

Section 2.2: Multiply and Divide Rational Expressions

**Objectives: Multiply rational expressions.
Divide rational expressions.**

When, multiplying and dividing rational expressions, we will use the same process as we do when multiplying and dividing fractions. Always be sure the answer is written in simplest form.

MULTIPLYING RATIONAL EXPRESSIONS

Example 1. Multiply, expressing the resulting fraction in its lowest terms.

$$\begin{aligned} \frac{15}{49} \cdot \frac{14}{45} & \quad \text{First, reduce by dividing out the common factors from numerator and denominator (15 and 7)} \\ = \frac{1}{7} \cdot \frac{2}{3} & \quad \text{Multiply the numerators together and the denominators together} \\ = \frac{2}{21} & \quad \text{Our Answer} \end{aligned}$$

When multiplying rational expressions, we first divide the numerators and denominators by any common factors. Then we multiply the remaining factors straight across.

Example 2. Multiply.

$$\begin{aligned} \frac{25x^2}{9y^8} \cdot \frac{24y^4}{55x^7} & \quad \text{Reduce coefficients by dividing out the common factors from the numerator and the denominator (3 and 5)} \\ & \quad \text{Reduce the variable terms by subtracting exponents} \\ & \quad (x^{2-7}) = x^{-5}; (y^{4-8}) = y^{-4} \\ & \quad \text{Negative exponent } (x^{-5}) \text{ moves to the denominator as } \frac{1}{x^5}; \\ & \quad \text{Likewise, } (y^{-4}) \text{ to the denominator as } \frac{1}{y^4} \\ = \frac{5}{3y^4} \cdot \frac{8}{11x^5} & \quad \text{Multiply across (numerators together; denominators together)} \\ = \frac{40}{33x^5y^4} & \quad \text{Our Answer} \end{aligned}$$

If the rational expression in either the numerator or the denominator is factorable, it must be factored first. That way, any common factors can be divided out before multiplying.

Example 3. Multiply.

$$\begin{aligned} & \frac{x^2 - 9}{x^2 + x - 20} \cdot \frac{x^2 - 8x + 16}{3x + 9} && \text{Factor each numerator and denominator} \\ & = \frac{(x+3)(x-3)}{(x-4)(x+5)} \cdot \frac{(x-4)(x-4)}{3(x+3)} && \text{Divide out the common factors } (x+3) \text{ and } (x-4) \\ & = \frac{x-3}{x+5} \cdot \frac{x-4}{3} && \text{Multiply across} \\ & = \frac{(x-3)(x-4)}{3(x+5)} && \text{Our Answer} \end{aligned}$$

DIVIDING RATIONAL EXPRESSIONS

When dividing rational expressions, we change the division problem into an equivalent multiplication problem. Multiply the first expression by the reciprocal of the divisor. In other words, keep the first expression, change the operation from division to multiplication, and “flip” the second expression. Then, multiply as shown in the examples above, writing the answer in simplest form.

Example 4. Divide.

$$\begin{aligned} & \frac{a^4 b^2}{a} \div \frac{b^4}{4} && \text{Multiply the first expression by the reciprocal of the second} \\ & && \text{expression} \\ & = \frac{a^4 b^2}{a} \cdot \frac{4}{b^4} && \text{Subtract exponents; negative exponents move to denominator} \\ & = \frac{a^3}{1} \cdot \frac{4}{b^2} && \text{Multiply across (numerators together; denominators together)} \\ & = \frac{4a^3}{b^2} && \text{Our Answer} \end{aligned}$$

Example 5. Divide.

$$\begin{aligned} \frac{x^2 - x - 12}{x^2 - 2x - 8} \div \frac{5x^2 + 15x}{x^2 + x - 2} & \quad \text{Multiply by the reciprocal} \\ = \frac{x^2 - x - 12}{x^2 - 2x - 8} \cdot \frac{x^2 + x - 2}{5x^2 + 15x} & \quad \text{Factor each numerator and denominator} \\ = \frac{(x-4)(x+3)}{(x+2)(x-4)} \cdot \frac{(x+2)(x-1)}{5x(x+3)} & \quad \text{Divide out the common factors } (x-4), (x+3), \text{ and } (x+2) \\ = \frac{1}{1} \cdot \frac{x-1}{5x} & \quad \text{Multiply across (numerators together; denominators together)} \\ = \frac{x-1}{5x} & \quad \text{Our Answer} \end{aligned}$$

The example below contains both multiplication and division. To perform these operations, we change division to multiplication by the reciprocal of the divisor, factor wherever possible, reduce if possible, and then multiply the remaining factors.

