

Section 1.2: Factoring by Grouping

Objective: Factor polynomials with four terms by grouping.

Whenever possible, we will always do when factoring a polynomial is factor out the greatest common factor (GCF). This GCF is often a monomial. For example, the GCF of $5xy + 10xz$ is the monomial $5x$, so we would factor as $5x(y + 2z)$. However, a GCF does not have to be a monomial; it could be a polynomial. To see this, consider the following two examples.

Example 1. Factor completely.

$$\begin{aligned} 3ax - 7bx & \text{ Both terms have } x \text{ in common, factor it out} \\ = x(3a - 7b) & \text{ Our Answer} \end{aligned}$$

Now we have a similar problem, but instead of the monomial x , we have the binomial $(2a + 5b)$ as the GCF.

Example 2. Factor completely.

$$\begin{aligned} 3a(2a + 5b) - 7b(2a + 5b) & \text{ Both terms have } (2a + 5b) \text{ in common, factor it out} \\ = (2a + 5b)(3a - 7b) & \text{ Our Answer} \end{aligned}$$

In the same way we factored out the GCF of x , we can factor out the GCF which is a binomial, $(2a + 5b)$.

FACTORING BY GROUPING

When a polynomial has a GCF of 1, it still may be factorable. Additional factoring strategies will be needed.

When a polynomial has **four** terms, we will attempt to factor it using a strategy called **grouping**.

Remember, factoring is the reverse of multiplying, so first we will look at a multiplication problem and then try to reverse the process.

Example 3. Multiply.

$$\begin{aligned} (2a + 3)(5b + 2) & \text{ Distribute } (2a + 3) \text{ to each term in the second parentheses} \\ = 5b(2a + 3) + 2(2a + 3) & \text{ Distribute each monomial} \\ = 10ab + 15b + 4a + 6 & \text{ Our Answer} \end{aligned}$$

The product has four terms. We arrived at this answer by looking at the two parts, $5b(2a + 3)$ and $2(2a + 3)$.

When we are factoring by grouping, we split the expression into two groups: the first two terms and the last two terms. Then we can factor the GCF out of each group of two terms. When we do this, our hope is what remains in the parentheses will match in both the left group and the right group. If they match, we can pull this matching binomial GCF out front, putting the rest in parentheses and the expression will be factored.

The next example is the same problem worked backwards, factoring instead of multiplying.

Example 4. Factor completely.

$$\begin{aligned}
 & 10ab + 15b + 4a + 6 && \text{Split expression into two groups} \\
 = & \boxed{10ab + 15b} \quad \boxed{+ 4a + 6} && \text{Factor the GCF from each group of two terms} \\
 = & \boxed{5b(2a + 3)} \quad \boxed{+ 2(2a + 3)} && (2a + 3) \text{ is common to both terms; Factor out this binomial GCF} \\
 = & (2a + 3)(5b + 2) && \text{Our Answer}
 \end{aligned}$$

Example 5. Factor completely.

$$\begin{aligned}
 & 6x^3 - 15x^2 + 2x - 5 && \text{Split expression into two groups} \\
 = & \boxed{6x^3 - 15x^2} \quad \boxed{+ 2x - 5} && \text{Factor the GCF from each group of two terms} \\
 = & \boxed{3x^2(2x - 5)} \quad \boxed{+ 1(2x - 5)} && (2x - 5) \text{ is common to both terms; Factor out this binomial GCF} \\
 = & (2x - 5)(3x^2 + 1) && \text{Our Answer}
 \end{aligned}$$

The key for grouping to work is after the GCF is factored out of the left and right groups, the two binomials must match exactly. If there is any difference between the two binomials, we either have to do some adjusting or it can't be factored using the grouping method.

Sometimes, we will need to factor the negative of the GCF of a grouping to be sure the remaining binomials match.

Example 6. Factor completely.

$$\begin{aligned}
 & 6x^2 + 9xy - 14x - 21y && \text{Split expression into two groups} \\
 = & \boxed{6x^2 + 9xy} \quad \boxed{- 14x - 21y} && \text{Factor the GCF from each group of two terms} \\
 = & \boxed{3x(2x + 3y)} \quad \boxed{- 7(2x + 3y)} && (2x + 3y) \text{ is common to both terms; Factor out this binomial GCF} \\
 = & (2x + 3y)(3x - 7) && \text{Our Answer}
 \end{aligned}$$

Example 7. Factor completely.

$$\begin{aligned}
 & 5xy - 8x - 10y + 16 \\
 = & \boxed{5xy - 8x} \quad \boxed{-10y + 16} \\
 = & \boxed{x(5y - 8)} \quad \boxed{-2(5y - 8)} \\
 = & (5y - 8)(x - 2)
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

$(5y - 8)$ is common to both terms; Factor out this binomial GCF

Our Answer

Example 8. Factor completely.

$$\begin{aligned}
 & 12ab - 14a - 6b + 7 \\
 = & \boxed{12ab - 14a} \quad \boxed{-6b + 7} \\
 = & \boxed{2a(6b - 7)} \quad \boxed{-1(6b - 7)} \\
 = & (6b - 7)(2a - 1)
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

