

Section 4.5: A General Factoring Strategy

Objective: Identify and use the correct method to factor various polynomials.

With so many different tools used to factor, it is easy to get lost as to which strategy to use when. Here, we will attempt to organize all the different factoring methods we have seen. A large part of deciding how to solve a problem is based on how many terms are in the problem. For all problem types, we will always try to factor out the GCF first.

Factoring Strategy (GCF First!!!!)

- **2 terms:** sum or difference of two squares:
 $a^2 - b^2 = (a + b)(a - b)$
 $a^2 + b^2 = \text{prime}$ (generally cannot be factored)
- **3 terms:** Watch for trinomials with leading coefficient of one and perfect square trinomials!
 $a^2 + 2ab + b^2 = (a + b)^2$
- **4 terms:** grouping

We will use the above strategy to factor each of the following examples. Here, the emphasis will be on which strategy to use rather than the steps used in that method.

Example 1. Factor completely.

$$x^2 - 23x + 42 \quad \text{GCF}=1; \text{ so nothing to factor out of all three terms}$$

Three terms; multiply to 42; sum to -23
 -2 and -21 ; write the factors

$$(x - 2)(x - 21) \quad \text{Our Solution}$$

Example 2. Factor completely.

$$z^2 + 6z - 9 \quad \text{GCF}=1; \text{ so nothing to factor out of all three terms}$$

Three terms; multiply to -9 ; sum to 6
Factors of -9 : $(-1)(9)$, $(1)(-9)$, $(3)(-3)$; none sum to 6

Prime (cannot be factored) Our Solution

Example 3. Factor completely.

$$4x^2 + 56xy + 196y^2 \quad \text{GCF first; factor out 4 from each term}$$
$$4(x^2 + 14xy + 49y^2) \quad \text{Three terms, } x^2 = (x)^2; 14xy = 2(x)(7y); 49y^2 = (7y)^2$$

Perfect square trinomial; use square roots from first and last terms and sign from the middle

$$4(x+7y)^2 \quad \text{Our Solution}$$

Example 4. Factor completely.

$$\begin{aligned} 5x^2y + 15xy - 35x^2 - 105x & \quad \text{GCF first; factor out } 5x \text{ from each term} \\ 5x(xy + 3y - 7x - 21) & \quad \text{Four terms; try grouping} \\ 5x[y(x+3) - 7(x+3)] & \quad (x+3) \text{ match} \\ 5x(x+3)(y-7) & \quad \text{Our Solution} \end{aligned}$$

Example 5. Factor completely.

$$\begin{aligned} 100x^2 - 400 & \quad \text{GCF first; factor out } 100 \text{ from each term} \\ 100(x^2 - 4) & \quad \text{Two terms; difference of two squares} \\ 100(x+2)(x-2) & \quad \text{Our Solution} \end{aligned}$$

Example 6. Factor completely.

$$\begin{aligned} 108x^3y^2 - 36x^2y^2 + 3xy^2 & \quad \text{GCF first; factor out } 3xy^2 \text{ from each term} \\ 3xy^2(36x^2 - 12x + 1) & \quad \text{Three terms; } 36x^2 = (6x)^2; 12x = 2(6x)(1); 1 = (1)^2 \\ & \quad \text{Perfect square trinomial; use square roots from first and last} \\ & \quad \text{terms and sign from the middle} \\ 3xy^2(6x-1)^2 & \quad \text{Our Solution} \end{aligned}$$

World View Note: Variables originated in ancient Greece where Aristotle would use a single capital letter to represent a number.

It is important to be comfortable and confident not just with using all the factoring methods, but also with deciding on which method to use. This is why your practice with these problems is very important!

4.5 Practice

Factor each expression completely.

- 1) $16x^2 + 48xy + 36y^2$
- 2) $n^2 - n$
- 3) $x^2 - 4xy + 3y^2$
- 4) $45u^2 - 150uv + 125v^2$
- 5) $64x^2 + 49y^2$
- 6) $m^2 - 4n^2$
- 7) $3m^3 - 6m^2n - 24n^2m$
- 8) $2x^3 + 6x^2y - 20y^2x$
- 9) $n^3 + 7n^2 + 10n$
- 10) $16a^2 - 9b^2$
- 11) $5x^2 + 2x$
- 12) $2x^2 - 10x + 12$
- 13) $3k^3 - 27k^2 + 60k$
- 14) $32x^2 - 18y^2$
- 15) $16x^2 - 8xy + y^2$
- 16) $v^2 + v$
- 17) $27m^2 - 48n^2$
- 18) $x^3 + 4x^2$
- 19) $9n^3 - 3n^2$
- 20) $2m^2 + 6mn - 20n^2$
- 21) $16x^2 + 1$
- 22) $9x^2 - 25y^2$
- 23) $mn + 3m - 4xn - 12x$
- 24) $24az - 18ah + 60yz - 45yh$
- 25) $20uv - 60u^3 - 5xv + 15xu^2$
- 26) $36b^2c - 24b^2d + 24xc - 16xd$

4.5 Answers

- 1) $4(2x+3y)^2$
- 2) $n(n-1)$
- 3) $(x-3y)(x-y)$
- 4) $5(3u-5v)^2$
- 5) Prime
- 6) $(m+2n)(m-2n)$
- 7) $3m(m+2n)(m-4n)$
- 8) $2x(x+5y)(x-2y)$
- 9) $n(n+2)(n+5)$
- 10) $(4a+3b)(4a-3b)$
- 11) $x(5x+2)$
- 12) $2(x-2)(x-3)$
- 13) $3k(k-5)(k-4)$
- 14) $2(4x+3y)(4x-3y)$
- 15) $(4x-y)^2$
- 16) $v(v+1)$
- 17) $3(3m+4n)(3m-4n)$
- 18) $x^2(x+4)$
- 19) $3n^2(3n-1)$
- 20) $2(m-2n)(m+5n)$
- 21) Prime
- 22) $(3x+5y)(3x-5y)$
- 23) $(m-4x)(n+3)$
- 24) $3(2a+5y)(4z-3h)$
- 25) $5(4u-x)(v-3u^2)$
- 26) $4(3b^2+2x)(3c-2d)$