

Section 2.3: Solving Systems of Equations by Addition

Objective: Solve systems of equations using the addition method.

When solving systems we have found that graphing is very limited when solving equations. We then considered a second method known as substitution. This is probably the most used idea in solving systems in various areas of algebra. However, substitution can get ugly if we don't have a lone variable. This leads us to our third method for solving systems of equations: the **addition method** (sometimes called the elimination method). We will set up the process in the following examples, then define the five step process we can use to solve by addition.

Example 1.

Solve the systems of equations by addition:

$$\begin{aligned} 3x - 4y &= 8 \\ 5x + 4y &= -24 \end{aligned}$$

$3x - 4y = 8$	Notice opposites in front of y 's. Add columns.
$5x + 4y = -24$	
$8x = -16$	
$\frac{8x}{8} = \frac{-16}{8}$	Solve for x , divide by 8
$x = -2$	We have our x !
$5(-2) + 4y = -24$	Plug into either original equation, simplify
$-10 + 4y = -24$	Add 10 to both sides
$+10 \qquad +10$	
$\frac{4y}{4} = \frac{-14}{4}$	Divide by 4
$y = -\frac{7}{2}$	Now we have our y !
$\left(-2, -\frac{7}{2}\right)$	Our Solution

In the previous example one variable had opposites in front of it, $-4y$ and $4y$. Adding these together eliminated the y completely. This allowed us to solve for the x . This is the idea behind the addition method. However, generally we won't have opposites in front of one of the variables. In this case we will manipulate the equations to get the opposites we want by multiplying one or both equations (on both sides!). This is shown in the next example.

Example 2.

Solve the systems of equations by addition: $-6x + 5y = 22$
 $2x + 3y = 2$

$$\begin{array}{r}
 -6x + 5y = 22 \\
 2x + 3y = 2 \\
 \hline
 3(2 + 3y) = (2)3 \\
 6x + 9y = 6 \\
 -6x + 5y = 22 \\
 \hline
 14y = 28 \\
 \hline
 \frac{14y}{14} = \frac{28}{14} \\
 y = 2 \\
 2x + 3(2) = 2 \\
 2x + 6 = 2 \\
 \hline
 \frac{2x}{2} = \frac{-4}{2} \\
 x = -2 \\
 (-2, 2)
 \end{array}$$

We can get opposites in front of x , by multiplying the second equation by 3, to get $-6x$ and $+6x$

Distribute to get the new second equation.

First equation still the same, add

Divide both sides by 14

We have our y !

Plug into one of the original equations, simplify

Subtract 6 from both sides

Divide both sides by 2

We also have our x !

Our Solution

When we looked at the x terms, $-6x$ and $2x$ we decided to multiply the $2x$ by 3 to get the opposites we were looking for. What we are looking for with our opposites is the least common multiple (LCM) of the coefficients. We also could have solved the above problem by looking at the terms with y , $5y$ and $3y$. The LCM of 3 and 5 is 15. So we would want to multiply both equations, the $5y$ by 3, and the $3y$ by -5 to get opposites, $15y$ and $-15y$.

This illustrates an important point: for some problems we will have to multiply both equations by a constant (on both sides) to get the opposites we want before adding.

Example 3.

Solve the systems of equations by addition: $3x + 6y = -9$
 $2x + 9y = -26$

$$\begin{array}{r}
 3x + 6y = -9 \\
 2x + 9y = -26
 \end{array}$$

We can get opposites in front of y , find LCM of 6 and 9

The LCM is 18. We will multiply to get $18y$ and $-18y$

$$3(3x+6y) = (-9)3 \quad \text{Multiply the first equation by 3, both sides!}$$

$$9x+18y = -27$$

$$-2(2x+9y) = (-26)(-2) \quad \text{Multiply the second equation by } -2, \text{ both sides!}$$

$$-4x-18y = 52$$

$$9x+18y = -27 \quad \text{Add the two new equations together}$$

$$\underline{-4x-18y = 52}$$

$$5x = 25 \quad \text{Divide both sides by 5}$$

$$\frac{5x}{5} = \frac{25}{5}$$

$$x = 5 \quad \text{We have our solution for } x$$

$$3(5)+6y = -9 \quad \text{Plug into either original equation; simplify}$$

$$15+6y = -9 \quad \text{Subtract 15 from both sides}$$

$$\underline{-15 \quad -15}$$

$$\frac{6y}{6} = \frac{-24}{6} \quad \text{Divide both sides by 6}$$

$$y = -4 \quad \text{Now we have our solution for } y$$

$$(5, -4) \quad \text{Our Solution}$$

It is important for each problem as we get started that all variables and constants are lined up before we start multiplying and adding equations. This is illustrated in the next example, which includes the five steps we will go through to solve a problem using addition.

Problem	$2x - 5y = -13$ $-3y + 4 = -5x$
1. Line up the variables and constants	Second Equation: $-3y + 4 = -5x$ $+5x - 4 \quad +5x - 4$ $5x - 3y = -4$
2. Multiply to get opposites (use LCM)	$2x - 5y = -13$ $5x - 3y = -4$ First Equation: multiply by -5 $-5(2x - 5y) = (-13)(-5)$ $-10x + 25y = 65$ Second Equation: multiply by 2 $2(5x - 3y) = (-4)2$ $10x - 6y = -8$

	$-10x + 25y = 65$ $10x - 6y = -8$
3. Add	$19y = 57$
4. Solve	$\frac{19y}{19} = \frac{57}{19}$ $y = 3$
5. Plug into either original and solve	$2x - 5(3) = -13$ $2x - 15 = -13$ $\quad +15 \quad +15$ <hr style="width: 10%; margin-left: 0;"/> $\frac{2x}{2} = \frac{2}{2}$ $x = 1$
Solution	$(1, 3)$

World View Note: The famous mathematical text, *The Nine Chapters on the Mathematical Art*, which was printed around 179 AD in China describes a formula very similar to Gaussian elimination which is very similar to the addition method.

