Section 2.1: Solving Systems of Equations by Graphing

Objective: Solve systems of equations by graphing and identifying the point of intersection.

We have solved equations like $3x-4=11$ by adding 4 to both sides and then dividing by 3 (solution is $x=5$). We also have methods to solve equations with more than one variable in them. It turns out that to solve for more than one variable we will need the same number of equations as variables. For example, to solve for two variables such as x and y we will need two equations. When we have several equations we are using to solve, we call the equations a **system of equations**. When solving a system of equations we are looking for a solution that works for all of these equations. In this discussion, we will limit ourselves to solving two equations with two unknowns. This solution is usually given as an ordered pair (x, y). The following example illustrates a solution working in both equations.

Example 1.

Show $(2,1)$ is the solution to the system $\begin{aligned} 3x-y&=5 \\ x+y&=3 \end{aligned}$

$(2,1)$	Identify x and y from the ordered pair
$x=2,\ y=1$	Plug these values into each equation
$3(2)-(1)=5$	First equation
$6-1=5$	Evaluate
$5=5$	True
$(2)+(1)=3$	Second equation, evaluate
$3=3$	True

As we found a true statement for both equations we know $(2,1)$ is the solution to the system. It is in fact the only combination of numbers that works in both equations. In this lesson we will be working to find this point given the equations. It seems to follow that if we use points to describe the solution, we can use graphs to find the solutions.

If the graph of a line is a picture of all the solutions, we can graph two lines on the same coordinate plane to see the solutions of both equations. We are interested in the point that is a solution for both lines, this would be where the lines intersect! If we can find the intersection of the lines we have found the solution that works in both equations.

Example 2.

Solve the system of equations by graphing: $\begin{array}{l} y=-\frac{1}{2}x+3 \\ y=\frac{3}{4}x-2 \end{array}$

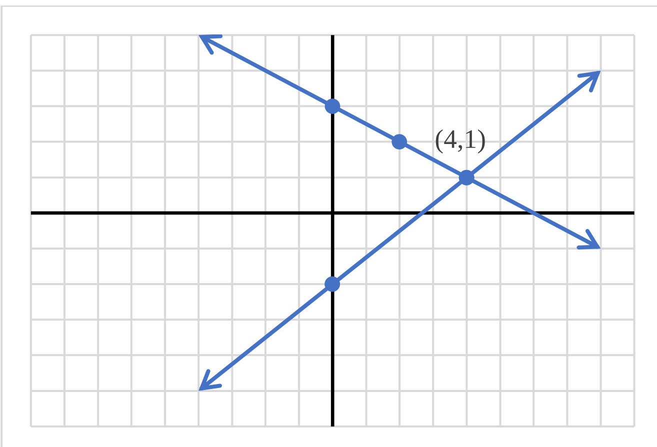
$$y=-\frac{1}{2}x+3$$ To graph we identify slopes and *y*-intercepts

$$y=\frac{3}{4}x-2$$

First: $m=-\frac{1}{2},\ b=3$

Second: $m=\frac{3}{4},\ b=-2$

Now we can graph both lines on the same plane.



To graph each equation, we start at the *y*-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

Remember a negative slope is downhill!

Find the intersection point, $(4,1)$

$(4,1)$ Our Solution

Often the equations will not be in slope-intercept form. We can solve both equations for y first to put the equation in slope-intercept form.

Example 3.

Solve the system of equations by graphing: $\begin{array}{l} 6x-3y=-9 \\ 2x+2y=-6 \end{array}$

$$\begin{array}{l} 6x-3y=-9 \\ 2x+2y=-6 \end{array}$$ Solve each equation for y

$$\begin{array}{rr} 6x-3y=-9 & 2x+2y=-6 \\ \underline{-6x \qquad -6x} & \underline{-2x \qquad -2x} \\ -3y=-6x-9 & 2y=-2x-6 \end{array}$$

