Section 1.1: Greatest Common Factor

Objectives: Find the greatest common factor of a polynomial.
Factor the GCF from a polynomial.

The inverse of multiplying polynomials together is factoring polynomials. There are many benefits of factoring a polynomial. We use factored polynomials to help us solve equations, study behaviors of graphs, work with fractions and more. Because so many concepts in algebra depend on us being able to factor polynomials, it is very important to have strong factoring skills.

In this lesson, we will focus on factoring using the Greatest Common Factor or GCF of a polynomial. When multiplying monomials by polynomials, such as $4x^2(2x^2-3x+8)$, we distribute to get a product of $8x^4-12x^3+32x^2$. In this lesson, we will work backwards, starting with $8x^4-12x^3+32x^2$ and factoring to write as the product $4x^2(2x^2-3x+8)$.

DETERMINING THE GREATEST COMMON FACTOR

We will first introduce this idea by finding the GCF of several numbers. To find a GCF of several numbers, we look for the largest number that can divide each number without leaving a remainder.

Example 1. Determine the GCF of 15, 24, and 27.

$\frac{15}{3}=5, \frac{24}{3}=8, \frac{27}{3}=9$ Each of the numbers can be divided by 3

GCF = 3 Our Answer

When there are variables in our problem, we can first find the GCF of the numbers as in Example 1 above. Then we take any variables that are in common to all terms. The variable part of the GCF uses the *smallest* power of each variable that appears in all terms. This idea is shown in the next example.

Example 2. Determine the GCF of $24x^4y^2z$, $18x^2y^4$, and $12x^3yz^5$.

$\frac{24}{6}=4, \frac{18}{6}=3, \frac{12}{6}=2$ Each number can be divided by 6

Use the lowest exponent for each common variable; each term contains x^2y.
Note that z is not part of the GCF because the term $18x^2y^4$ does not contain the variable z.

GCF = $6x^2y$ Our Answer

FACTORING THE GREATEST COMMON FACTOR

Now we will learn to factor the GCF from a polynomial with two or more terms. Remember that factoring is the inverse process of multiplying. In particular, factoring the GCF reverses the distributive property of multiplication.

To factor the GCF from a polynomial, we first identify the GCF of all the terms. The GCF is the factor that goes in front of the parentheses. Then we divide each term of the given polynomial by the GCF. For the second factor, enclose the quotients within the parentheses. In the final answer, the GCF is outside the parentheses and the remaining quotients are enclosed within the parentheses.

Example 3. Factor using the GCF.

$4x^2-20x-16$ GCF of $4x^2$, $-20x$, and 16 is 4; divide each term by 4

$$\frac{4x^2}{4}=x^2,\ \frac{-20x}{4}=-5x,\ \frac{-16}{4}=-4$$

The quotients are the terms left inside the parentheses; keep the GCF outside the parentheses

$=4(x^2-5x-4)$ Our Answer

With factoring, we can always check our answers by multiplying (distributing); the resulting product should be the original expression.

Example 4. Factor using the GCF.

$25x^4-15x^3+20x^2$ GCF of $25x^4$, $-15x^3$, and $20x^2$ is $5x^2$; divide each term by $5x^2$

$$\frac{25x^4}{5x^2}=5x^2,\ \frac{-15x^3}{5x^2}=-3x,\ \frac{20x^2}{5x^2}=4$$

These quotients are the terms left inside the parentheses; keep the GCF outside the parentheses

$=5x^2(5x^2-3x+4)$ Our Answer

Example 5. Factor using the GCF.

$3x^3y^2z+5x^4y^3z^5-4xy^4$ GCF of $3x^3y^2z$, $5x^4y^3z^5$, and $-4xy^4$ is xy^2; divide each term by xy^2

$$\frac{3x^3y^2z}{xy^2}=3x^2z,\ \frac{5x^4y^3z^5}{xy^2}=5x^3yz^5,\ \frac{-4xy^4}{xy^2}=-4y^2$$

These quotients are the terms left inside the parentheses; keep the GCF outside the parentheses

$=xy^2(3x^2z+5x^3yz^5-4y^2)$ Our Answer

Example 6. Factor using the GCF.

$21x^3+14x^2+7x$	GCF of $21x^3$, $14x^2$, and $7x$ is $7x$; divide each term by $7x$
	$\dfrac{21x^3}{7x}=3x^2, \dfrac{14x^2}{7x}=2x, \dfrac{7x}{7x}=1$
	The factors are the GCF and the result of the division; These quotients are the terms left inside the parentheses; keep the GCF outside the parentheses.
$=7x(3x^2+2x+1)$	Our Answer

It is important to note in the previous example, that when the GCF was $7x$ and $7x$ was also one of the terms, so dividing resulting in a quotient of 1. Factoring will never make terms disappear. Anything divided by itself is 1; be sure not to forget to put the 1 into the solution.

Often the second line is not shown in the work of factoring the GCF. We can simply identify the GCF and put it in front of the parentheses containing the remaining factors as shown in the following example.

Example 7. Factor using the GCF.

