Section 3.3: Scientific Notation

Objective: Multiply and divide expressions using scientific notation and exponent properties.

One application of exponent properties comes from scientific notation. Scientific notation is used to represent very large or very small numbers. An example of really large numbers would be the distance that light travels in a year in miles.
An example of really small numbers would be the mass of a single hydrogen atom in grams. Doing basic operations, such as multiplication and division with these numbers, would normally be very cumbersome. However, our exponent properties make this process much simpler.
First, we will take a look at what scientific notation is. Scientific notation has two parts: the **mantissa**, a, which is a number between one and ten (it can be equal to one, but not ten), and that number multiplied by ten to some exponent, n, where n is an integer.

Scientific Notation: $\pm a \times 10^n$ **where** $1 \le a < 10$

The exponent, n, is very important to how we convert between scientific notation and normal numbers, or standard notation. The exponent tells us how many times we will multiply by 10. Multiplying by 10 moves the decimal point one place to the right. So the exponent will tell us how many times the decimal point moves between scientific notation and standard notation. To decide which direction to move the decimal (left or right), we simply need to remember that positive exponents mean we have a big number (bigger than ten) in standard notation, and negative exponents mean we have a small number (less than one) in standard notation.
Keeping this in mind, we can easily make conversions between standard notation and scientific notation.

Example 1. Convert to scientific notation.

14,200	Put decimal after first nonzero number
1.42	Exponent is how many places decimal moved; 4 places
$\times 10^4$	Positive exponent; standard notation is big
1.42×10^4	Our Solution

Example 2. Convert to scientific notation.

0.0042	Put decimal after first nonzero number
4.2	Exponent is how many places decimal moved, 3 places
$\times 10^{-3}$	Negative exponent; standard notation is small

4.2×10^{-3} Our Solution

Example 3. Convert to standard notation.

3.21×10^{5} Positive exponent means big number,
Move decimal right 5 places
321,000 Our Solution

Example 4. Convert to standard notation.

7.4×10^{-3} Negative exponent means small number
Move decimal left 3 places
0.0074 Our Solution

Converting between standard notation and scientific notation is important in understanding how scientific notation works and what it does. Here, our main interest is to be able to multiply and divide numbers in scientific notation using exponent properties. First, do the operation with the front numbers (multiply or divide); then use exponent properties to simplify the 10's. Scientific notation is the only time where we will be allowed to have negative exponents in our final solution. The negative exponent simply informs us that we are dealing with small numbers. Consider the following examples.

Example 5. Simplify and write the answer in scientific notation.

$(2.1\times10^{-7})(3.7\times10^{5})$ Deal with numbers and 10's separately
$(2.1)(3.7)=7.77$ Multiply numbers
$10^{-7}10^{5}=10^{-2}$ Use product rule on 10's; add exponents
7.77×10^{-2} Our Solution

Example 6. Simplify and write the answer in scientific notation.

$\frac{4.96\times10^{4}}{3.1\times10^{-3}}$ Deal with numbers and 10's separately
$\frac{4.96}{3.1}=1.6$ Divide numbers
$\frac{10^{4}}{10^{-3}}=10^{7}$ Use quotient rule; subtract exponents; be careful with negatives
Be careful with negatives, $4-(-3)=4+3=7$
1.6×10^{7} Our Solution

Example 7. Simplify and write the answer in scientific notation.

$(1.8\times10^{-4})^{3}$ Use power rule to deal with numbers and 10's separately
$1.8^{3}=5.832$ Evaluate 1.8^{3}

$(10^{-4})^3 = 10^{-12}$ Our Solution

Often when we multiply or divide in scientific notation, the end result is not in scientific notation. We will have to convert the front number or mantissa to scientific notation; then combine the 10's using the product property of exponents to add the exponents. This is shown in the following examples.

Example 8. Simplify and write the answer in scientific notation.

$(4.7\times10^{-3})(6.1\times10^{9})$	Deal with numbers and 10's separately
$(4.7)(6.1) = 28.67$	Multiply numbers
2.867×10^{1}	Convert this number to scientific notation
$\mathbf{10}^{1}10^{-3}10^{9} = 10^{7}$	Use product rule; add exponents; use 10^{1} from conversion
2.867×10^{7}	Our Solution

World View Note: Archimedes (287 BC - 212 BC), the Greek mathematician, developed a system for representing large numbers using a system very similar to scientific notation. He used his system to calculate the number of grains of sand it would take to fill the universe. His conclusion was 1063 grains of sand, because he figured the universe to have a diameter of 10^{14} stadia or about 2 light years.

