Section 3.7: Solving Radical Equations

Objective: Solve equations with radicals and check for extraneous solutions.

In this section, we solve equations that have roots in the problem. As you might expect, to clear a root we can raise both sides to an exponent. Thus, to clear a square root, we can raise both sides to the second power. To clear a cube root, we can raise both sides to the third power.

There is one catch to solving radical equations. Sometimes we end up with proposed solutions that do not actually work in the original equation. This will only happen if the index on the root is even, and it will not happen all the time for those roots. So, for radical equations solved by raising both sides to an *even* power, we must check our answers by substituting each result into the original equation. If a proposed solution does not work, it is called an *extraneous* solution, and is not included in the final solution.

NOTE: When solving a radical equation with an *even* index, always check your answers!

Example 1. Solve the equation.

$\sqrt{7x+2}=4$	Even index; we will have to check all results
$\left(\sqrt{7x+2}\right)^2=4^2$	Square both sides, simplify exponents
$7x+2=16$	Solve
$\underline{-2\quad -2}$	Subtract 2 from both sides
$\frac{7x}{7}=\frac{14}{7}$	Divide both sides by 7.
$x=2$	Need to check this result in the original equation
$\sqrt{7(2)+2}=4$	Multiply
$\sqrt{14+2}=4$	Add
$\sqrt{16}=4$	Square root
$4=4$	True, it works
$x=2$	Our Solution

Example 2. Solve the equation.

$\sqrt[3]{x-1}=-4$	Odd index; we don't need to check our results
$\left(\sqrt[3]{(x-1)}\right)^3=(-4)^3$	Cube both sides, simplify exponents
$x-1=-64$	Solve
$+1 \quad +1$	Add 1 to both sides
$x=-63$	Our Solution

Example 3. Solve the equation.

$\sqrt[4]{3x+6}=-3$	Even index; we will have to check all results
$\left(\sqrt[4]{3x+6}\right)^4=(-3)^4$	Raise both sides to the fourth power
$3x+6=81$	Solve
$-6 \quad -6$	Subtract 6 from both sides
$\frac{3x}{3}=\frac{75}{3}$	Divide both sides by 3
$x=25$	Need to check this result in the original equation
$\sqrt[4]{3(25)+6}=-3$	Multiply
$\sqrt[4]{75+6}=-3$	Add
$\sqrt[4]{81}=-3$	Simplify the radical
$3=-3$	False, extraneous solution; thus, $x=25$ is not a solution
No Solution	Our Solution

If the radical is not alone on one side of the equation, we will have to isolate the radical before we raise it to an exponent.

Example 4. Solve the equation.

$x+\sqrt{4x+1}=5$	Even index, we will have to check all results
$-x \qquad -x$	Isolate radical by subtracting *x* from both sides
$\sqrt{4x+1}=5-x$	Square both sides
$\left(\sqrt{4x+1}\right)^2=(5-x)^2$	Evaluate exponents, recall $(a-b)^2=a^2-2ab+b^2$

$4x+1=25-10x+x^2$	Rewrite equation equal to zero
$-4x-1 \quad\quad -4x-1$	Subtract $4x$ and 1 from both sides; reorder terms
$0=x^2-14x+24$	Factor
$0=(x-12)(x-2)$	Set each factor equal to zero
$x-12=0$ or $x-2=0$	Solve each equation
$+12 \quad +12 \quad\quad +2 \quad +2$	
$x=12$ or $x=2$	Need to check both results by substituting each into the original equation
$(12)+\sqrt{4(12)+1}=5$	Check $x=12$ first; multiply inside the radical
$12+\sqrt{48+1}=5$	Add inside the root sign
$12+\sqrt{49}=5$	Take the square root
$12+7=5$	Add
$19=5$	False, extraneous solution; thus $x=12$ is not a solution
$(2)+\sqrt{4(2)+1}=5$	Check $x=2$ second; multiply inside the radical
$2+\sqrt{8+1}=5$	Add inside the root sign
$2+\sqrt{9}=5$	Take the square root
$2+3=5$	Add
$5=5$	True, it works
$x=2$	Our Solution

The above example illustrates that as we square both sides of the equation we could end up with a quadratic equation. In this case, we must set the equation to zero and solve by factoring. We will have to check both solutions if the root in the problem was even (for example, a square root or a fourth root). Sometimes both values work, sometimes only one value works, and sometimes neither value works.

