Section 2.3: Least Common Denominator

Objective: Identify the least common denominator and build equivalent fractions using that common denominator.

In the previous section, we performed the operations of multiplication and division of rational expressions. We also need to learn to perform addition and subtraction of rational expressions.

In this section, we will focus on finding the LCD of two or more rational expressions. We will then express each rational expression as an equivalent one with the LCD as the denominator. We will hold off on actually adding or subtracting until the next section.

FINDING THE LEAST COMMON DENOMINATOR (LCD)

The process used to find the LCD of rational expressions is based on the process used to find the LCD of integers.

Example 1. Find the Least Common Denominator.

Find the LCD of $\frac{1}{8}$ and $\frac{5}{6}$	Consider multiples of the larger denominator: $8, 16, 24, \ldots$ 24 is the first multiple of 8 that is also divisible by 6
LCD is 24	Our Answer

When finding the LCD of several monomials, we first find the LCD of the coefficients. The variable part of the LCD uses the *highest* exponent of each unique variable.

Example 2. Find the Least Common Denominator.

Find the LCD of $\frac{1}{4x^2y^5}$ and $\frac{7}{6x^4y^3z^6}$	First find the LCD of coefficients 4 and 6: 12 is the LCD of 4 and 6 because it is the smallest number that both 4 and 6 divide into without a remainder Then find the LCD of variables: Use the highest exponent for each variable: x^4, y^5, and z^6
LCD is $12x^4y^5z^6$	Our Answer

The same idea is used when finding the LCD of polynomials that have more than one term. First factor each polynomial and then identify all the factors to be used (attaching highest exponent if necessary).

Example 3. Find the Least Common Denominator.

Find the LCD of $\dfrac{4}{x^2+2x-3}$ and $\dfrac{9}{x^2-x-12}$

$x^2+2x-3=(x-1)(x+3)$	Factor each polynomial
$x^2-x-12=(x-4)(x+3)$	
$(x-1)(x+3)$ and $(x-4)(x+3)$	LCD uses all unique factors with the highest exponent; Notice $(x+3)$ is common to both expressions; we only include it once
LCD is $(x-1)(x+3)(x-4)$	Our Answer

Notice we only used the factor (x+3) once in our LCD because it only appears as a factor once in either polynomial.

We will need to repeat a factor or use an exponent on a factor if there are multiple like factors associated with one of the polynomials in completely factored form.

Example 4. Find the Least Common Denominator.

Find the LCD of $\dfrac{3x}{x^2-10x+25}$ and $\dfrac{1}{x^2-14x+45}$

$x^2-10x+25=(x-5)(x-5)=(x-5)^2$	Factor each polynomial
$x^2-14x+45=(x-5)(x-9)$	
$(x-5)^2$ and $(x-5)(x-9)$	LCD uses all unique factors with the highest exponent
LCD is $(x-5)^2(x-9)$	Our Answer

The previous example could have also been done by factoring the first polynomial to $(x-5)(x-5)$ and not expressing it as $(x-5)^2$. We still would have included $(x-5)$ twice in the LCD because it showed up twice in one of the polynomials. However, expressing the factors using exponents allows us to use the same pattern (including the highest exponent in the LCD) that we previously used with monomials.

BUILDING EQUIVALENT EXPRESSIONS WITH THE LCD AS DENOMINATOR

Once we know the LCD, our goal will be to build up fractions so that they have matching denominators. Whenever we alter the denominator of a fraction by multiplying to get the LCD, we must multiply by the same factor in the numerator of that fraction in order to keep the fraction equivalent to its original value. We can build up a fraction to an equivalent one with a specified denominator by multiplying the numerator and denominator by any factors that are part of the LCD, but not part of the original denominator.

Example 5. Build the rational expression to an equivalent expression with the specified denominator. State the value of the missing numerator.

$\frac{5a}{3a^2b}=\frac{?}{6a^5b^3}$ Identify what factors we need to match denominators:

The missing factor is $2a^3b^2$ because $3\cdot 2=6$ and we need three more factors of a and two more factors of b

$\frac{5a}{3a^2b}\left(\frac{2a^3b^2}{2a^3b^2}\right)$ Multiply both numerator and denominator by missing factor

$=\frac{10a^4b^2}{6a^5b^3}$ Find the value of the numerator

$10a^4b^2$ Our Answer

Example 6. Build the rational expression to an equivalent expression with the specified denominator. State the value of the missing numerator.