Just as with graphing and substitution, it is possible to have no solution or infinite solutions with addition. Just as with substitution, if the variables all disappear from our problem, a true statement will indicate infinite solutions and a false statement will indicate no solution.

Example 4.

Solve the systems of equations by addition: $2x - 5y = 3$
 $-6x + 15y = -9$

$$\begin{array}{ll} 2x - 5y = 3 & \text{To get opposites in front of } x, \text{ multiply first} \\ -6x + 15y = -9 & \text{equation by 3} \end{array}$$

$$\begin{array}{ll} 3(2x - 5y) = (3)3 & \text{Distribute} \\ 6x - 15y = 9 & \end{array}$$

$$\begin{array}{ll} 6x - 15y = 9 & \text{Add equations together} \\ -6x + 15y = -9 & \\ \hline 0 = 0 & \text{True statement} \end{array}$$

Infinite solutions Our Solution

Example 5.

Solve the systems of equations by addition: $4x - 6y = 8$
 $6x - 9y = 15$

$$4x - 6y = 8 \quad \text{LCM for } x \text{ 's is } 12$$

$$6x - 9y = 15$$

$$3(4x - 6y) = (8)3 \quad \text{Multiply first equation by } 3$$

$$12x - 18y = 24$$

$$-2(6x - 9y) = (15)(-2) \quad \text{Multiply second equation by } -2$$

$$-12x + 18y = -30$$

$$12x - 18y = 24 \quad \text{Add both new equations together}$$

$$\underline{-12x + 18y = -30}$$

$$0 = -6 \quad \text{False statement}$$

No Solution Our Solution

We have covered three different methods that can be used to solve a system of two equations with two variables. While all three can be used to solve any system, graphing works well for small integer solutions. Substitution works well when we have a lone variable, and addition works well when the other two methods fail. As each method has its own strengths, it is important you are familiar with all three methods.

2.3 Practice

Solve each system by the addition (or elimination) method.

1) $4x + 2y = 0$
 $-4x - 9y = -28$

2) $-7x + y = -10$
 $-9x - y = -22$

3) $-9x + 5y = -22$
 $9x - 5y = 13$

4) $-x - 2y = -7$
 $x + 2y = 7$

5) $-6x + 9y = 3$
 $6x - 9y = -9$

6) $5x - 5y = 15$
 $-10x + 10y = -30$

7) $4x - 6y = -10$
 $4x - 6y = -14$

8) $-3x + 3y = -12$
 $-3x + 9y = -24$

9) $-x - 5y = 28$
 $-x + 4y = -17$

10) $-10x - 5y = 0$
 $-10x - 10y = -30$

11) $2x - y = 5$
 $5x + 2y = -28$

12) $-5x + 6y = -17$
 $x - 2y = 5$

13) $10x + 6y = 24$
 $-6x + y = 4$

14) $x + 3y = -12$
 $10x + 6y = -10$

- 15) $2x + 4y = 25$
 $4x - 12y = 7$
- 16) $-6x + 4y = 12$
 $12x + 6y = 18$
- 17) $-7x + 4y = -4$
 $10x - 8y = -8$
- 18) $-6x + 4y = 4$
 $-3x - y = 26$
- 19) $5x + 10y = 20$
 $-6x - 5y = -3$
- 20) $-9x - 5y = -19$
 $3x - 7y = -11$
- 21) $-7x - 3y = 12$
 $-6x - 5y = 20$
- 22) $-5x + 4y = 4$
 $-7x - 10y = -10$
- 23) $9x - 2y = -18$
 $5x - 7y = -10$
- 24) $3x + 7y = -8$
 $4x + 6y = -4$
- 25) $9x + 6y = -21$
 $-10x - 9y = 28$
- 26) $-4x - 5y = 12$
 $-10x + 6y = 30$
- 27) $-7x + 5y = -18$
 $-3x - 3y = 12$
- 28) $8x + 7y = -24$
 $6x + 3y = -18$
- 29) $-8x - 8y = -8$
 $10x + 9y = 1$
- 30) $-7x + 10y = 13$
 $4x + 9y = 22$

$$31) \begin{cases} 9y = 7 - x \\ -18y + 4x = -26 \end{cases}$$

$$32) \begin{cases} 2x + 5y = -9 \\ 6x - 10y = 3 \end{cases}$$

2.3 Answers

- 1) $(-2, 4)$
- 2) $(2, 4)$
- 3) No solution
- 4) Infinite number of solutions
- 5) No solution
- 6) Infinite number of solutions
- 7) No solution
- 8) $(2, -2)$
- 9) $(-3, -5)$
- 10) $(-3, 6)$
- 11) $(-2, -9)$
- 12) $(1, -2)$
- 13) $(0, 4)$
- 14) $(\frac{7}{4}, -\frac{55}{12})$
- 15) $(\frac{41}{5}, \frac{43}{20})$
- 16) $(0, 3)$
- 17) $(4, 6)$
- 18) $(-6, -8)$
- 19) $(-2, 3)$
- 20) $(1, 2)$
- 21) $(0, -4)$
- 22) $(0, 1)$
- 23) $(-2, 0)$
- 24) $(2, -2)$
- 25) $(-1, -2)$
- 26) $(-3, 0)$
- 27) $(-\frac{1}{6}, -\frac{23}{6})$
- 28) $(-3, 0)$
- 29) $(-8, 9)$
- 30) $(1, 2)$
- 31) $(-2, 1)$
- 32) $(-\frac{3}{2}, -\frac{6}{5})$