Section 2.7: Motion and Work Applications

Objective: Solve application problems by creating a rational equation to model the problem.

In this section, we will solve a variety of application problems using rational equations.

UNIFORM MOTION PROBLEMS

Solving motion problems where time traveled is the main focus usually involves rational equations. The distance formula, $d = rt$, is still utilized. However, since the focus is time, the formula is rewritten as $t = \dfrac{d}{r}$. If the times are known to be equal, we can use a proportion.

Example 1. Use a rational equation to solve.

In the time it takes for a car to travel 120 miles, a train can travel 180 miles. If the train's rate is 20 miles per hour faster than the car's rate, what is the average rate for each?

	distance	rate	time
Train	180	$r + 20$	$\dfrac{180}{r+20}$
Car	120	r	$\dfrac{120}{r}$

We do not know rate, r, or time, t, traveled by either the train or the car. But we do know the distances traveled, and that they traveled for the same amount of time.

Since the times traveled by the train and car are the same, we set them equal to each other.

$$\frac{180}{r+20} = \frac{120}{r}$$

We have a proportion;
Set the cross products equal

$$180r = (120)(r + 20)$$

Distribute on the right side

$$180r = 120r + 2400$$
$$\underline{-120r \qquad -120r}$$
$$\frac{60r}{60} = \frac{2400}{60}$$

Add $(-120r)$ to both sides

Divide both sides by 60

$$r = 40$$ Speed of the car

$$r + 20 = (40) + 20 = 60$$ Speed of the train

car: 40 mph; train: 60 mph   Our Solutions

Another type of motion problem concerns a boat traveling in a river either with the current or against the current (or an airplane flying with the wind or against the wind). If a boat is traveling downstream, the current will push it, increasing the rate by the speed of the current. If a boat is traveling upstream, the current will pull against it, decreasing the rate by the speed of the current. Applications of these types are shown in the examples below.

Example 2. Use a rational equation to solve.

A man rows downstream for 30 miles, then turns around. He travels 20 miles upstream in the same amount of time. In still water, his boat averages 15 miles per hour. What is the speed of the water's current?

	distance	rate	time
downstream	30	$15+x$	$\dfrac{30}{15+x}$
upstream	20	$15-x$	$\dfrac{20}{15-x}$

Let x represent the speed of the water's current.

Downstream, the boat's rate is increased by the speed of the current.

Upstream, the boat's rate is decreased by the speed of the current.

We know the time traveled is the same in both directions.

We set the times equal to each other.

$$\frac{30}{15+x}=\frac{20}{15-x}$$ We have a proportion; Set the cross products equal

$$(30)(15-x)=(20)(15+x)$$ Simplify by distributing on both sides

$$450-30x=300+20x$$ Add $(+30x)$ to both sides
$$\underline{+30x \quad\quad +30x}$$
$$450=300+50x$$ Add (-300) to both sides
$$\underline{-300 \quad -300}$$
$$\frac{150}{50}=\frac{50x}{50}$$ Divide both sides by 50

$$x=3$$ This is the speed of the current

The speed of the water's current is 3 mph.   Our Solution

Example 3. Use a rational equation to solve.

Two planes left an airport at the same time with the same average speed in still air. The first plane traveled to a remote island, a distance of 450 miles, with a tailwind. The other plane flew to a mountain resort, a distance of 250 miles, in the same amount of time with a head wind. The wind current's speed for both planes was 55 miles per hour. What would be each plane's average speed in still air?

	distance	rate	time
With a tailwind	450	$x+55$	$\dfrac{450}{x+55}$
With a head wind	250	$x-55$	$\dfrac{250}{x-55}$

Let x represent the speed of the planes in still air*:*

With a tailwind, the speed of the plane in still air is increased by the wind current's speed of 55 mph.

With a head wind, the speed of the plane in still air is decreased by the wind current's speed of 55 mph.

We know the time traveled is the same so set the times equal.

$$\frac{450}{x+55} = \frac{250}{x-55}$$

We have a proportion;
Set the cross products equal

$$450(x-55) = 250(x+55)$$

Simplify by distributing on both sides

$$450x - 24750 = 250x + 13750$$
$$\underline{-250x \qquad\qquad -250x}$$

Add $(-250x)$ to both sides

$$200x - 24750 = 13750$$
$$\underline{+24750 \quad +24750}$$

Add 24750 to both sides

$$\frac{200x}{200} = \frac{38500}{200}$$

Divide both sides by 200

$$x = 192.5$$

This is each plane's average speed in the air.