$\frac{x-2}{x+4}=\frac{?}{x^2+7x+12}$ Factor to identify factors we need to match denominators:

$(x+4)(x+3)$ The missing factor is $(x+3)$

$\frac{x-2}{x+4}\left(\frac{x+3}{x+3}\right)$ Multiply numerator and denominator by missing factor

$=\frac{(x-2)(x+3)}{(x+4)(x+3)}$ Multiply (FOIL) the numerator

$=\frac{x^2+x-6}{(x+4)(x+3)}$ Find the value of the numerator

x^2+x-6 Our Answer

As the above example illustrates, we multiply out the numerators, but keep the denominators in factored form. The reason for multiplying out only the numerators is that in order to add and subtract rational expressions, we need to combine like terms in the numerators. However, once the like terms have been added/subtracted in the numerator, we factor so that we can see if the expression can be reduced. Since we reduce factors, both the numerator and the denominator should be in completely factored format, and common factors can be divided out. We will discuss this process in detail in the next section.

In the previous examples, we were given a specified denominator and asked to build up to an equivalent expression with that given denominator. Now we will consider more than one rational expression at a time. First, we will identify the Least Common Denominator of the given expressions. Then, we will build up each rational expression to an equivalent one with that LCD as the denominator. In this section, we are not adding and subtracting fractions, just building them up to a common denominator.

Example 7. Build up each rational expression so they have a common denominator.

$$\frac{5a}{4b^3c} \text{ and } \frac{3c}{6a^2b}$$

First, identify the LCD; in this case the least common denominator is $12a^2b^3c$

Determine what factors each fraction's denominator is missing:

First is missing $3a^2$ and second is missing $2b^2c$

$$\frac{5a}{4b^3c}\left(\frac{3a^2}{3a^2}\right) \text{ and } \frac{3c}{6a^2b}\left(\frac{2b^2c}{2b^2c}\right)$$

Multiply each corresponding fraction's numerator and denominator by that denominator's missing factors to get equivalent fractions

$$\frac{15a^3}{12a^2b^3c} \text{ and } \frac{6b^2c^2}{12a^2b^3c}$$

Our Answers

Example 8. Build up each rational expression so they have a common denominator.

$$\frac{5x}{x^2-5x-6} \text{ and } \frac{x-2}{x^2+4x+3}$$

$$(x-6)(x+1) \qquad (x+1)(x+3)$$

Factor denominators to find LCD

Use factors to find LCD:

LCD is $(x-6)(x+1)(x+3)$

Determine what factors each fraction's denominator is missing:

First: $(x+3)$ Second: $(x-6)$

$$\frac{5x}{(x-6)(x+1)}\left(\frac{x+3}{x+3}\right) \text{ and } \frac{x-2}{(x+1)(x+3)}\left(\frac{x-6}{x-6}\right)$$

Multiply each corresponding fraction's numerator and denominator by that denominator's missing factors to get equivalent fractions

$$\frac{5x^2+15x}{(x-6)(x+1)(x+3)} \text{ and } \frac{x^2-8x+12}{(x-6)(x+1)(x+3)}$$

Our Answers

Practice Exercises
Section 2.3: Least Common Denominator

Build the rational expression to an equivalent expression with the specified denominator. State the value of the missing numerator.

1) $\frac{3}{8}=\frac{?}{48}$

2) $\frac{a}{5}=\frac{?}{5a}$

3) $\frac{a}{x}=\frac{?}{xy}$

4) $\frac{5}{2x^2}=\frac{?}{8x^3y}$

5) $\frac{2}{3a^3b^2c}=\frac{?}{9a^5b^2c^4}$

6) $\frac{4}{3a^5b^2c^4}=\frac{?}{9a^5b^2c^4}$

7) $\frac{2}{x+4}=\frac{?}{x^2-16}$

8) $\frac{x+1}{x-3}=\frac{?}{x^2-6x+9}$

9) $\frac{x-4}{x+2}=\frac{?}{x^2+5x+6}$

10) $\frac{x-6}{x+3}=\frac{?}{x^2-2x-15}$

Find the Least Common Denominator.

