Section 3.5: Multiplying Polynomials

Objective: Multiply polynomials.

Multiplying polynomials can take several different forms based on what we are multiplying. We will first look at multiplying monomials; then we will multiply monomials by polynomials; and finish with multiplying polynomials by polynomials.

Multiplying monomials is done by multiplying the numbers or coefficients; then adding the exponents on like variable factors. This is shown in the next example.

Example 1. Simplify.

$(4x^3y^4z)(2x^2y^6z^3)$	Multiply numbers and add exponents for x, y, and z
$8x^5y^{10}z^4$	Our Solution

In the previous example, it is important to remember that the z has an exponent of 1 when no exponent is written. Thus, for our answer, the z has an exponent of $1+3=4$. Be very careful with exponents in polynomials. If we are adding or subtracting polynomials, the exponents will stay the same, but when we multiply (or divide) the exponents will be changing.

Next, we consider multiplying a monomial by a polynomial. We have seen this operation before when distributing through parentheses. Here we will see the exact same process.

Example 2. Simplify.

$4x^3(5x^2-2x+5)$	Distribute the $4x^3$; multiply numbers; add exponents
$20x^5-8x^4+20x^3$	Our Solution

Next, we have another example with more variables. When distributing, the exponents on a are added and the exponents on b are added.

Example 3. Simplify.

$2a^3b(3ab^2-4a)$	Distribute; multiply numbers; add exponents
$6a^4b^3-8a^4b$	Our Solution

There are several different methods for multiplying polynomials. Often students prefer the method they are first taught. Here, two methods will be discussed.

Both methods will be used to perform the same two multiplication problems.

Multiply by Distributing

Just as we distribute a monomial through parentheses, we can distribute an entire polynomial. As we do this, we take each term of the second polynomial and put it in front of the first polynomial.

Example 4. Simplify.

$(4x+7y)(3x-2y)$	Distribute $(4x+7y)$ through parentheses
$3x(\mathbf{4x+7y})-2y(\mathbf{4x+7y})$	Distribute the $3x$ and $-2y$
$12x^2+21xy-8xy-14y^2$	Combine like terms $21xy-8xy$
$12x^2+13xy-14y^2$	Our Solution

This example illustrates an important point that the negative/subtraction sign stays with the $2y$. On the second step, the negative is also distributed through the last set of parentheses.

Multiplying by distributing can easily be extended to problems with more terms. First, distribute the front parentheses onto each term; then distribute again.

Example 5. Simplify.

$(2x-5)(4x^2-7x+3)$	Distribute $(2x-5)$ through parentheses
$4x^2(\mathbf{2x-5})-7x(\mathbf{2x-5})+3(\mathbf{2x-5})$	Distribute again through each parentheses
$8x^3-20x^2-14x^2+35x+6x-15$	Combine like terms
$8x^3-34x^2+41x-15$	Our Solution

This process of multiplying by distributing can easily be reversed to do an important procedure known as factoring. Factoring will be addressed in a future lesson.

Multiply by FOIL

Another form of multiplying is known as FOIL. Using the FOIL method we multiply each term in the first binomial by each term in the second binomial. The letters of FOIL help us remember every combination. F stands for First, and we multiply the first term of each binomial. O stands for Outside, and we multiply the outside two terms. I stands for Inside, and we multiply the inside two terms. L stands for Last, and we multiply the last term of each binomial. This is shown in the next example.

Example 6. Simplify.

$(4x+7y)(3x-2y)$	Use FOIL to multiply
$(4x)(3x)=12x^2$	F – First terms $(4x)\,(3x)$
$(4x)(-2y)=-8xy$	O – Outside terms $(4x)\,(-2y)$

$(7y)(3x)=21xy$	I – Inside terms $(7y)\,(3x)$
$(7y)(-2y)=-14y^2$	L – Last terms $(7y)\,(-2y)$
$12x^2-8xy+21xy-14y^2$	Combine like terms $-8xy+21xy$
$12x^2+13xy-14y^2$	Our Solution

In reality, the FOIL method is a shortcut for distributing each of the terms in the first set of parentheses by all of the terms in the second set of parentheses. In the previous example, the first term, $4x$, is distributed through the $(3x-2y)$ and then the second term, $7y$, is distributed through the $(3x-2y)$. By distributing in this manner, it possible to multiply polynomials containing more than two terms.

Example 7. Simplify.