Subtract x terms

Put x terms first

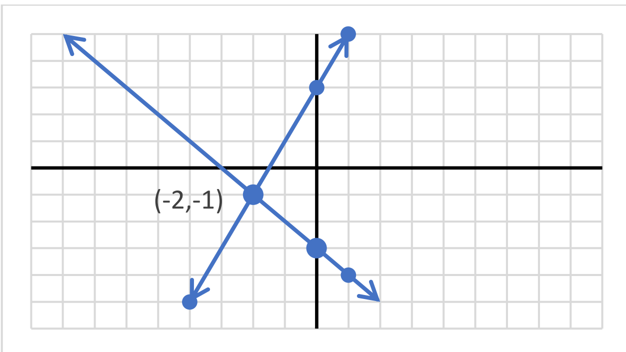
$$\frac{-3y}{-3}=\frac{-6x}{-3}-\frac{9}{-3} \qquad \frac{2y}{2}=\frac{-2x}{2}-\frac{6}{2}$$ Divide by coefficient of y

$$y=2x+3 \qquad y=-x-3$$ Identify slope and y-intercepts

$$\text{First: } m=\frac{2}{1}, b=3$$

$$\text{Second: } m=-\frac{1}{1}, b=-3$$

Now we can graph both lines on the same plane



To graph each equation, we start at the y-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

Remember a negative slope is downhill!

Find the intersection point, $(-2,-1)$

$(-2,-1)$ Our Solution

As we are graphing our lines, it is possible to have one of two unexpected results. These are shown and discussed in the next two examples.

Example 4.

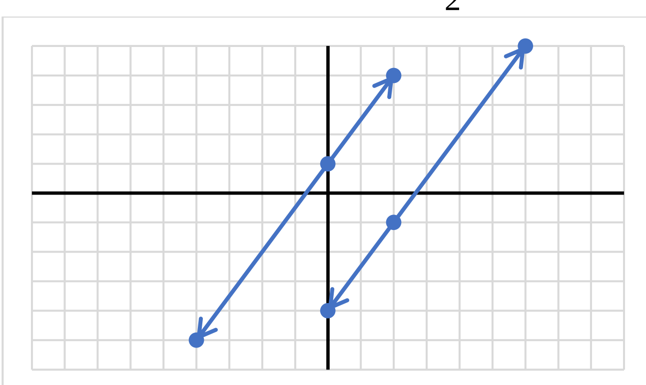
Solve the system of equations by graphing:
$$y=\frac{3}{2}x-4$$
$$y=\frac{3}{2}x+1$$

$$y=\frac{3}{2}x-4$$
$$y=\frac{3}{2}x+1$$
Identify slope and y-intercept of each equation

$$\text{First: } m=\frac{3}{2}, b=-4$$

$$\text{Second: } m=\frac{3}{2}, b=1$$

Now we can graph both lines on the same plane



To graph each equation, we start at the y-intercept and use the slope $\frac{\text{rise}}{\text{run}}$ to get the next point and connect the dots.

The two lines do not intersect! They are parallel! If the lines do not intersect we know that there is no point that works in both equations. There is no solution.

$\varnothing$ No Solution

You can graph lines using intercepts as well.

Example 5.

Solve the system of equations by graphing: $\begin{aligned} 2x - 6y &= 12 \\ 3x - 9y &= 18 \end{aligned}$

$2x - 6y = 12$ Let us graph using intercepts.

$2x - 6(0) = 12$ *x*-intercept (let $y = 0$)

$2x = 12$

$x = 6$

$(6, 0)$ Our *x*-intercept

$2(0) - 6y = 12$

$-6y = 12$

$y = -2$

$(0, -2)$ Our *y*-intercept

$3x - 9y = 18$ Let us graph using intercepts again.

$3x - 9(0) = 18$ *x*-intercept (let $y = 0$)

$3x = 18$

$x = 6$

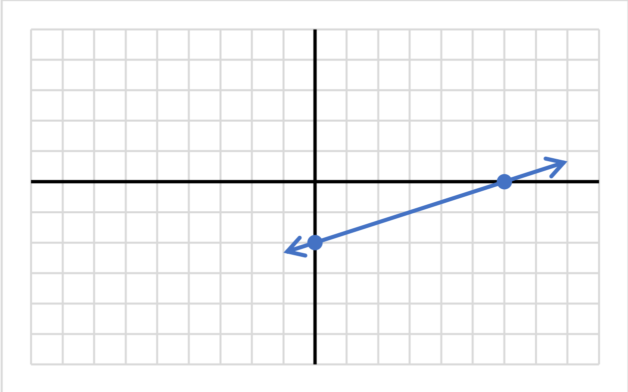
$(6, 0)$ Our *x*-intercept

$3(0) - 9y = 18$

$-9y = 18$

$y = -2$

$(0, -2)$ Our *y*-intercept



Both equations are the same line! As one line is directly on top of the other line, we can say that the lines "intersect" at all the points! Here we say we have infinite solutions.