$18a^4b^3-27a^3b^3+9a^2b^3$	GCF is $9a^2b^3$, divide each term by $9a^2b^3$
$=9a^2b^3(2a^2-3a+1)$	Our Answer

Again, in the previous problem when dividing $9a^2b^3$ by itself, the result is 1. Be very careful that each term is accounted for in your final solution.

GREATEST COMMON FACTOR EQUAL TO 1

Sometimes an expression has a GCF of 1. If there is no common factor other than 1, the polynomial expression cannot be factored using the GCF. This is shown in the following example.

Example 8. Factor using the GCF.

$8ab-17c+49$	GCF is 1 because there are no other factors in common to all 3 terms
cannot be factored using the GCF	Our Answer

FACTORING THE *NEGATIVE* OF THE GCF

If the first term of a polynomial has a negative coefficient, always make the GCF negative in order to make the first term inside the parentheses have a positive coefficient. See Example 9 on the next page.

Example 9. Factor using the GCF.

$-12x^5y^2 + 6x^4y^4 - 8x^3y^5$ GCF of $\left(-12x^5y^2\right)$, $\left(6x^4y^4\right)$, and $\left(-8x^3y^5\right)$ is $(-2x^3y^2)$;
because the first term is negative;
divide each term by $(-2x^3y^2)$

$$\frac{-12x^5y^2}{-2x^3y^2} = 6x^2, \frac{6x^4y^4}{-2x^3y^2} = -3xy^2, \frac{-8x^3y^5}{-2x^3y^2} = 4y^3$$

The results are what is left inside the parentheses

$= -2x^3y^2(6x^2 - 3xy^2 + 4y^3)$ Our Answer

We will always begin factoring by looking for a Greatest Common Factor and factoring it out if there is one. In the rest of this chapter, we will learn other factoring techniques that might be used to write a polynomial as a product of prime polynomials.

Practice Exercises
Section 1.1: Greatest Common Factor

Factor using the GCF.
If the GCF is 1, state that the polynomial "cannot be factored using the GCF".

1) $15x+20$

2) $12-8x$

3) $9x-9$

4) $3x^2+5x$

5) $10x^3-18x$

6) $7ab-35a^2b$

7) $9+8x^2$

8) $4x^3y^2+8x^3$

9) $24x^2y^5-18x^3y^2$

10) $-3a^2b+6a^3b^2$

11) $5x^3-7$

12) $-32n^9+32n^6+40n^5$

13) $20x^4-30x+30$

14) $21p^6+30p^2+27$

15) $28m^4+40m^3+8$

16) $-10x^4+20x^2+12x$

17) $30b^9+5ab-15a^2$

18) $27y^7+12xy^2+9y^2$

19) $-48a^2b^2-56a^3b-56a^5b$

20) $30m^6+15mn^2-25$

21) $20x^8y^2z^2+15x^5y^2z+35x^3y^3z$

22) $3p+12q-15q^2r^2$

23) $50x^2y+10y^2+70xz^2$

24) $30x^5y^4z^3+50y^4z^5-10xy^4z^3$

25) $30pqr-5pq+5q$

26) $28b+14b^2+35b^3+7b^5$

27) $-18n^5+3n^3-21n+3$

28) $30a^8+6a^5+27a^3+21a^2$

29) $-40x^{11}-20x^{12}+50x^{13}-50x^{14}$

30) $-24x^6-4x^4+12x^3+4x^2$

31) $-32mn^8+4m^6n+12mn^4+16mn$

32) $-10y^7+6y^{10}-4xy^{10}-8xy^8$

ANSWERS to Practice Exercises
Section 1.1: Greatest Common Factor

1) $5(3x+4)$
2) $4(3-2x)$
3) $9(x-1)$
4) $x(3x+5)$
5) $2x(5x^2-9)$
6) $7ab(1-5a)$
7) cannot be factored using the GCF
8) $4x^3(y^2+2)$
9) $6x^2y^2(4y^3-3x)$
10) $-3a^2b(1-2ab)$
11) cannot be factored using the GCF
12) $-8n^5(4n^4-4n-5)$
13) $10(2x^4-3x+3)$
14) $3(7p^6+10p^2+9)$
15) $4(7m^4+10m^3+2)$
16) $-2x(5x^3-10x-6)$
17) $5(6b^9+ab-3a^2)$
18) $3y^2(9y^5+4x+3)$
19) $-8a^2b(6b+7a+7a^3)$
20) $5(6m^6+3mn^2-5)$
21) $5x^3y^2z(4x^5z+3x^2+7y)$
22) $3(p+4q-5q^2r^2)$
23) $10(5x^2y+y^2+7xz^2)$
24) $10y^4z^3(3x^5+5z^2-x)$
25) $5q(6pr-p+1)$
26) $7b(4+2b+5b^2+b^4)$
27) $-3(6n^5-n^3+7n-1)$
28) $3a^2(10a^6+2a^3+9a+7)$
29) $-10x^{11}(4+2x-5x^2+5x^3)$
30) $-4x^2(6x^4+x^2-3x-1)$
31) $-4mn(8n^7-m^5-3n^3-4)$
32) $-2y^7(5-3y^3+2xy^3+4xy)$