Section 3.1: Exponent Properties

Objective: Simplify expressions using the properties of exponents.

Problems with exponents can often be simplified using a few basic exponent properties. Exponents represent repeated multiplication. We will use this fact to discover the important properties.

World View Note: The word exponent comes from the Latin "expo" meaning "out of" and "ponere" meaning "place". While there is some debate, it seems that the Babylonians living in Iraq were the first to do work with exponents (dating back to the 23rd century BC or earlier).

Example 1. Simplify.

a^3a^2	Expand exponents to multiplication problem
$(aaa)(aa)$	Now we have $5a$'s being multiplied together
a^5	Our Solution

A quicker method to arrive at our answer would have been to just add the exponents: $a^3a^2 = a^{3+2} = a^5$ This is known as the **product rule of exponents**.

Product Rule of Exponents: $\boldsymbol{a^m a^n = a^{m+n}}$

The product rule of exponents can be used to simplify many problems. We will add the exponents on like bases. This is shown in the following examples.

Example 2. Simplify.

$3^2 \cdot 3^6 \cdot 3$	Same base; add exponents $2+6+1$
3^9	Our Solution

Example 3. Simplify.

$2x^3y^5z \cdot 5xy^2z^3$	Multiply $2 \cdot 5$; add exponents on x, y and z
$10x^4y^7z^4$	Our Solution

Rather than multiplying, we will now try to divide with exponents.

Example 4. Simplify.

$\dfrac{a^5}{a^2}$	Expand exponents

$\dfrac{aaaaa}{aa}$	Divide out two of the a's
aaa	Convert to exponents
a^3	Our Solution

A quicker method to arrive at the solution would have been to just subtract the exponents: $\frac{a^5}{a^2} = a^{5-2} = a^3$. This is known as the quotient rule of exponents.

Quotient Rule of Exponents: $\boldsymbol{\dfrac{a^m}{a^n} = a^{m-n}}$

The quotient rule of exponents can similarly be used to simplify exponent problems by subtracting exponents on like bases. This is shown in the following examples.

Example 5. Simplify.

$\dfrac{7^{13}}{7^5}$	Same base; subtract exponents
7^8	Our Solution

Example 6. Simplify.

$\dfrac{5a^3b^5c^2}{2ab^3c}$	Subtract exponents a, b and c
$\dfrac{5}{2}a^2b^2c$	Our Solution

A third property we will look at will have an exponent expression raised to a second exponent. This is investigated in the following example.

Example 7. Simplify.

$\left(a^2\right)^3$	Notice a^2 three times
$a^2 \cdot a^2 \cdot a^2$	Add exponents
a^6	Our solution

A quicker method to arrive at the solution would have been to just multiply the exponents: $\left(a^2\right)^3 = a^{2\cdot 3} = a^6$. This is known as the power of a power rule of exponents.

Power of a Power Rule of Exponents: $\boldsymbol{\left(a^m\right)^n = a^{mn}}$

This property is often combined with two other properties which we will investigate now.

Example 8. Simplify.

$(ab)^3$	Notice (ab) three times
$(ab)(ab)(ab)$	Three a 's and three b 's can be written with exponents
a^3b^3	Our solution

A quicker method to arrive at the solution would have been to take the exponent of three and put it on each factor in parentheses: $(ab)^3 = a^3b^3$. This is known as the power of a product rule of exponents.

Power of a Product Rule of Exponents: $\boldsymbol{(ab)^m = a^m b^m}$

It is important to be careful to only use the power of a product rule with multiplication inside parentheses. This property does NOT work if there is addition or subtraction.

Warning!

$(a+b)^m \neq a^m + b^m$ These are **NOT** equal; beware of this error!

Another property that is very similar to the power of a product rule is considered next.

Example 9. Simplify.

$\left(\frac{a}{b}\right)^3$	Notice the fraction three times
$\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$	Multiply fractions across the top and bottom and use exponents
$\frac{a^3}{b^3}$	Our solution

A quicker method to arrive at the solution would have been to put the exponent on every factor in both the numerator and denominator: $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$. This is known as the power of a quotient rule of exponents.

