Section 4.2: Factoring by Grouping

Objective: Factor polynomials with four terms using grouping.

The first thing we will always do, when factoring, is try to factor out a GCF. This GCF is often a monomial. For example, in the problem $5xy+10xz$, the GCF is the monomial $5x$; so, the factored expression is $5x(y+2z)$. However, a GCF does not have to be a monomial; it could be a binomial. To see this, consider the following two examples.

Example 1. Factor completely.

$3ax-7bx$	Both have x in common; factor out x
$x(3a-7b)$	Our Solution

In the next example, we have the same type of problem as in the example above; but, instead of x, the GCF is $(2a+5b)$.

Example 2. Factor completely.

$3a(2a+5b)-7b(2a+5b)$	Both have $(2a+5b)$ in common; factor out $(2a+5b)$
$(2a+5b)(3a-7b)$	Our Solution

In Example 2, we factored out a GCF that is a binomial, $(2a+5b)$. We will use this process of factoring a binomial GCF when the original polynomial has four terms and its GCF is 1.

When we need to factor a polynomial with four terms whose GCF is 1, we will have to use another strategy to factor. We will use a process known as **grouping**. We use grouping when factoring a polynomial with four terms. Remember, factoring is the reverse of multiplying, so first we will look at a multiplication problem and then try to reverse the process.

Example 3. Multiply.

$(2a+3)(5b+2)$	Distribute $(2a+3)$ into second parentheses
$5b(2a+3)+2(2a+3)$	Distribute each monomial
$10ab+15b+4a+6$	Our Solution

The product has four terms in it. We arrived at the solution by looking at the two parts, $5b(2a+3)$ and $2(2a+3)$. When we are factoring by grouping we will always divide the problem into two parts: the first two terms and the last two terms. Then, we can factor the GCF out of both the left and right groups. When we do this, our hope is what remains in the parentheses will match on both the left term and the right term. If they match, we can pull this matching GCF out front, putting the rest in parentheses, and the expression will be factored. The next example is the same problem worked backwards, factoring instead of multiplying.

Example 4. Factor completely.

$10ab+15b+4a+6$	Split expressions into two groups
$\boxed{10ab+15b}\boxed{+4a+6}$	GCF on left is $5b$; GCF on right is 2
$\boxed{5b(2a+3)}\boxed{+2(2a+3)}$	$(2a+3)$ is the same; factor out this GCF
$(2a+3)(5b+2)$	Our Solution

The key, for grouping to work, is after the GCF is factored out of the left and right groups, the two binomials must match exactly. If there is any difference between the two, we either have to do some adjusting or it cannot be factored using the grouping method. Consider the following example.

Example 5. Factor completely.

$6x^2+9xy-14x-21y$	Split expression into two groups
$\boxed{6x^2+9xy}\boxed{-14x-21y}$	GCF on left is $3x$; GCF on right is 7
$\boxed{3x(2x+3y)}\boxed{+7(-2x-3y)}$	The signs in the parentheses don't match!

When the signs don't match in both terms, we can easily make them match by factoring the negative of the GCF on the right side. Instead of 7 we will use -7. This will change the signs inside the second parentheses. In general, if the third term of the four - term expression is subtracted, then factor out the negative of the GCF for the second group of two terms.

$\boxed{3x(2x+3y)}\boxed{-7(2x+3y)}$	$(2x+3y)$ is the same; factor out this GCF
$(2x+3y)(3x-7)$	Our Solution

Often we can recognize early that we need to use the negative of the GCF when factoring. If the first term of the first binomial is positive in the problem, we will also want the first term of the second binomial to be positive. If the first term of the second binomial is negative, then we will use the negative of the GCF to be sure they match.

Example 6. Factor completely.

$5xy-8x-10y+16$	Split expression into two groups
$\boxed{5xy-8x}\boxed{-10y+16}$	GCF on left is x; GCF on right is -2
$\boxed{x(5y-8)}\boxed{-2(5y-8)}$	$(5y-8)$ is the same; factor out this GCF
$(5y-8)(x-2)$	Our Solution

Sometimes when factoring the GCF out of the left or right group there is no GCF to factor out other than one. In this case we will use either the GCF of 1 or -1. Often this is all we need to be sure the two binomials match.

Example 7. Factor completely.