192.5 mph   Our Solution

WORK PROBLEMS

Work problems typically involve finding the time it takes for two individuals working together to complete a job. The first person can complete the job in A hours working alone, and the second person can complete that job in B hours working alone. Each person is

performing a fractional part of the work. We let the time spent working together be denoted by x. The first person's portion of the work is $\dfrac{x}{A}$ and the second person's portion of the work is $\dfrac{x}{B}$. Working together one whole job is completed. Thus, we add the two parts together and set the equation equal to 1 to signify that the whole job is finished. We use the rational equation below for this type of problem.

WORK EQUATION:

$$\frac{x}{A} + \frac{x}{B} = 1$$

x = the time spent working together
A = time spent by one person working alone
B = time spent by another person working alone

The equation above is used to solve the work problems included in this section.

Example 4. Use a rational equation to solve.

Pat can paint a room in 3 hours. Les can paint the same room in 6 hours. How long will it take them to do the job together?

Pat: 3 hours, Les: 6 hours	The time working together is unknown, or x
	Use the work equation to form the rational equation:
$\dfrac{x}{3} + \dfrac{x}{6} = 1$	Multiply each term by LCD 6
$\dfrac{x(6)}{3} + \dfrac{x(6)}{6} = 1(6)$	Reduce; clear fractions
$2x + x = 6$	Combine like terms
$3x = 6$	Divide by 3 on both sides of the equation
$\dfrac{3x}{3} = \dfrac{6}{3}$	
$x = 2$	Our solution for x
The job takes them 2 hours working together.	Our Solution

Example 5. Use a rational equation to solve.

Adam can clean a room in 3 hours. His sister Maria can clean it in 12 hours. How long will it take them to do the job together?

Adam: 3 hours, Maria: 12 hours   The time working together is unknown, or x

Use the work equation to form the rational equation:

$$\frac{x}{3} + \frac{x}{12} = 1 \quad \text{Multiply each term by LCD, } 12$$

$$\frac{x(12)}{3} + \frac{x(12)}{12} = 1(12) \quad \text{Reduce; clear fractions}$$

$$4x + x = 12 \quad \text{Combine like terms}$$

$$5x = 12 \quad \text{Divide by } 5 \text{ on both sides of the equation}$$

$$\frac{5x}{5} = \frac{12}{5}$$

$$x = \frac{12}{5} = 2\frac{2}{5} \quad \text{Our solution for } x$$

The job takes them $2\frac{2}{5}$ hours   Our Solution

working together.

Practice Exercises
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Use a rational equation to solve.

1) A powerboat travels upstream for 50 miles in the same amount of time it travels 80 miles downstream. If the speed of the powerboat in still water is 65 mph, find the rate of the current.

2) Mary drove 840 miles to Texas in the same amount of time that Sue drove 770 miles to Louisiana. Mary was traveling 5 miles per hour faster than Sue. How fast was each traveling?

3) Steve runs uphill 3 miles in the time that it takes Mark to run 5 miles downhill. If Steve is traveling 4 miles per hour slower than Mark, how fast is each one running?

4) Alberta went on a kayaking trip. She traveled 2 miles upstream in the same amount of time that she spent kayaking 6 miles downstream. The water's current was 2 miles per hour. What was her average rate of travel in still water?

5) A car travels 240 miles in the same amount of time that a motorcyclist travels 360 miles. The car is traveling an average of 20 miles per hour slower. How fast is each traveling?

6) If Andre can do a piece of work alone in 6 days and Bonita can do it alone in 4 days, how long will it take the two working together to complete the job?

7) Cedric can do a piece of work in 4 days and Dominic can do it in half the time. How long will it take them to do the work together?

8) A cistern can be filled by one pipe in 20 minutes and by another in 30 minutes. How long will it take both pipes together to fill the tank?

9) If Sam can do a certain job in 3 days, while it takes Fred 6 days to do the same job, how long will it take them, working together, to complete the job?

10) Tim can finish a certain job in 10 hours. It takes his wife JoAnn only 8 hours to do the same job. If they work together, how long will it take them to complete the job?

ANSWERS to Practice Exercises
Section 2.7: Motion and Work Applications

1) The rate of the current is 15 mph.

2) Mary was traveling 60 mph and Sue was traveling 55 mph.

3) Steve runs at 6 mph and Mark runs at 10 mph.

4) Alberta's average rate in still water is 4 mph.

5) The car is traveling 40 mph and the motorcycle is traveling 60 mph.

6) The job takes 2.4 days working together.

7) The job takes $1\dfrac{1}{3}$ days working together.

8) The tank is filled in 12 minutes.

9) The job takes 2 days working together.

10) The job takes $4\dfrac{4}{9}$ hours working together.