Section 4.1: Factoring Using the Greatest Common Factor

Objective: Find the greatest common factor of a polynomial and factor it out of the expression.

The inverse of multiplying polynomials together is factoring polynomials. There are many benefits of a polynomial being factored. We use factored polynomials to help us solve equations, learn behaviors of graphs, work with fractions and more. Because so many concepts in algebra depend on us being able to factor polynomials, it is very important to have very strong factoring skills.

In this lesson, we will focus on factoring using the greatest common factor or GCF of a polynomial. When we multiplied polynomials, we multiplied monomials by polynomials by distributing, such as $4x^2(2x^2-3x+8)=8x^4-12x^3+32x^2$. In this lesson, we will work the same problem backwards. For example, we will start with $8x^4-12x^3+32x^2$ and try and work backwards to the $4x^2(2x^2-3x+8)$.

To do this, we have to be able to first identify what is the GCF of a polynomial. We will introduce this idea by looking at finding the GCF of several numbers. To find the GCF of several numbers, we are looking for the largest number that can divide each number without leaving a remainder. This can often be done with quick mental math. See the example below.

Example 1. Determine the greatest common factor.

Find the GCF of 15, 24, and 27

$\frac{15}{3}=5, \frac{24}{3}=6, \frac{27}{3}=9$ Each number can be divided by 3

$\text{GCF}=3$ Our Solution

When there are variables in our problem, we can first find the GCF of the numbers using mental math. Then, we take any variables that are in common with each term, using the lowest exponent. This is shown in the next example.

Example 2. Determine the greatest common factor.

Find the GCF of $24x^4y^2z$, $18x^2y^4$, and $12x^3yz^5$

$\frac{24}{6}=4, \frac{18}{6}=3, \frac{12}{6}=2$ Each number can be divided by 6.

x^2y x and y are in all 3; use lowest exponents

$$\text{GCF} = 6x^2y \quad \text{Our Solution}$$

To factor out a GCF from a polynomial, we first need to identify the GCF of all the terms. This is the part that goes in front of the parentheses. Then we divide each term by the GCF, and the quotients go inside the parentheses. This is shown in the following examples.

Example 3. Factor using the greatest common factor.

$$4x^2 - 20x + 16 \quad \text{GCF is 4; divide each term by 4}$$

$$\frac{4x^2}{4} = x^2, \frac{-20x}{4} = -5x, \frac{16}{4} = 4 \quad \text{Result is what is left in parentheses}$$

$$4(x^2 - 5x + 4) \quad \text{Our Solution}$$

With factoring, we can always check our solutions by multiplying (or distributing), and the product should be the original expression.

Example 4. Factor using the greatest common factor.

$$25x^4 - 15x^3 + 20x^2 \quad \text{GCF is } 5x^2\text{; divide each term by } 5x^2$$

$$\frac{25x^4}{5x^2} = 5x^2, \frac{-15x^3}{5x^2} = -3x, \frac{20x^2}{5x^2} = 4 \quad \text{Result is what is left in parentheses}$$

$$5x^2(5x^2 - 3x + 4) \quad \text{Our Solution}$$

Example 5. Factor using the greatest common factor.

$$3x^3y^2z + 5x^4y^3z^5 - 4xy^4 \quad \text{GCF is } xy^2\text{; divide each term by } xy^2$$

$$\frac{3x^3y^2z}{xy^2} = 3x^2z, \frac{-5x^4y^3z^5}{xy^2} = 5x^3yz^5, \frac{-4xy^4}{xy^2} = -4y^2 \quad \text{Result is what is left in parentheses}$$

$$xy^2(3x^2z + 5x^3yz^5 - 4y^2) \quad \text{Our Solution}$$

World View Note: The first recorded algorithm for finding the greatest common factor comes from the Greek mathematician Euclid around the year 300 BC!

Example 6. Factor using the greatest common factor.

$$21x^3 + 14x^2 + 7x \quad \text{GCF is } 7x\text{; divide each term by } 7x$$

$$\frac{21x^3}{7x} = 3x^2, \frac{14x^2}{7x} = 2x, \frac{7x}{7x} = 1 \quad \text{Result is what is left in parentheses}$$

$$7x(3x^2 + 2x + 1) \quad \text{Our Solution}$$

It is important to note in the previous example, that when the GCF was $7x$ and $7x$ was one of the terms, dividing gave an answer of 1. Students often try to factor out the $7x$ and get zero which is incorrect. Factoring will never make terms disappear. Anything divided by itself is 1, so be sure to not forget to put the 1 into the solution.

In the next example, we will factor out the negative of the GCF. Whenever the first term of a polynomial is negative, we will factor out the negative of the GCF.

Example 7. Factor using the negative of the greatest common factor.