$12ab-14a-6b+7$	Split expression into two groups
$\boxed{12ab-14a}\boxed{-6b+7}$	GCF on left is $2a$; negative of GCF on right is -1
$\boxed{2a(6b-7)}\boxed{-1(6b-7)}$	$(6b-7)$ is the same; factor out this GCF
$(6b-7)(2a-1)$	Our Solution

Example 8. Factor completely.

$6x^3-15x^2+2x-5$	Split expression into two groups
$\boxed{6x^3-15x^2}\boxed{+2x-5}$	GCF on left is $3x^2$; GCF on right is 1
$\boxed{3x^2(2x-5)}\boxed{+1(2x-5)}$	$(2x-5)$ is the same; factor out this GCF
$(2x-5)(3x^2+1)$	Our Solution

Example 9. Factor completely.

$12x^3+6x^2+5x+10$	Split expression into two groups
$\boxed{12x^3+6x^2}\boxed{+5x+10}$	GCF on left is $6x^2$; GCF on right is 5
$\boxed{6x^2(2x+1)}\boxed{+5(x+2)}$	Factors are not the same. This will happen even if we rearrange the terms. Therefore, it cannot be factored by grouping.
$12x^3+6x^2+5x+10$	Our Solution, Prime

4.2 Practice

Factor each expression completely.

1) $40r^3 - 8r^2 - 25r + 5$
2) $35x^3 - 10x^2 - 56x + 16$
3) $3n^3 - 2n^2 - 9n + 6$
4) $14v^3 + 10v^2 - 7v - 5$
5) $15b^3 + 21b^2 - 35b - 49$
6) $6x^3 - 48x^2 + 5x - 40$
7) $3x^3 + 15x^2 + 2x + 10$
8) $9x^3 + 3x^2 + 4x + 8$
9) $35x^3 - 28x^2 - 20x + 16$
10) $7n^3 + 21n^2 - 5n - 15$
11) $7xy - 49x + 5y - 35$
12) $42r^3 - 49r^2 + 18r - 21$
13) $32xy + 40x^2 + 12y + 15x$
14) $15ab - 6a + 5b^3 - 2b^2$
15) $16xy - 56x + 2y - 7$
16) $3mn - 8m + 15n - 40$
17) $x^3 - 5x^2 + 7x - 21$
18) $5mn + 2m - 25n - 10$
19) $40xy + 35x - 8y^2 - 7y$
20) $6a^2 + 3a - 4b^2 + 2b$
21) $32uv - 20u + 24v - 15$
22) $4uv + 14u^2 + 12v + 42u$
23) $10xy + 25x + 12y + 30$
24) $24xy - 20x - 30y^3 + 25y^2$
25) $3uv - 6u^2 - 7v + 14u$
26) $56ab - 49a - 16b + 14$
27) $2xy - 8x^2 + 7y^3 - 28y^2x$
28) $28p^3 + 21p^2 + 20p + 15$
29) $16xy - 6x^2 + 8y - 3x$
30) $8xy + 56x - y - 7$

4.2 Answers

1) $(8r^2-5)(5r-1)$
2) $(5x^2-8)(7x-2)$
3) $(n^2-3)(3n-2)$
4) $(2v^2-1)(7v+5)$
5) $(3b^2-7)(5b+7)$
6) $(6x^2+5)(x-8)$
7) $(3x^2+2)(x+5)$
8) Prime
9) $(7x^2-4)(5x-4)$
10) $(7n^2-5)(n+3)$
11) $(7x+5)(y-7)$
12) $(7r^2+3)(6r-7)$
13) $(8x+3)(4y+5x)$
14) $(3a+b^2)(5b-2)$
15) $(8x+1)(2y-7)$
16) $(m+5)(3n-8)$
17) Prime
18) $(m-5)(5n+2)$
19) $(5x-y)(8y+7)$
20) Prime
21) $(4u+3)(8v-5)$
22) $2(u+3)(2v+7u)$
23) $(5x+6)(2y+5)$
24) $(4x-5y^2)(6y-5)$
25) $(3u-7)(v-2u)$
26) $(7a-2)(8b-7)$
27) $(2x+7y^2)(y-4x)$
28) $(7p^2+5)(4p+3)$
29) $(2x+1)(8y-3x)$
30) $(8x-1)(y+7)$