Power of a Quotient Rule of Exponents: $\boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$

The power of a power, product and quotient rules are often used together to simplify expressions. This is shown in the following examples.

Example 10. Simplify.

$(x^3yz^2)^4$ Put exponent of 4 on each factor; multiply powers

$x^{12}y^4z^8$ Our Solution

Example 11. Simplify.

$\left(\dfrac{a^3b}{c^8d^5}\right)^2$ Put exponent of 2 on each factor; multiply powers

$\dfrac{a^6b^2}{c^{16}d^{10}}$ Our Solution

As we multiply exponents, it is important to remember these properties apply to exponents and not the bases. An expression such as 5^3 does not mean we multiply 5 by 3; instead we multiply 5 three times, $5\times5\times5=125$. This is shown in the next example.

Example 12. Simplify.

$(4x^2y^5)^3$ Put exponent of 3 on each factor; multiply powers

$4^3x^6y^{15}$ Evaluate 4^3

$64x^6y^{15}$ Our Solution

In the previous example we did not put the 3 on the 4 and multiply to get 12. This would have been incorrect. Never multiply a base by the exponent. These properties pertain to exponents only and not the bases.

In this lesson we have discussed 5 different exponent properties. These rules are summarized in the following table.

Rules of Exponents

Product Rule of Exponents	$a^ma^n = a^{m+n}$
Quotient Rule of Exponents	$\dfrac{a^m}{a^n} = a^{m-n}$
Power of a Power Rule of Exponents	$(a^m)^n = a^{mn}$
Power of a Product Rule of Exponents	$(ab)^m = a^mb^m$
Power of a Quotient Rule of Exponents	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

These five properties are often combined in the same problem. Often there is a bit of flexibility as to which property is used first. However, order of operations still applies to a problem. For this reason it is the suggestion of the author to simplify inside parentheses first;

then simplify any exponents (using power rules); and finally simplify any multiplication or division (using product and quotient rules). This is illustrated in the next few examples.

Example 13. Simplify.

$(4x^3y\cdot 5x^4y^2)^3$	In parentheses simplify using product rule; add exponents
$(20x^7y^3)^3$	In parentheses simplify using product rule; add exponents
$20^3x^{21}y^9$	Evaluate 20^3
$8000x^{21}y^9$	Our Solution

Example 14. Simplify.

$7a^3(2a^4)^3$	Parentheses are already simplified; use power rule
$7a^3(8a^{12})$	Use product rule; add exponents; multiply numbers
$56a^{15}$	Our Solution

Example 15. Simplify.

$\dfrac{3a^3b\cdot 10a^4b^3}{2a^4b^2}$	Simplify numerator with product rule; add exponents
$\dfrac{30a^7b^4}{2a^4b^2}$	Now use the quotient rule to subtract exponents
$15a^3b^2$	Our Solution

Example 16. Simplify.

$\dfrac{3m^8n^{12}}{(m^2n^3)^3}$	Use power rule in denominator
$\dfrac{3m^8n^{12}}{m^6n^9}$	Use quotient rule
$3m^2n^3$	Our Solution

Example 17. Simplify.

$\left(\dfrac{3ab^2(2a^4b^2)^3}{6a^5b^7}\right)^2$	Simplify inside parentheses first; use power rule in numerator

$$\left(\frac{3ab^2(8a^{12}b^6)}{6a^5b^7}\right)^2 \quad \text{Simplify numerator; use product rule}$$

$$\left(\frac{24a^{13}b^8}{6a^5b^7}\right)^2 \quad \text{Simplify; use quotient rule}$$

$$(4a^8b)^2 \quad \text{Parentheses are simplified; use power rule}$$

$$16a^{16}b^2 \quad \text{Our Solution}$$

Clearly these problems can quickly become quite involved. Remember to follow order of operations as a guide; simplify inside parentheses first; then use power rules followed by the product and quotient rules.

3.1 Practice

Simplify each expression.

