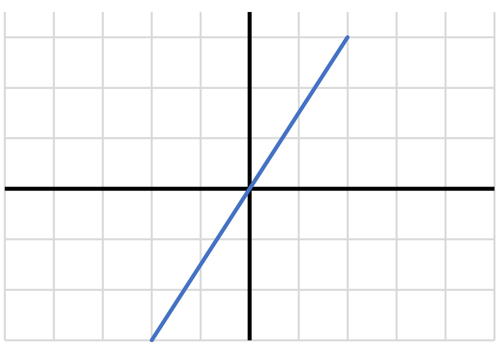

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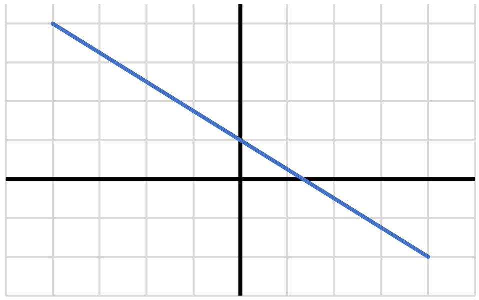

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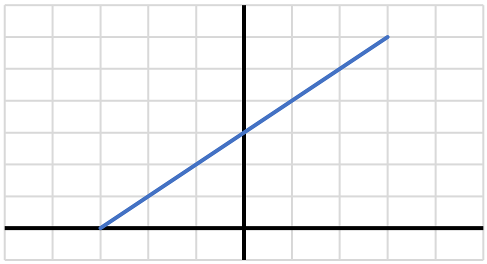

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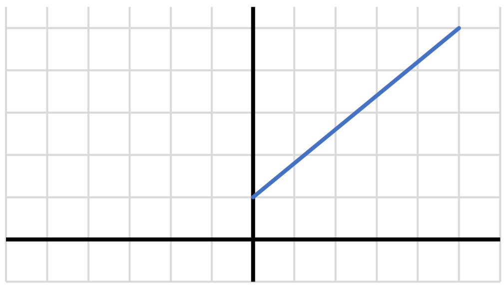

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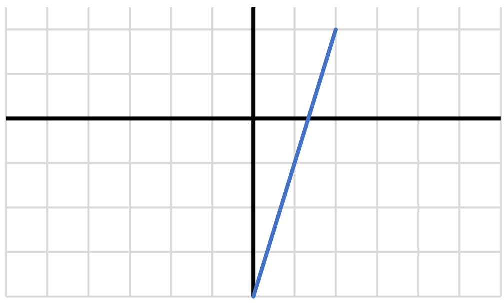

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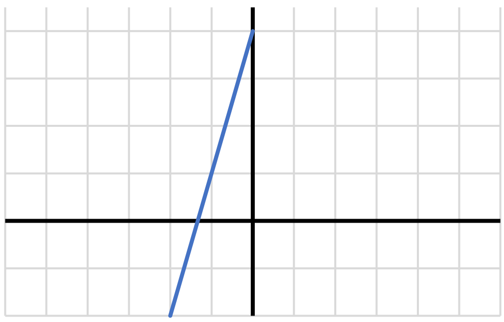

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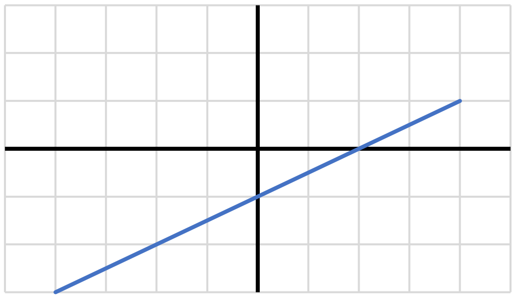
40) 

41) $y = 70 - 5x$ with slope -5 and y-intercept $(0, 70)$. The slope represents the average rate of change. As the hour increases by 1, the temperature decreases by 5. The y-intercept represents the temperature at 0 hours, or the initial temperature.

42) $y = 2.95x + 4.95$ with slope 2.95 and y-intercept $(0, 4.95)$. The slope represents the average rate of change. As the month increases by 1, the total cost increases by 2.95. The y-intercept represents the total cost at 0 months, or the initial cost.