$(2x-5)\,(4x^2-7x+3)$	Distribute $2x$ and -5 to each term of the trinomial in the second set of parentheses
$(2x)(4x^2)+(2x)(-7x)+(2x)(3)-5(4x^2)-5(-7x)-5(3)$	Multiply out each term
$8x^3-14x^2+6x-20x^2+35x-15$	Combine like terms
$8x^3-34x^2+41x-15$	Our Solution

When we are multiplying a monomial by a polynomial by a polynomial, we can first multiply the polynomials; then distribute the monomial last. This is shown in the last example.

Example 8. Simplify.

$3(2x-4)(x+5)$	Multiply the binomials; use FOIL
$3(2x^2+10x-4x-20)$	Combine like terms
$3(2x^2+6x-20)$	Distribute the 3
$6x^2+18x-60$	Our Solution

A common error students do is distribute the three at the start into both parentheses. While we can distribute the 3 into the $(2x-4)$ factor, distributing into both would be wrong. Be careful of this error. This is why it is suggested to multiply the binomials first; then distribute the monomial last.

3.5 Practice

Find each product and simplify your answers.

1) $6(p-7)$
2) $4k(8k+4)$
3) $2(6x+3)$
4) $3n^2(6n+7)$
5) $5m^4(4m+4)$
6) $3(4r-7)$
7) $(4n+6)(8n+8)$
8) $(2x+1)(x-4)$
9) $(8b+3)(7b-5)$
10) $(r+8)(4r+8)$
11) $(4x+5)(2x+3)$
12) $(7n-6)(n+7)$
13) $(3v-4)(5v-2)$
14) $(6a+4)(a-8)$
15) $(6x-7)(4x+1)$
16) $(5x-6)(4x-1)$
17) $(5x+y)(6x-4y)$
18) $(2u+3v)(8u-7v)$
19) $(x+3y)(3x+4y)$
20) $(8u+6v)(5u-8v)$
21) $(7x+5y)(8x+3y)$
22) $(5a+8b)(a-3b)$
23) $(r-7)(6r^2-r+5)$
24) $(4x+8)(4x^2+3x+5)$
25) $(6n-4)(2n^2-2n+5)$
26) $(2b-3)(4b^2+4b+4)$
27) $(6x+3y)(6x^2-7xy+4y^2)$
28) $(3m-2n)(7m^2+6mn+4n^2)$
29) $(8n^2+4n+6)(6n^2-5n+6)$
30) $(2a^2+6a+3)(7a^2-6a+1)$
31) $(5k^2+3k+3)(3k^2+3k+6)$

32) $(7u^2 + 8uv - 6v^2)(6u^2 + 4uv + 3v^2)$

33) $3(3x - 4)(2x + 1)$

34) $5(x - 4)(2x - 3)$

35) $3(2x + 1)(4x - 5)$

36) $2(4x + 1)(2x - 6)$

37) $7(x - 5)(x - 2)$

38) $5(2x - 1)(4x + 1)$

39) $6(4x - 1)(4x + 1)$

40) $3(2x + 3)(6x + 9)$

3.5 Answers

1) $6p-42$
2) $32k^2+16k$
3) $12x+6$
4) $18n^3+21n^2$
5) $20m^5+20m^4$
6) $12r-21$
7) $32n^2+80n+48$
8) $2x^2-7x-4$
9) $56b^2-19b-15$
10) $4r^2+40r+64$
11) $8x^2+22x+15$
12) $7n^2+43n-42$
13) $15v^2-26v+8$
14) $6a^2-44a-32$
15) $24x^2-22x-7$
16) $20x^2-29x+6$
17) $30x^2-14xy-4y^2$
18) $16u^2+10uv-21v^2$
19) $3x^2+13xy+12y^2$
20) $40u^2-34uv-48v^2$
21) $56x^2+61xy+15y^2$
22) $5a^2-7ab-24b^2$
23) $6r^3-43r^2+12r-35$
24) $16x^3+44x^2+44x+40$
25) $12n^3-20n^2+38n-20$
26) $8b^3-4b^2-4b-12$
27) $36x^3-24x^2y+3xy^2+12y^3$
28) $21m^3+4m^2n-8n^3$
29) $48n^4-16n^3+64n^2-6n+36$
30) $14a^4+30a^3-13a^2-12a+3$
31) $15k^4+24k^3+48k^2+27k+18$
32) $42u^4+76u^3v+17u^2v^2-18v^4$
33) $18x^2-15x-12$
34) $10x^2-55x+60$

35) $24x^2 - 18x - 15$

36) $16x^2 - 44x - 12$

37) $7x^2 - 49x + 70$

38) $40x^2 - 10x - 5$

39) $96x^2 - 6$

40) $36x^2 + 108x + 81$