

## Section 1.4: Factor Trinomials Whose Leading Coefficient is not 1

**Objective:** Factor trinomials using the *ac* method when the leading coefficient of the polynomial is not 1.

When factoring trinomials, we use the *ac method* to split the middle term and then factor by grouping. The *ac method* gets its name from the general trinomial expression,  $ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are the numbers (coefficients) in front of  $x^2$  and  $x$  terms, and the constant at the end, respectively.

The *ac method* is named *ac* because we multiply  $a \cdot c$  to find out what we want the two numbers to multiply to. In the previous lesson, we always multiplied to just  $c$  because there was no number written in front of  $x^2$ . This meant the leading coefficient was 1 and we were multiplying  $1 \cdot c$ , which is  $c$ . Now we will have a number other than 1 as the leading coefficient; so, we will be looking for two numbers that multiply to  $a \cdot c$  and that add to  $b$ .

### FACTORIZING TRINOMIALS OF THE FORM $ax^2 + bx + c$ WHERE $a \neq 1$

**Example 1.** Factor completely.

$$\begin{aligned} &3x^2 + 11x + 6 \\ &= 3x^2 + 9x + 2x + 6 \\ &= 3x(x+3) + 2(x+3) \\ &= (x+3)(3x+2) \end{aligned}$$

Multiply  $a \cdot c$ :  $(3)(6) = 18$

Find factors that multiply to 18 and add to 11:

Use 9 and 2

Replace  $11x$  with  $9x + 2x$

Factor by grouping

Our Answer

When  $a = 1$ , we are able to use the shortcut discussed in the previous section, using the numbers that split the middle term to write the binomial factors. Example 1 above illustrates that the shortcut does not work when  $a \neq 1$ . **We must go through all the steps of grouping in order to factor the expression.**

**Example 2.** Factor completely.

$$\begin{aligned} &8x^2 - 2x - 15 \\ &= 8x^2 - 12x + 10x - 15 \\ &= 4x(2x-3) + 5(2x-3) \\ &= (2x-3)(4x+5) \end{aligned}$$

Multiply  $a \cdot c$ :  $(8)(-15) = -120$

Find factors that multiply to  $-120$  and add to  $-2$ :

Use  $-12$  and  $10$

Replace  $-2x$  with  $-12x + 10x$

Factor by grouping

Our Answer

**Example 3.** Factor completely.

$$\begin{aligned} &10x^2 - 27x + 5 \\ &= 10x^2 - 25x - 2x + 5 \\ &= 5x(2x - 5) - 1(2x - 5) \\ &= (2x - 5)(5x - 1) \end{aligned}$$

Multiply  $a \cdot c$ :  $(10)(5) = 50$   
 Find factors that multiply to 50 and add to  $-27$ :  
 Use  $-25$  and  $-2$   
 Replace  $-27x$  with  $-25x - 2x$   
 Factor by grouping  
 Our Answer

The same process works with polynomials in two variables.

**Example 4.** Factor completely.

$$\begin{aligned} &4x^2 - xy - 5y^2 \\ &= 4x^2 + 4xy - 5xy - 5y^2 \\ &= 4x(x + y) - 5y(x + y) \\ &= (x + y)(4x - 5y) \end{aligned}$$

Multiply  $a \cdot c$ :  $(4)(-5) = -20$   
 Find factors that multiply to  $-20$  and add to  $-1$ :  
 Use 4 and  $-5$   
 Replace  $-xy$  with  $4xy - 5xy$   
 Factor by grouping  
 Our Answer

## FACTORIZING USING MORE THAN ONE STRATEGY

As always, when factoring we will first look for a GCF before using any other method, including the *ac method*. Factoring out the GCF first also has the added bonus of making the numbers smaller so the *ac method* becomes easier.

**Example 5.** Factor completely.

$$\begin{aligned} &18x^3 + 33x^2 - 30x \\ &= 3x[6x^2 + 11x - 10] \\ &= 3x[6x^2 + 15x - 4x - 10] \\ &= 3x[3x(2x + 5) - 2(2x + 5)] \\ &= 3x(2x + 5)(3x - 2) \end{aligned}$$

GCF =  $3x$ ; factor from each term  
 Multiply  $a \cdot c$ :  $(6)(-10) = -60$   
 Find factors that multiply to  $-60$  and add to 11:  
 Use 15 and  $-4$   
 Replace  $11x$  with  $15x - 4x$   
 Factor by grouping  
 Our Answer

As was the case with trinomials when  $a = 1$ , not all trinomials can be factored. If there is no combination of numbers that multiplies and adds up to the correct numbers, then we cannot factor the polynomial and we say the polynomial is *prime*.