World View Note: The Babylonians were the first to work with systems of equations with two variables. However, their work with systems was quickly passed by the Greeks who would solve systems of equations with three or four variables and, around 300 AD, developed methods for solving systems with any number of unknowns!

2.1 Practice

Solve each system of equations by graphing.

1) $y = -x + 1$
$y = -5x - 3$

2) $y = -\frac{5}{4}x - 2$
$y = -\frac{1}{4}x + 2$

3) $y = -3$
$y = -x - 4$

4) $y = -x - 2$
$y = \frac{2}{3}x + 3$

5) $y = -\frac{3}{4}x + 1$
$y = -\frac{3}{4}x + 2$

6) $y = 2x + 2$
$y = -x - 4$

7) $y = \frac{1}{3}x + 2$
$y = -\frac{5}{3}x - 4$

8) $y = 2x - 4$
$y = \frac{1}{2}x + 2$

9) $y = \frac{5}{3}x + 4$
$y = -\frac{2}{3}x - 3$

10) $y = \frac{1}{2}x + 4$
$y = \frac{1}{2}x + 1$

11) $\begin{array}{l} x+3y=-9 \\ 5x+3y=3 \end{array}$

12) $\begin{array}{l} x+4y=-12 \\ 2x+y=4 \end{array}$

13) $\begin{array}{l} x-y=4 \\ 2x+y=-1 \end{array}$

14) $\begin{array}{l} 6x+y=-3 \\ x+y=2 \end{array}$

15) $\begin{array}{l} 2x+3y=-6 \\ 2x+y=2 \end{array}$

16) $\begin{array}{l} 3x+2y=2 \\ 3x+2y=-6 \end{array}$

17) $\begin{array}{l} 2x+y=2 \\ x-y=4 \end{array}$

18) $\begin{array}{l} x+2y=6 \\ 5x-4y=16 \end{array}$

19) $\begin{array}{l} 2x+y=-2 \\ x+3y=9 \end{array}$

20) $\begin{array}{l} x-y=3 \\ 5x+2y=8 \end{array}$

21) $\begin{array}{l} 2y=4x+6 \\ y=2x+3 \end{array}$

22) $\begin{array}{l} -2y+x=4 \\ 2=-x+\frac{1}{2}y \end{array}$

23) $\begin{array}{l} 2x-y=-1 \\ 0=-2x-y-3 \end{array}$

24) $\begin{array}{l} -2y=-4-x \\ -2y=-5x+4 \end{array}$

25) $\begin{array}{l} 3+y=-x \\ -4-6x=-y \end{array}$

26) $\begin{aligned} 16 &= -x - 4y \\ -2x &= -4 - 4y \end{aligned}$

27) $\begin{aligned} -y + 7x &= 4 \\ -y - 3 + 7x &= 0 \end{aligned}$

28) $\begin{aligned} -4 + y &= x \\ x + 2 &= -y \end{aligned}$

29) $\begin{aligned} -12 + x &= 4y \\ 12 - 5x &= 4y \end{aligned}$

30) $\begin{aligned} 12x - 3y &= 9 \\ y &= 4x - 3 \end{aligned}$

2.1 Answers

1) $(-1, 2)$
2) $(-4, 3)$
3) $(-1, -3)$
4) $(-3, 1)$
5) No Solution
6) $(-2, -2)$
7) $(-3, 1)$
8) $(4, 4)$
9) $(-3, -1)$
10) No Solution
11) $(3, -4)$
12) $(4, -4)$
13) $(1, -3)$
14) $(-1, 3)$
15) $(3, -4)$
16) No Solution
17) $(2, -2)$
18) $(4, 1)$
19) $(-3, 4)$
20) $(2, -1)$
21) Infinite Solutions
22) $(-4, -4)$
23) $(-1, -1)$
24) $(2, 3)$
25) $(-1, -2)$
26) $(-4, -3)$
27) No Solution
28) $(-3, 1)$
29) $(4, -2)$
30) Infinite Solutions