Example 6. Multiply and divide as indicated.

$$\begin{aligned} \frac{a^2 + 7a + 10}{a^2 + 6a + 5} \cdot \frac{a+1}{4a+8} \div \frac{(a-1)}{(a+2)} & \quad \text{Factor each numerator and denominator} \\ = \frac{(a+5)(a+2)}{(a+5)(a+1)} \cdot \frac{(a+1)}{4(a+2)} \div \frac{(a-1)}{(a+2)} & \quad \text{Multiply by the reciprocal of last fraction} \\ = \frac{(a+5)(a+2)}{(a+5)(a+1)} \cdot \frac{(a+1)}{4(a+2)} \cdot \frac{(a+2)}{(a-1)} & \quad \text{Divide out the common factors } (a+5), (a+2), \text{ and } (a+1) \\ = \frac{a+2}{4(a-1)} & \quad \text{Our Answer} \end{aligned}$$

Practice Exercises

Section 2.2: Multiply and Divide Rational Expressions

Multiply.

1) $\frac{8x^2}{9} \cdot \frac{9}{2}$

2) $\frac{9n}{2n} \cdot \frac{7}{5n}$

3) $\frac{5x^2}{4} \cdot \frac{6}{5}$

4) $\frac{7(m-6)}{m-6} \cdot \frac{5m(7m-5)}{7(7m-5)}$

5) $\frac{6x(x+4)}{x-3} \cdot \frac{(x-3)(x-6)}{6x(x-6)}$

6) $\frac{25n+25}{5} \cdot \frac{4}{30n+30}$

7) $\frac{v-1}{4} \cdot \frac{4}{v^2-11v+10}$

8) $\frac{x^2-6x-7}{x+5} \cdot \frac{x+5}{x-7}$

Divide.

9) $\frac{8x}{3x} \div \frac{4}{7}$

10) $\frac{9m}{5m^2} \div \frac{7}{2}$

11) $\frac{10p}{5} \div \frac{8}{10}$

12) $\frac{7}{10(n+3)} \div \frac{n-2}{(n+3)(n-2)}$

13) $\frac{7r}{7r(r+10)} \div \frac{r-6}{(r-6)^2}$

14) $\frac{9}{b^2-b-12} \div \frac{b-5}{b^2-b-12}$

15) $\frac{x-10}{35x+21} \div \frac{7}{35x+21}$

16) $\frac{8k}{24k^2-40k} \div \frac{1}{15k-25}$

Perform the indicated operation.

17) $\frac{1}{a-6} \cdot \frac{8a+80}{8}$

18) $\frac{p-8}{p^2-12p+32} \div \frac{1}{p-10}$

19) $(n-8) \cdot \frac{6}{10n-80}$

20) $\frac{x^2-7x+10}{x-2} \cdot \frac{x+10}{x^2-x-20}$

21) $\frac{4m+36}{m+9} \cdot \frac{m-5}{5m^2}$

22) $\frac{2r}{r+6} \div \frac{2r}{7r+42}$

The Practice Exercises are continued on the next page.

Practice Exercises: Section 2.2 (continued)

Perform the indicated operation.

