

## Section 3.3: Add and Subtract Radical Expressions

**Objective:** Add and subtract like radicals by first simplifying each radical.

Adding and subtracting radical expressions is very similar to adding and subtracting variable expressions. If two or more radical expressions have the same indices and the same radicands, they are called *like radicals*. Consider the similarities between the following two examples.

**Example 1.** Perform the indicated operations.

$$\begin{aligned} &5x + 3x - 2x \\ &= 6x \end{aligned}$$

Combine like terms

Our Answer

**Example 2.** Perform the indicated operations.

$$\begin{aligned} &5\sqrt{11} + 3\sqrt{11} - 2\sqrt{11} \\ &= 6\sqrt{11} \end{aligned}$$

Combine like radicals

Our Answer

Notice that when we combined the radical terms with  $\sqrt{11}$  it was just like combining variable terms with  $x$ . When adding and subtracting like radicals, we add and subtract the coefficients in front of the radical, and the radical stays the same.

**Example 3.** Perform the indicated operations.

$$\begin{aligned} &7\sqrt[5]{6} + 4\sqrt[5]{3} - 9\sqrt[5]{3} + \sqrt[5]{6} \\ &= 8\sqrt[5]{6} - 5\sqrt[5]{3} \end{aligned}$$

Combine like radicals  $7\sqrt[5]{6} + \sqrt[5]{6}$  and  $4\sqrt[5]{3} - 9\sqrt[5]{3}$

Our Answer

We cannot combine these radical expressions any more because the radicals are not like radical terms.

Often radical expressions do not look *like* at first. However, if we simplify the radicals, we may find we do in fact have like radicals. This process is shown in the examples on the next page.

**Example 4.** Perform the indicated operations.

$$\begin{aligned}
 & 5\sqrt{45} + 6\sqrt{18} - 2\sqrt{98} + \sqrt{20} && \text{Simplify radicals, finding perfect square factors} \\
 & = 5\sqrt{9 \cdot 5} + 6\sqrt{9 \cdot 2} - 2\sqrt{49 \cdot 2} + \sqrt{4 \cdot 5} && \text{Take square roots where possible} \\
 & = 5 \cdot 3\sqrt{5} + 6 \cdot 3\sqrt{2} - 2 \cdot 7\sqrt{2} + 2\sqrt{5} && \text{Multiply coefficients} \\
 & = 15\sqrt{5} + 18\sqrt{2} - 14\sqrt{2} + 2\sqrt{5} && \text{Combine like radicals} \\
 & = 17\sqrt{5} + 4\sqrt{2} && \text{Our Answer}
 \end{aligned}$$

**Example 5.** Perform the indicated operations.

$$\begin{aligned}
 & 4\sqrt[3]{54} - 9\sqrt[3]{16} + 5\sqrt[3]{9} && \text{Simplify radicals, finding perfect cube factors} \\
 & = 4\sqrt[3]{27 \cdot 2} - 9\sqrt[3]{8 \cdot 2} + 5\sqrt[3]{9} && \text{Take cube roots where possible} \\
 & = 4 \cdot 3\sqrt[3]{2} - 9 \cdot 2\sqrt[3]{2} + 5\sqrt[3]{9} && \text{Multiply coefficients} \\
 & = 12\sqrt[3]{2} - 18\sqrt[3]{2} + 5\sqrt[3]{9} && \text{Combine like radicals } 12\sqrt[3]{2} - 18\sqrt[3]{2} \\
 & = -6\sqrt[3]{2} + 5\sqrt[3]{9} && \text{Our Answer}
 \end{aligned}$$

## Practice Exercises

### Section 3.3: Add and Subtract Radical Expressions

Perform the indicated operation.