If there is more than one square root in a problem we will clear all the roots at the same time. This means we must first make sure that one root is isolated on one side of the equal sign before squaring both sides.

Example 5. Solve the equation.

$\sqrt{3x-8}-\sqrt{x}=0$	Even index, we will have to check all results
$\underline{\quad +\sqrt{x} \quad +\sqrt{x}}$	Isolate first root by adding $\sqrt{x}$ to both sides
$\sqrt{3x-8}=\sqrt{x}$	Square both sides
$\left(\sqrt{3x-8}\right)^2=\left(\sqrt{x}\right)^2$	Evaluate exponents
$3x-8=x$	Solve the equation
$\underline{-3x \qquad -3x}$	Subtract $3x$ from both sides
$\frac{-8}{-2}=\frac{-2x}{-2}$	Divide both sides by -2
$4=x$	Need to check result in original equation
$\sqrt{3(4)-8}-\sqrt{4}=0$	Multiply inside the root sign
$\sqrt{12-8}-\sqrt{4}=0$	Subtract inside the root sign
$\sqrt{4}-\sqrt{4}=0$	Take roots
$2-2=0$	Subtract
$0=0$	True, it works
$x=4$	Our Solution

When the index of the roots is not 2, we need to raise both sides of the equation to the power that corresponds to that index.

Example 6. Solve the equation.

$\sqrt[4]{x-1}=\sqrt[4]{8}$	Even index, we will have to check all results
$\left(\sqrt[4]{x-1}\right)^4=\left(\sqrt[4]{8}\right)^4$	Raise both sides to the fourth power
$x-1=8$	Evaluate exponents
$\underline{\quad +1 \quad +1}$	Add 1 to both sides of the equation
$x=9$	Need to check result in original equation
$\sqrt[4]{(9)-1}=\sqrt[4]{8}$	Subtract
$\sqrt[4]{8}=\sqrt[4]{8}$	True, it works
$x=9$	Our Solution

Example 7. Solve the equation.

$\sqrt[3]{x^2+5}=\sqrt[3]{x^2-4x+1}$	Odd index, we don't need to check our results
$\left(\sqrt[3]{x^2+5}\right)=\left(\sqrt[3]{x^2-4x+1}\right)^3$	Raise both sides to the third power
$x^2+5=x^2-4x+1$	Subtract x^2 on both sides of the equation
$-x^2 \qquad -x^2$	
$5=-4x+1$	Subtract 1 from both sides of the equation
$-1 \qquad -1$	
$\frac{4}{-4}=\frac{-4x}{-4}$	Divide both sides by -4
$-1=x$	
$x=-1$	Our Solution

Practice Exercises
Section 3.7: Solving Radical Equations

Solve.

1) $\sqrt{2x+3}-3=0$

2) $\sqrt{5x+1}-4=0$

3) $\sqrt{6x-5}-x=0$

4) $\sqrt{x+2}-\sqrt{3x}=0$

5) $3+x=\sqrt{6x+13}$

6) $x-1=\sqrt{7-x}$

7) $\sqrt{3-3x}-1=2x$

8) $\sqrt{2x+2}=\sqrt{5x-1}$

9) $\sqrt{4x+5}-\sqrt{x+4}=0$

10) $\sqrt{3x+4}-\sqrt{x+2}=0$

11) $\sqrt{2x-4}-\sqrt{x+3}=0$

12) $\sqrt[3]{3x+1}=-2$

13) $\sqrt[4]{x-3}=2$

14) $\sqrt[4]{7x-5}=-2$

15) $\sqrt[5]{6x-2}=-2$

16) $\sqrt[3]{2x-1}=\sqrt[3]{7x+9}$

ANSWERS to Practice Exercises
Section 3.7: Solving Radical Equations

1) 3

2) 3

3) $1, 5$

4) 1

5) ± 2

6) 3

7) $\frac{1}{4}$

8) 1

9) $-\frac{1}{3}$

10) -1

11) 7

12) -3

13) 19

14) no solution

15) -5

16) -2