Example 9. Simplify and write the answer in scientific notation.

$\dfrac{2.014\times10^{-3}}{3.8\times10^{-7}}$	Deal with numbers and 10's separately
$\dfrac{2.014}{3.8} = 0.53$	Divide numbers
$0.53 = 5.3\times10^{-1}$	Change this number to scientific notation
$\dfrac{\mathbf{10}^{-1}10^{-3}}{10^{-7}} = 10^{3}$	Use product and quotient rule; use 10^{-1} from the conversion Be careful with signs: $(-1)+(-3)-(-7) = (-1)+(-3)+7 = 3$
5.3×10^{3}	Our Solution

3.3 Practice

Write each number in scientific notation.

1) 885
2) 0.000744
3) 0.081
4) 1.09
5) 0.039
6) 15000

Write each number in standard notation.

7) 8.7×10^{5}
8) 2.56×10^{2}
9) 9×10^{-4}
10) 5×10^{4}
11) 2×10^{0}
12) 6×10^{-5}

Simplify. Write each answer in scientific notation and when necessary, round the mantissa to 3 decimal places.

13) $(7\times10^{-1})(2\times10^{-3})$
14) $(2\times10^{-6})(8.8\times10^{-5})$
15) $(5.26\times10^{-5})(3.16\times10^{-2})$
16) $(5.1\times10^{6})(9.84\times10^{-1})$
17) $(2.6\times10^{-2})(6\times10^{-2})$
18) $\dfrac{7.4\times10^{4}}{1.7\times10^{-4}}$
19) $\dfrac{4.9\times10^{1}}{2.7\times10^{-3}}$
20) $\dfrac{7.2\times10^{-1}}{7.32\times10^{-1}}$
21) $\dfrac{5.33\times10^{-6}}{9.62\times10^{-2}}$
22) $\dfrac{3.2\times10^{-3}}{5.02\times10^{0}}$
23) $(5.5\times10^{-5})^{2}$
24) $(9.6\times10^{3})^{-4}$

25) $(7.8\times10^{-2})^{5}$

26) $(5.4\times10^{6})^{-3}$

27) $(8.03\times10^{4})^{-4}$

28) $(6.88\times10^{-4})(4.23\times10^{1})$

29) $\dfrac{6.1\times10^{-6}}{5.1\times10^{-4}}$

30) $\dfrac{8.4\times10^{5}}{7\times10^{-2}}$

31) $(3.6\times10^{0})(6.1\times10^{-3})$

32) $(3.15\times10^{3})(8\times10^{-1})$

33) $(1.8\times10^{-5})^{-3}$

34) $\dfrac{9.58\times10^{-2}}{1.14\times10^{-3}}$

35) $\dfrac{9\times10^{4}}{7.83\times10^{-2}}$

36) $(8.3\times10^{1})^{5}$

37) $\dfrac{3.22\times10^{-3}}{7\times10^{-6}}$

38) $\dfrac{5\times10^{6}}{6.69\times10^{2}}$

3.3 Answers

1) 8.85×10^{2}
2) 7.44×10^{-4}
3) 8.1×10^{-2}
4) 1.09×10^{0}
5) 3.9×10^{-2}
6) 1.5×10^{4}
7) 870000
8) 256
9) 0.0009
10) 50000
11) 2
12) 0.00006
13) 1.4×10^{-3}
14) 1.76×10^{-10}
15) 1.662×10^{-6}
16) 5.018×10^{6}
17) 1.56×10^{-3}
18) 4.353×10^{8}
19) 1.815×10^{4}
20) 9.836×10^{-1}
21) 5.541×10^{-5}
22) 6.375×10^{-4}
23) 3.025×10^{-9}
24) 1.177×10^{-16}
25) 2.887×10^{-6}
26) 6.351×10^{-21}
27) 2.405×10^{-20}
28) 2.91×10^{-2}
29) 1.196×10^{-2}
30) 1.2×10^{7}
31) 2.196×10^{-2}
32) 2.52×10^{3}
33) 1.715×10^{14}
34) 8.404×10^{1}

35) 1.149×10^{6}

36) 3.939×10^{9}

37) 4.6×10^{2}

38) 7.474×10^{3}

39) 3.692×10^{-7}