11) $\frac{1}{2a^3},\frac{7}{6a^4b^2},\frac{3}{4a^3b^5}$

12) $\frac{4}{5x^2y},\frac{9}{25x^3y^5z}$

13) $\frac{5}{x^2-3x},\frac{1}{x-3},\frac{2}{x}$

14) $\frac{7}{4x-8},\frac{3}{x-2},\frac{1}{4}$

15) $\frac{2}{x+2},\frac{8}{x-4}$

16) $\frac{5}{x},\frac{1}{x-7},\frac{9x}{x+1}$

17) $\frac{4}{x^2-25},\frac{6}{x+5}$

18) $\frac{7x}{x^2-9},\frac{3}{x^2-6x+9}$

19) $\frac{10}{x^2+3x+2},\frac{1}{x^2+5x+6}$

20) $\frac{2}{x^2-7x+10},\frac{20}{x^2-2x-15},\frac{3x^2}{x^2+x-6}$

The Practice Exercises are continued on the next page.

Practice Exercises: Section 2.3 (continued)

Find the LCD and build up each rational expression so they have a common denominator.

21) $\dfrac{3a}{5b^2}, \dfrac{2}{10a^3b}$

22) $\dfrac{3x}{x-4}, \dfrac{2}{x+2}$

23) $\dfrac{x+2}{x-3}, \dfrac{x-3}{x+2}$

24) $\dfrac{5}{x^2-6x}, \dfrac{2}{x}, \dfrac{-3}{x-6}$

25) $\dfrac{x}{x^2-16}, \dfrac{3x}{x^2-8x+16}$

26) $\dfrac{5x+1}{x^2-3x-10}, \dfrac{4}{x-5}$

27) $\dfrac{x+1}{x^2-36}, \dfrac{2x+3}{x^2+12x+36}$

28) $\dfrac{3x+1}{x^2-x-12}, \dfrac{2x}{x^2+4x+3}$

29) $\dfrac{4x}{x^2-x-6}, \dfrac{x+2}{x-3}$

30) $\dfrac{3x}{x^2-6x+8}, \dfrac{x-2}{x^2+x-20}, \dfrac{5}{x^2+3x-10}$

ANSWERS to Practice Exercises
Section 2.3: Least Common Denominator

1) 18
2) a^2
3) ay
4) $20xy$
5) $6a^2c^3$
6) 12
7) $2x-8$
8) x^2-2x-3
9) x^2-x-12
10) $x^2-11x+30$

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11) $12a^4b^5$
12) $25x^3y^5z$
13) $x(x-3)$
14) $4(x-2)$
15) $(x+2)(x-4)$
16) $x(x-7)(x+1)$
17) $(x+5)(x-5)$
18) $(x-3)^2(x+3)$
19) $(x+1)(x+2)(x+3)$
20) $(x-2)(x-5)(x+3)$

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 2.3 (continued)

21) $\dfrac{6a^4}{10a^3b^2}, \dfrac{2b}{10a^3b^2}$

22) $\dfrac{3x^2+6x}{(x-4)(x+2)}, \dfrac{2x-8}{(x-4)(x+2)}$

23) $\dfrac{x^2+4x+4}{(x-3)(x+2)}, \dfrac{x^2-6x+9}{(x-3)(x+2)}$

24) $\dfrac{5}{x(x-6)}, \dfrac{2x-12}{x(x-6)}, \dfrac{-3x}{x(x-6)}$

25) $\dfrac{x^2-4x}{(x-4)^2(x+4)}, \dfrac{3x^2+12x}{(x-4)^2(x+4)}$

26) $\dfrac{5x+1}{(x-5)(x+2)}, \dfrac{4x+8}{(x-5)(x+2)}$

27) $\dfrac{x^2+7x+6}{(x-6)(x+6)^2}, \dfrac{2x^2-9x-18}{(x-6)(x+6)^2}$

28) $\dfrac{3x^2+4x+1}{(x-4)(x+3)(x+1)}, \dfrac{2x^2-8x}{(x-4)(x+3)(x+1)}$

29) $\dfrac{4x}{(x-3)(x+2)}, \dfrac{x^2+4x+4}{(x-3)(x+2)}$

30) $\dfrac{3x^2+15x}{(x-4)(x-2)(x+5)}, \dfrac{x^2-4x+4}{(x-4)(x-2)(x+5)}, \dfrac{5x-20}{(x-4)(x-2)(x+5)}$