**Example 6.** Factor completely.

$3x^2 + 2x - 7$       Multiply  $a \cdot c$ :  $(3)(-7) = -21$   
 $-3(7)$ ,  $-7(3)$ ,  $-1(21)$ , and  $-21(1)$  are the only ways to  
 multiply to  $-21$ , but none of these pairs sums to 2

prime      Our Answer

## Practice Exercises

### Section 1.4: Factoring Trinomials Whose Leading Coefficient is not 1

Factor completely.

1)  $5x^2 + 13x + 6$

2)  $2x^2 - 5x + 2$

3)  $3r^2 - 4r - 4$

4)  $4r^2 + 3r - 7$

5)  $2x^2 - x + 3$

6)  $4k^2 - 17k + 4$

7)  $2b^2 - b - 3$

8)  $6p^2 + 11p - 7$

9)  $4r^2 + r - 3$

10)  $2x^2 + 19x + 35$

11)  $3x^2 - 17x + 20$

12)  $5n^2 - 4n - 20$

13)  $7x^2 - 48x + 36$

14)  $7n^2 - 44n + 12$

15)  $-7x^2 - 15x - 2$

16)  $7v^2 - 24v - 16$

17)  $5a^2 - 13a - 28$

18)  $7x^2 + 29x - 30$

19)  $5k^2 - 26k + 24$

20)  $-3r^2 - 16r - 21$

21)  $3u^2 + 13uv - 10v^2$

22)  $3x^2 + 17xy + 10y^2$

23)  $7x^2 - 2xy - 5y^2$

24)  $5x^2 + 28xy - 49y^2$

25)  $5u^2 + 31uv - 28v^2$

26)  $6x^2 - 39x - 21$

27)  $-10a^2 + 54a + 36$

28)  $21k^2 - 87k - 90$

29)  $-21n^2 - 45n + 54$

30)  $14x^2 - 60x + 16$

31)  $6x^2 + 29x + 20$

32)  $4x^2 + 9xy + 2y^2$

33)  $4m^2 + 6mn + 6n^2$

34)  $4m^2 - 9mn - 9n^2$

35)  $4x^2 - 6xy + 30y^2$

36)  $4x^2 + 13xy + 3y^2$

37)  $18u^2 - 3uv - 36v^2$

38)  $12x^2 + 62xy + 70y^2$

39)  $16x^2 + 60xy + 36y^2$

40)  $24x^2 - 52xy + 8y^2$

**ANSWERS to Practice Exercises**  
**Section 1.4: Factoring Trinomials Whose**  
**Leading Coefficient is not 1**

- |                    |                         |
|--------------------|-------------------------|
| 1) $(5x+3)(x+2)$   | 21) $(3u-2v)(u+5v)$     |
| 2) $(2x-1)(x-2)$   | 22) $(3x+2y)(x+5y)$     |
| 3) $(3r+2)(r-2)$   | 23) $(7x+5y)(x-y)$      |
| 4) $(r-1)(4r+7)$   | 24) $(5x-7y)(x+7y)$     |
| 5) prime           | 25) $(5u-4v)(u+7v)$     |
| 6) $(k-4)(4k-1)$   | 26) $3(2x+1)(x-7)$      |
| 7) $(2b-3)(b+1)$   | 27) $-2(5a+3)(a-6)$     |
| 8) $(3p+7)(2p-1)$  | 28) $3(7k+6)(k-5)$      |
| 9) $(r+1)(4r-3)$   | 29) $-3(7n-6)(n+3)$     |
| 10) $(2x+5)(x+7)$  | 30) $2(7x-2)(x-4)$      |
| 11) $(3x-5)(x-4)$  | 31) $(x+4)(6x+5)$       |
| 12) prime          | 32) $(x+2y)(4x+y)$      |
| 13) $(7x-6)(x-6)$  | 33) $2(2m^2+3mn+3n^2)$  |
| 14) $(7n-2)(n-6)$  | 34) $(m-3n)(4m+3n)$     |
| 15) $-(7x+1)(x+2)$ | 35) $2(2x^2-3xy+15y^2)$ |
| 16) $(7v+4)(v-4)$  | 36) $(x+3y)(4x+y)$      |
| 17) $(5a+7)(a-4)$  | 37) $3(3u+4v)(2u-3v)$   |
| 18) $(7x-6)(x+5)$  | 38) $2(2x+7y)(3x+5y)$   |
| 19) $(5k-6)(k-4)$  | 39) $4(x+3y)(4x+3y)$    |
| 20) $-(3r+7)(r+3)$ | 40) $4(x-2y)(6x-y)$     |