1) $4 \cdot 4^4 \cdot 4^4$
2) $4 \cdot 4^4 \cdot 4^2$
3) $4 \cdot 2^2$
4) $3 \cdot 3^3 \cdot 3^2$
5) $3m \cdot 4mn$
6) $3x \cdot 4x^2$
7) $2m^4n^2 \cdot 4nm^2$
8) $x^2y^4 \cdot xy^2$
9) $(3^3)^4$
10) $(4^3)^4$
11) $(4^4)^2$
12) $(3^2)^3$
13) $(2u^3v^2)^2$
14) $(xy)^3$
15) $(2a^4)^4$
16) $(2xy)^4$
17) $\frac{4^5}{4^3}$
18) $\frac{3^7}{3^3}$
19) $\frac{3^2}{3}$
20) $\frac{3^4}{3}$
21) $\frac{3nm^2}{3n}$
22) $\frac{x^2y^4}{4xy}$
23) $\frac{4x^3y^4}{3xy^3}$
24) $\frac{xy^3}{4xy}$

25) $\left(x^3y^4 \cdot 2x^2y^3\right)^2$

26) $\left(u^2v^2 \cdot 2u^4\right)^3$

27) $2x\left(x^4y^4\right)^4$

28) $\dfrac{3vu^5 \cdot 2v^3}{uv^2 \cdot 2u^3v}$

29) $\dfrac{2x^7y^5}{3x^3y \cdot 4x^2y^3}$

30) $\dfrac{2ba^7 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}$

31) $\left(\dfrac{(2x)^3}{x^3}\right)^2$

32) $\dfrac{2a^2b^2a^7}{(ba^4)^2}$

33) $\left(\dfrac{2y^{17}}{(2x^2y^4)^4}\right)^3$

34) $\dfrac{yx^2 \cdot (y^4)^2}{2y^4}$

35) $\left(\dfrac{2mn^4 \cdot 2m^4n^4}{mn^4}\right)^3$

36) $\dfrac{n^3(n^4)^2}{2mn}$

37) $\dfrac{2xy^5 \cdot 2x^2y^3}{2xy^4 \cdot y^3}$

38) $\dfrac{(2y^3x^2)^2}{2x^2y^4 \cdot x^2}$

39) $\dfrac{q^3r^2 \cdot (2p^2q^2r^3)^2}{2p^3}$

40) $\dfrac{2x^4y^5 \cdot 2z^{10}x^2y^7}{(xy^2z^2)^4}$

41) $\left(\dfrac{zy^3 \cdot z^3x^4y^4}{x^3y^3z^3}\right)^4$

42) $\left(\dfrac{2q^3p^3r^4 \cdot 2p^3}{(qrp^3)^2}\right)^4$

3.1 Answers

1) $4^9 = 262,144$
2) $4^7 = 16,384$
3) $2^4 = 16$
4) $3^6 = 729$
5) $12m^2n$
6) $12x^3$
7) $8m^6n^3$
8) x^3y^6
9) $3^{12} = 531,441$
10) $4^{12} = 16,777,216$
11) $4^8 = 65,536$
12) $3^6 = 729$
13) $4u^6v^4$
14) x^3y^3
15) $16a^{16}$
16) $16x^4y^4$
17) $4^2 = 16$
18) $3^4 = 81$
19) 3
20) $3^3 = 27$
21) m^2
22) $\frac{xy^3}{4}$
23) $\frac{4x^2y}{3}$
24) $\frac{y^2}{4}$
25) $4x^{10}y^{14}$
26) $8u^{18}v^6$
27) $2x^{17}y^{16}$
28) $3uv$
29) $\frac{x^2y}{6}$

30) $\frac{4a^2}{3}$

31) 64

32) $2a$

33) $\frac{y^3}{512x^{24}}$

34) $\frac{y^5x^2}{2}$

35) $64m^{12}n^{12}$

36) $\frac{n^{10}}{2m}$

37) $2x^2y$

38) $2y^2$

39) $2q^7r^8p$

40) $4x^2y^4z^2$

41) $x^4y^{16}z^4$

42) $256q^4r^8$