Section 3.6: Special Products

Objective: Recognize and use special product rules of a sum and a difference and perfect squares to multiply polynomials.

There are a few shortcuts that we can take when multiplying polynomials. If we can recognize them, the shortcuts can help us arrive at the solution much faster. These shortcuts will also be useful to us as our study of algebra continues.

The first shortcut is often called a **sum and a difference**. A sum and a difference is easily recognized as the numbers and variables are exactly the same, but the sign in the middle is different (one sum, one difference). To illustrate the shortcut, consider the following example, where we multiply using the distributing method.

Example 1. Simplify.

$(a+b)(a-b)$	Distribute $(a+b)$
$a(a+b)-b(a+b)$	Distribute a and $-b$
$a^2+ab-ab-b^2$	Combine like terms $ab-ab$
a^2-b^2	Our Solution

The important part of this example is that the middle terms subtracted to zero. Rather than going through all this work, when we have a sum and a difference, we will jump right to our solution by squaring the first term and squaring the last term, putting a subtraction between them. This is illustrated in the following example.

Example 2. Simplify.

$(x-5)(x+5)$	Recognize sum and difference
	Square both x and 5; put subtraction between the squares
x^2-25	Our Solution

This is much quicker than going through the work of multiplying and combining like terms. Often students ask if they can just multiply out using another method and not learn the shortcut. These shortcuts are going to be very useful when we get to factoring polynomials, or reversing the multiplication process. For this reason it is very important to be able to recognize these shortcuts. More examples are shown below.

Example 3. Simplify.

$(3x+7)(3x-7)$	Recognize sum and difference
	Square both $3x$ and 7; put subtraction between the squares
$9x^2-49$	Our Solution

Example 4. Simplify.

$(2x-6y)(2x+6y)$	Recognize sum and difference
	Square both $2x$ and $6y$; put subtraction between the squares
$4x^2-36y^2$	Our Solution

It is interesting to note that while we can multiply and get an answer like a^2-b^2 (with subtraction), it is impossible to multiply binomial expressions and end up with a product such as a^2+b^2 (with addition).

There is also a shortcut to multiply **a perfect square**, which is a binomial raised to the power two. The following example illustrates multiplying a perfect square.

Example 5. Simplify.

$(a+b)^2$	Square or multiply $(a+b)$ by itself
$(a+b)(a+b)$	Distribute $(a+b)$
$a(a+b)+b(a+b)$	Distribute again through final parentheses
$a^2+ab+ab+b^2$	Combine like terms $ab+ab$
$a^2+2ab+b^2$	Our Solution

This problem also helps us find our shortcut for multiplying. The first term in the answer is the **square of the first term** in the problem. The middle term is **2 times the first term times the second term**. The last term is the **square of the last term**. This can be shortened to square the first, twice the product, and square the last. If we can remember this shortcut, we can square any binomial. This is illustrated in the following example.

Example 6. Simplify.

$(x-5)^2$	Recognize perfect square
x^2	Square the first
$2(x)(-5)=-10x$	Twice the product
$(-5)^2=25$	Square the last
$x^2-10x+25$	Our Solution

Be very careful when squaring a binomial to **NOT** distribute the square through the parentheses. A common error is to do the following: $(x-5)^2=x^2-25$. Notice that both of these are missing the middle term, $-10x$. This is why it is important to use the shortcut to help us find the correct solution. Another important observation is that the second term in the solution always has the same sign as the second term in the problem. This is illustrated in the next examples.

Example 7. Simplify.

$(2x+5)^2$	Recognize perfect square
$(2x)^2 = 4x^2$	Square the first
$2(2x)(5) = 20x$	Twice the product
$5^2 = 25$	Square the last
$4x^2 + 20x + 25$	Our Solution

Example 8. Simplify.

$(3x-7y)^2$	Recognize perfect square
	Square the first; twice the product; square the last
$9x^2 - 42xy + 49y^2$	Our Solution

Example 9. Simplify.