1)  $2\sqrt{5} + 2\sqrt{5} + 2\sqrt{5}$

18)  $-2\sqrt{18} - 3\sqrt{8} - \sqrt{20} + 2\sqrt{20}$

2)  $-3\sqrt{6} - 3\sqrt{3} - 2\sqrt{3}$

19)  $-3\sqrt{18} - \sqrt{8} + 5\sqrt{8} + 2\sqrt{8}$

3)  $-3\sqrt{2} + 3\sqrt{5} + 3\sqrt{5}$

20)  $-2\sqrt{24} - 2\sqrt{6} + 2\sqrt{6} + 2\sqrt{20}$

4)  $-2\sqrt{6} - \sqrt{3} + 3\sqrt{6}$

21)  $-3\sqrt{8} - \sqrt{5} - 3\sqrt{6} + 2\sqrt{18}$

5)  $-2\sqrt{6} - 2\sqrt{6} - \sqrt{6}$

22)  $3\sqrt{24} - 3\sqrt{27} + 2\sqrt{6} + 2\sqrt{8}$

6)  $-3\sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$

23)  $2\sqrt{6} - \sqrt{54} - 3\sqrt{27} - \sqrt{3}$

7)  $3\sqrt{6} + 3\sqrt{5} + 2\sqrt{5}$

24)  $-2\sqrt[3]{16} + 2\sqrt[3]{16} + 2\sqrt[3]{2}$

8)  $-\sqrt{5} + 2\sqrt{3} - 2\sqrt{3}$

25)  $3\sqrt[3]{135} - \sqrt[3]{81} - \sqrt[3]{135}$

9)  $2\sqrt{2} - 3\sqrt{18} - \sqrt{2}$

26)  $2\sqrt[4]{243} - 2\sqrt[4]{243} - \sqrt[4]{3}$

10)  $-\sqrt{54} - 3\sqrt{6} + 3\sqrt{27}$

27)  $-3\sqrt[4]{4} + 3\sqrt[4]{324} + 2\sqrt[4]{64}$

11)  $-3\sqrt{6} - \sqrt{12} + 3\sqrt{3}$

28)  $3\sqrt[4]{2} - 2\sqrt[4]{2} - \sqrt[4]{243}$

12)  $4\sqrt{5} - \sqrt{5} - 2\sqrt{48}$

29)  $2\sqrt[4]{6} + 2\sqrt[4]{4} + 3\sqrt[4]{6}$

13)  $3\sqrt{2} + 2\sqrt{8} - 3\sqrt{18}$

30)  $-\sqrt[4]{324} + 3\sqrt[4]{324} - 3\sqrt[4]{4}$

14)  $2\sqrt{20} + 2\sqrt{20} - \sqrt{3}$

31)  $-2\sqrt[4]{243} - \sqrt[4]{96} + 2\sqrt[4]{96}$

15)  $3\sqrt{18} - \sqrt{2} - 3\sqrt{2}$

32)  $2\sqrt[4]{2} + 2\sqrt[4]{3} + 3\sqrt[4]{64} - \sqrt[4]{3}$

16)  $3\sqrt{27} + 2\sqrt{3} - \sqrt{12}$

33)  $2\sqrt[4]{48} - 3\sqrt[4]{405} - 3\sqrt[4]{48} - \sqrt[4]{162}$

17)  $-2\sqrt{2} - \sqrt{2} + 3\sqrt{8} + 3\sqrt{6}$

34)  $-3\sqrt[5]{6} - \sqrt[5]{64} + 2\sqrt[5]{192} - 2\sqrt[5]{64}$

## ANSWERS to Practice Exercises

### Section 3.3: Add and Subtract Radical Expressions

1)  $6\sqrt{5}$

18)  $-12\sqrt{2} + 2\sqrt{5}$

2)  $-3\sqrt{6} - 5\sqrt{3}$

19)  $3\sqrt{2}$

3)  $-3\sqrt{2} + 6\sqrt{5}$

20)  $-4\sqrt{6} + 4\sqrt{5}$

4)  $\sqrt{6} - \sqrt{3}$

21)  $-\sqrt{5} - 3\sqrt{6}$

5)  $-5\sqrt{6}$

22)  $8\sqrt{6} - 9\sqrt{3} + 4\sqrt{2}$

6)  $4\sqrt{3}$

23)  $-\sqrt{6} - 10\sqrt{3}$

7)  $3\sqrt{6} + 5\sqrt{5}$

24)  $2\sqrt[3]{2}$

8)  $-\sqrt{5}$

25)  $6\sqrt[3]{5} - 3\sqrt[3]{3}$

9)  $-8\sqrt{2}$

26)  $-\sqrt[4]{3}$

10)  $-6\sqrt{6} + 9\sqrt{3}$

27)  $10\sqrt[4]{4}$

11)  $-3\sqrt{6} + \sqrt{3}$

28)  $\sqrt[4]{2} - 3\sqrt[4]{3}$

12)  $3\sqrt{5} - 8\sqrt{3}$

29)  $5\sqrt[4]{6} + 2\sqrt[4]{4}$

13)  $-2\sqrt{2}$

30)  $3\sqrt[4]{4}$

14)  $8\sqrt{5} - \sqrt{3}$

31)  $-6\sqrt[4]{3} + 2\sqrt[4]{6}$

15)  $5\sqrt{2}$

32)  $2\sqrt[4]{2} + \sqrt[4]{3} + 6\sqrt[4]{4}$

16)  $9\sqrt{3}$

33)  $-2\sqrt[4]{3} - 9\sqrt[4]{5} - 3\sqrt[4]{2}$

17)  $3\sqrt{2} + 3\sqrt{6}$

34)  $\sqrt[5]{6} - 6\sqrt[5]{2}$