$(6b - 7)$ is common to both terms; Factor out this binomial GCF

Our Answer

Example 9. Factor completely.

$$\begin{aligned}
 & 4a^2 + 6ab - 14ab^2 - 21b^3 \\
 = & \boxed{4a^2 + 6ab} \quad \boxed{-14ab^2 - 21b^3} \\
 = & \boxed{2a(2a + 3b)} \quad \boxed{-7b^2(2a + 3b)} \\
 = & (2a + 3b)(2a - 7b^2)
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

$(2a + 3b)$ is common to both terms; Factor out this binomial GCF

Our Answer

Example 10. Factor completely.

$$\begin{aligned}
 & 8xy - 12y - 10x + 15 \\
 = & \boxed{8xy - 12y} \quad \boxed{-10x + 15} \\
 = & \boxed{4y(2x - 3)} \quad \boxed{-5(2x - 3)} \\
 = & (2x - 3)(4y - 5)
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

$(2x - 3)$ is common to both terms; Factor out this binomial GCF

Our Answer

Sometimes the terms in the expression must be rearranged in order for factoring by grouping to work.

Example 11. Factor completely.

$$\begin{aligned}
 & 6xy + 4 + 3x + 8y \\
 = & \boxed{6xy + 4} \quad \boxed{+3x + 8y} \\
 = & \boxed{2(3xy + 2)} \quad \boxed{+1(3x + 8y)}
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

The remaining factors are not the same; rearrange the terms.

$$\begin{aligned}
 & 6xy + 3x + 8y + 4 \\
 = & \boxed{6xy + 3x} \quad \boxed{+8y + 4} \\
 = & \boxed{3x(2y + 1)} \quad \boxed{+4(2y + 1)}
 \end{aligned}$$

Split expression into two groups

Factor the GCF from each group of two terms

$(2y + 1)$ is common to both terms; Factor out this binomial GCF

$$= (2y + 1)(3x + 4)$$

Our Answer

Practice Exercises

Section 1.2: Factoring by Grouping

Factor completely.

1) $x^3 + 3x^2 + 4x + 12$

2) $x^3 - 3x^2 + 6x - 18$

3) $x^3 - 5x^2 - 2x + 10$

4) $x^3 + x^2 - 3x - 3$

5) $40r^3 - 8r^2 - 25r + 5$

6) $35x^3 - 10x^2 - 56x + 16$

7) $3n^3 - 2n^2 - 9n + 6$

8) $14v^3 + 10v^2 - 7v - 5$

9) $15b^3 + 21b^2 - 35b - 49$

10) $6x^3 - 48x^2 + 5x - 40$

11) $3x^3 + 15x^2 + 2x + 10$

12) $35x^3 - 28x^2 - 20x + 16$

13) $7n^3 + 21n^2 - 5n - 15$

14) $7xy - 49x + 5y - 35$

15) $42r^3 - 49r^2 + 18r - 21$

16) $32xy + 40x^2 + 12y + 15x$

17) $15ab - 6a + 5b^3 - 2b^2$

18) $16xy - 56x + 2y - 7$

19) $3mn - 8m + 15n - 40$

20) $5mn + 2m - 25n - 10$

21) $40xy + 35x - 8y^2 - 7y$

22) $32uv - 20u + 24v - 15$

23) $10xy + 30 + 25x + 12y$

24) $24xy + 25y^2 - 20x - 30y^3$

25) $3uv + 14u - 6u^2 - 7v$

26) $56ab + 14 - 49a - 16b$

27) $2xy - 8x^2 + 7y^3 - 28xy^2$

28) $28p^3 + 21p^2 + 20p + 15$

29) $16xy - 3x - 6x^2 + 8y$

30) $8xy + 56x - y - 7$

ANSWERS to Practice Exercises
Section 1.2: Factoring by Grouping

1) $(x+3)(x^2+4)$

2) $(x-3)(x^2+6)$

3) $(x-5)(x^2-2)$

4) $(x+1)(x^2-3)$

5) $(5r-1)(8r^2-5)$

6) $(7x-2)(5x^2-8)$

7) $(3n-2)(n^2-3)$

8) $(7v+5)(2v^2-1)$

9) $(5b+7)(3b^2-7)$

10) $(x-8)(6x^2+5)$

11) $(x+5)(3x^2+2)$

12) $(5x-4)(7x^2-4)$

13) $(n+3)(7n^2-5)$

14) $(y-7)(7x+5)$

15) $(6r-7)(7r^2+3)$

16) $(4y+5x)(8x+3)$

17) $(5b-2)(3a+b^2)$

18) $(2y-7)(8x+1)$

19) $(3n-8)(m+5)$

20) $(5n+2)(m-5)$

21) $(8y+7)(5x-y)$

22) $(8v-5)(4u+3)$

23) $(2y+5)(5x+6)$

24) $(6y-5)(4x-5y^2)$

25) $(v-2u)(3u-7)$

26) $(8b-7)(7a-2)$

27) $(y-4x)(2x+7y^2)$

28) $(4p+3)(7p^2+5)$

29) $(8y-3x)(2x+1)$

30) $(y+7)(8x-1)$