$(5a+9b)^2$	Recognize perfect square
	Square the first; twice the product; square the last
$25a^2 + 90ab + 81b^2$	Our Solution

These two formulas will be important to commit to memory. The more familiar we are with them, the easier factoring, or multiplying in reverse, will be. The final example covers both types of problems (sum and difference and two perfect squares - one positive, one negative). Be sure to notice the difference between the examples and how each formula is used.

Example 10. Simplify each expression.

$(4x-7)(4x+7)$	$(4x+7)^2$	$(4x-7)^2$
$16x^2 - 49$	$16x^2 + 56x + 49$	$16x^2 - 56x + 49$

World View Note: There are also formulas for higher powers of binomials as well, such as $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. While French mathematician Blaise Pascal often gets credit for working with these expansions of binomials in the 17th century, Chinese mathematicians had been working with them almost 400 years earlier!

3.6 Practice

Find each product and simplify your answers.

1) $(x+8)(x-8)$
2) $(a-4)(a+4)$
3) $(1+3p)(1-3p)$
4) $(x-3)(x+3)$
5) $(1-7n)(1+7n)$
6) $(8m+5)(8m-5)$
7) $(5n-8)(5n+8)$
8) $(2x+3)(2x-3)$
9) $(4x+8)(4x-8)$
10) $(b-7)(b+7)$
11) $(4y-x)(4y+x)$
12) $(7a+7b)(7a-7b)$
13) $(4m-8n)(4m+8n)$
14) $(3y-3x)(3y+3x)$
15) $(6x-2y)(6x+2y)$
16) $(1+5n)^2$
17) $(a+5)^2$
18) $(v+4)^2$
19) $(x-8)^2$
20) $(1-6n)^2$
21) $(p+7)^2$
22) $(7k-7)^2$
23) $(7-5n)^2$
24) $(4x-5)^2$
25) $(5m-8)^2$
26) $(3a+3b)^2$
27) $(5x+7y)^2$
28) $(4m-n)^2$
29) $(2x+2y)^2$
30) $(8x+5y)^2$
31) $(5+2r)^2$

32) $(m-7)^2$

33) $(2+5x)^2$

34) $(8n+7)\,(8n-7)$

35) $(4v-7)\,(4v+7)$

36) $(b+4)\,(b-4)$

37) $(n-5)\,(n+5)$

38) $(7x+7)^2$

39) $(4k+2)^2$

40) $(3a-8)(3a+8)$

3.6 Answers

1) $x^2 - 64$
2) $a^2 - 16$
3) $1 - 9p^2$
4) $x^2 - 9$
5) $1 - 49n^2$
6) $64m^2 - 25$
7) $25n^2 - 64$
8) $4x^2 - 9$
9) $16x^2 - 64$
10) $b^2 - 49$
11) $16y^2 - x^2$
12) $49a^2 - 49b^2$
13) $16m^2 - 64n^2$
14) $9y^2 - 9x^2$
15) $36x^2 - 4y^2$
16) $1 + 10n + 25n^2$
17) $a^2 + 10a + 25$
18) $v^2 + 8v + 16$
19) $x^2 - 16x + 64$
20) $1 - 12n + 36n^2$
21) $p^2 + 14p + 49$
22) $49k^2 - 98k + 49$
23) $49 - 70n + 25n^2$
24) $16x^2 - 40x + 25$
25) $25m^2 - 80m + 64$
26) $9a^2 + 18ab + 9b^2$
27) $25x^2 + 70xy + 49y^2$
28) $16m^2 - 8mn + n^2$
29) $4x^2 + 8xy + 4y^2$
30) $64x^2 + 80xy + 25y^2$
31) $25 + 20r + 4r^2$
32) $m^2 - 14m + 49$
33) $4 + 20x + 25x^2$
34) $64n^2 - 49$

35) $16v^2 - 49$
36) $b^2 - 16$
37) $n^2 - 25$
38) $49x^2 + 98x + 49$
39) $16k^2 + 16k + 4$
40) $9a^2 - 64$