23) $\frac{3x-6}{12x-24} \cdot (x+3)$

33) $(10m^2 + 100m) \cdot \frac{18m^3 - 36m^2}{20m^2 - 40m}$

24) $\frac{2n^2 - 12n - 54}{n+7} \div (2n+6)$

34) $\frac{n-7}{n^2 - 2n - 35} \div \frac{9n+54}{10n+50}$

25) $\frac{b+2}{40b^2 - 24b} (5b-3)$

35) $\frac{7p^2 + 25p + 12}{6p+48} \cdot \frac{3p-8}{21p^2 - 44p - 32}$

26) $\frac{21v^2 + 16v - 16}{3v+4} \div \frac{35v-20}{v-9}$

36) $\frac{7x^2 - 66x + 80}{49x^2 + 7x - 72} \div \frac{7x^2 + 39x - 70}{49x^2 + 7x - 72}$

27) $\frac{n-7}{6n-12} \cdot \frac{12-6n}{n^2 - 13n + 42}$

37) $\frac{10b^2}{30b+20} \cdot \frac{30b+20}{2b^2 + 10b}$

28) $\frac{x^2 + 11x + 24}{6x^3 + 18x^2} \cdot \frac{6x^3 + 6x^2}{x^2 + 5x - 24}$

38) $\frac{35n^2 - 12n - 32}{49n^2 - 91n + 40} \cdot \frac{7n^2 + 16n - 15}{5n+4}$

29) $\frac{27a+36}{9a+63} \div \frac{6a+8}{2}$

39) $\frac{7r^2 - 53r - 24}{7r+2} \div \frac{49r+21}{49r+14}$

30) $\frac{k-7}{k^2 - k - 12} \cdot \frac{7k^2 - 28k}{8k^2 - 56k}$

40) $\frac{12x+24}{10x^2 + 34x + 28} \cdot \frac{15x+21}{5}$

31) $\frac{x^2 - 12x + 32}{x^2 - 6x - 16} \cdot \frac{7x^2 + 14x}{7x^2 + 21x}$

41) $\frac{x^2 - 1}{2x-4} \cdot \frac{x^2 - 4}{x^2 - x - 2} \div \frac{x^2 + x - 2}{3x-6}$

32) $\frac{9x^3 + 54x^2}{x^2 + 5x - 14} \cdot \frac{x^2 + 5x - 14}{10x^2}$

42) $\frac{a^3 + b^3}{a^2 + 3ab + 2b^2} \cdot \frac{3a - 6b}{3a^2 - 3ab + 3b^2} \div \frac{a^2 - 4b^2}{a + 2b}$

ANSWERS to Practice Exercises
Section 2.2: Multiply and Divide Rational Expressions

1) $4x^2$

5) $x+4$

2) $\frac{63}{10n}$

6) $\frac{2}{3}$

3) $\frac{3x^2}{2}$

7) $\frac{1}{v-10}$

4) $5m$

8) $x+1$

9) $\frac{14}{3}$

13) $\frac{r-6}{r+10}$

10) $\frac{18}{35m}$

14) $\frac{9}{b-5}$

11) $\frac{5p}{2}$

15) $\frac{x-10}{7}$

12) $\frac{7}{10}$

16) 5

17) $\frac{a+10}{a-6}$

20) $\frac{x+10}{x+4}$

18) $\frac{p-10}{p-4}$

21) $\frac{4(m-5)}{5m^2}$

19) $\frac{3}{5}$

22) 7

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 2.2 (continued)

23) $\frac{x+3}{4}$

24) $\frac{n-9}{n+7}$

25) $\frac{b+2}{8b}$

26) $\frac{v-9}{5}$

27) $-\frac{1}{n-6}$

28) $\frac{x+1}{x-3}$

29) $\frac{1}{a+7}$

30) $\frac{7}{8(k+3)}$

31) $\frac{x-4}{x+3}$

32) $\frac{9(x+6)}{10}$

33) $9m^2(m+10)$

34) $\frac{10}{9(n+6)}$

35) $\frac{p+3}{6(p+8)}$

36) $\frac{x-8}{x+7}$

37) $\frac{5b}{b+5}$

38) $n+3$

39) $r-8$

40) $\frac{18}{5}$

41) $\frac{3}{2}$

42) $\frac{1}{a+2b}$