$-12x^5y^2 + 6x^4y^4 - 8x^3y^5$ Negative of the GCF is $-2x^3y^2$; divide each term by $-2x^3y^2$

$\frac{-12x^5y^2}{-2x^3y^2} = 6x^2, \frac{6x^4y^4}{-2x^3y^2} = -3xy^2, \frac{-8x^3y^5}{-2x^3y^2} = 4y^3$ Result is what is left in parentheses

$-2x^3y^2(6x^2 - 3xy^2 + 4y^3)$ Our Solution

Often the second step is not shown in the work of factoring the GCF. We can simply identify the GCF and put it in front of the parentheses as shown in the following two examples.

Example 8. Factor using the greatest common factor.

$18a^4b^3 - 27a^3b^3 + 9a^2b^3$ GCF is $9a^2b^3$; divide each term by $9a^2b^3$

$9a^2b^3(2a^2 - 3a + 1)$ Our Solution

Again, in the previous example, when dividing $9a^2b^3$ by itself, the answer is 1, not zero. Be very careful that each term is accounted for in your final solution.

It is possible to have a problem where the GCF is 1. If the GCF is 1, then the polynomial cannot be factored. In this case, we state that the polynomial is **prime**. This is shown in the following example.

Example 9. Factor using the greatest common factor.

$8ab - 17c + 49$ GCF is 1 because there are no other factors in common to all 3 terms

$8ab - 17c + 49$ Our Solution: Prime

4.1 Practice

Factor each polynomial using the greatest common factor. If the first term of the polynomial is negative, then factor out the negative of the greatest common factor. If the expression cannot be factored, state that it is *prime*.

1) $9+8b^2$
2) $x-5$
3) $45x^2-25$
4) $-1-2n^2$
5) $56-35p$
6) $50x-80y$
7) $8ab-35a^2b$
8) $27x^2y^5-72x^3y^2$
9) $-3a^2b+6a^3b^2$
10) $8x^3y^2+4x^3$
11) $-5x^2-5x^3-15x^4$
12) $-32n^9+32n^6+40n^5$
13) $20x^4-30x+30$
14) $21p^6+30p^2+27$
15) $20x^4-30x+30$
16) $-10x^4+20x^2+12x$
17) $30b^9+5ab-15a^2$
18) $27y^7+12y^2x+9y^2$
19) $-48a^2b^2-56a^3b-56a^5b$
20) $30m^6+15mn^2-25$
21) $20x^8y^2z^2+15x^5y^2z+35x^3y^3z$
22) $3p+12q-15q^2r^2$
23) $50x^2y+10y^2+70xz^2$
24) $30y^4z^3x^5+50y^4z^5-10y^4z^3x$
25) $30qpr-5qp+5q$
26) $28b+14b^2+35b^3+7b^5$
27) $-18n^5+3n^3-21n+3$
28) $30a^8+6a^5+27a^3+21a^2$
29) $-40x^{11}-20x^{12}+50x^{13}-50x^{14}$
30) $-24x^6-4x^4+12x^3+4x^2$
31) $-32mn^8+4m^6n+12mn^4+16mn$

4.1 Answers

1) prime
2) prime
3) $5(9x^2-5)$
4) $-1\left(1+2n^2\right)$
5) $7\,(8-5p)$
6) $10\,(5x-8y)$
7) $ab(8-35a)$
8) $9x^2y^2\,(3y^3-8x)$
9) $-3a^2b\,(1-2ab)$
10) $4x^3\,(2y^2+1)$
11) $-5x^2\,(1+x+3x^2)$
12) $-8n^5\,(4n^4-4n-5)$
13) $10\,(2x^4-3x+3)$
14) $3\,(7p^6+10p^2+9)$
15) $4\,(7m^4+10m^3+2)$
16) $-2x\,(5x^3-10x-6)$
17) $5\,(6b^9+ab-3a^2)$
18) $3y^2\,(9y^5+4x+3)$
19) $-8a^2b(6b+7a+7a^3)$
20) $5\,(6m^6+3mn^2-5)$
21) $5x^3y^2z\,(4x^5z+3x^2+7y)$
22) $3\,(p+4q-5q^2r^2)$
23) $10\,(5x^2y+y^2+7xz^2)$
24) $10y^4z^3\,(3x^5+5z^2-x)$
25) $5q\,(6pr-p+1)$
26) $7b\,(4+2b+5b^2+b^4)$
27) $-3\,(6n^5-n^3+7n-1)$
28) $3a^2\,(10a^6+2a^3+9a+7)$
29) $-10x^{11}(4+2x-5x^2+5x^3)$
30) $-4x^2(6x^4+x^2-3x-1)$
31) $-4mn\,(8n^7-m^5-3n^3-4)$