Section 4.4: Parabolas

Objective: Graph parabolas using the vertex, *x*-intercepts, and *y*-intercept.

Just as the graph of a linear equation $y = mx + b$ can be drawn, the graph of a quadratic equation $y = ax^2 + bx + c$ can be drawn. The graph is simply a picture showing what pairs of values x and y can be used to make the equation true. For a linear equation, the graph is a line but for a quadratic equation, the graph is a U shaped curve called a **parabola**.

GRAPHING A PARABOLA BY CREATING A TABLE OF VALUES

One way to draw the graph of a quadratic equation is to make a table of values and evaluate the equation for each *x*-value we choose. The completed table gives us a set of points to graph. Remember that points are ordered pairs in the form of (x, y); so, each *x*-value and its corresponding *y*-value are a point to be graphed.

Example 1. Graph the parabola $y = x^2 - 4x + 3$.

Make a table of values. We will test five *x*-values to get an idea of the shape of the graph:

$$y = x^2 - 4x + 3$$

x	y
0	
1	
2	
3	
4	

Plug 0 in for x and evaluate: $y = (0)^2 + 4(0) + 3 = 0 - 0 + 3 = 3$

Plug 1 in for x and evaluate: $y = (1)^2 - 4(1) + 3 = 1 - 4 + 3 = 0$

Plug 2 in for x and evaluate: $y = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -1$

Plug 3 in for x and evaluate: $y = (3)^2 - 4(3) + 3 = 9 - 12 + 3 = 0$

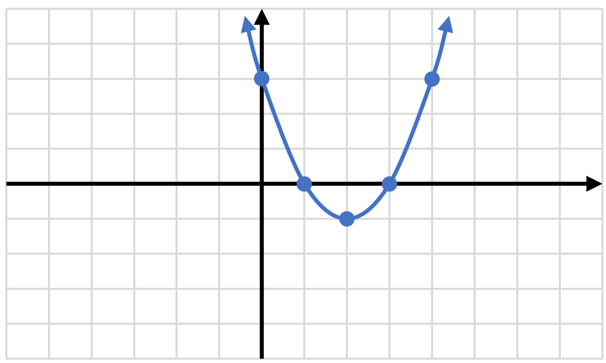
Plug 4 in for x and evaluate: $y = (4)^2 - 4(4) + 3 = 16 - 16 + 3 = 3$

The completed table is shown below:

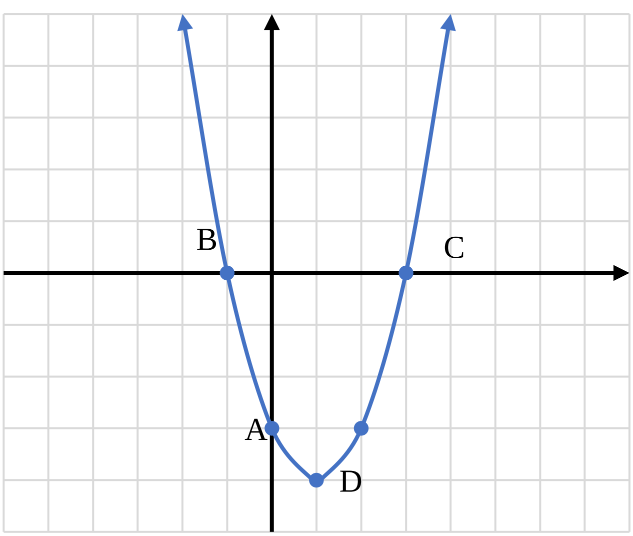
$$y = x^2 - 4x + 3$$

x	y
0	3
1	0
2	-1
3	0
4	3

Graph by plotting the points $(0,3)$, $(1,0)$, $(2,-1)$, $(3,0)$ and $(4,3)$. Connect the points with a smooth, U shaped curve.



The above method to graph a parabola works for any quadratic equation; however, it can be very tedious to find all the points that would be necessary to get the correct bend and shape. For this reason, we identify several key points on a graph to help us graph parabolas more efficiently. These key points are described below.



y**-intercept** (Point A): where the graph crosses the vertical y-axis.

x**-intercepts** (Points B and C): where the graph crosses the horizontal x-axis.

Vertex (Point D): the turning point where the graph changes directions.

GRAPHING A PARABOLA USING THE VERTEX, *X*-INTERCEPTS, AND *Y*-INTERCEPT

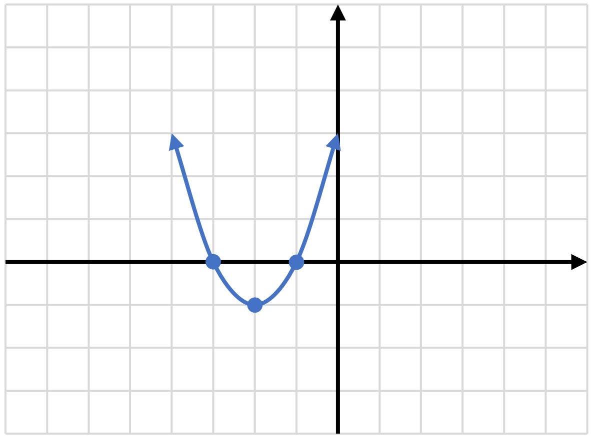
We will use the following method to find each of the points on our parabola.

To graph the parabola $y = ax^2 + bx + c$:

1. **Find the y-intercept:** Find the y-intercept by evaluating when $x = 0$; this always simplifies to $y = c$.
2. **Find the x-intercepts:** Find any x-intercepts by setting $y = 0$ and solving the equation $0 = ax^2 + bx + c$.
   If the solutions are real numbers, there are two x-intercepts. It is also possible to have only one x-intercept or no x-intercepts (if the solutions are complex numbers).
3. **Find the vertex:** Let $x = \frac{-b}{2a}$ to find the x-coordinate of the vertex. Then plug this x-value into the equation to find its corresponding *y*-value, which is the *y*-coordinate of the vertex.
4. **Determine whether the parabola opens upward or downward:**
   If a is a *positive* number, then the vertex will be the *minimum* point of the parabola and the graph will open *upward* (U-shaped).
   If a is a *negative* number, then the vertex will be the *maximum* point of the parabola and the graph will open *downward* (upside down U-shaped).
5. **Plot the points and connect with a smooth U-shaped curve.**

Example 2. Graph the parabola

$y = x^2 + 4x + 3$	Find the key points
$y = 3$	*y*-intercept is $y = c$, point $(0,3)$
$0 = x^2 + 4x + 3$	To find the x-intercepts, we solve the equation
$0 = (x+3)(x+1)$	Factor completely
$x + 3 = 0$ or $x + 1 = 0$	Set each factor equal to zero
$-3 = -3$ $\quad$ $-1 = -1$	Solve each equation
$x = -3$ or $x = -1$	Our x-intercepts, points $(-3,0)$ and $(-1,0)$
$x = \dfrac{-4}{2(1)} = \dfrac{-4}{2} = -2$	To find the vertex, first use $x = \dfrac{-b}{2a}$
$y = (-2)^2 + 4(-2) + 3$	Plug this value into the equation to find the y-coordinate
$y = 4 - 8 + 3$	Evaluate
$y = -1$	y-value of vertex
$(-2,-1)$	Vertex as a point



Graph points $(0,3)$, $(-3,0)$, and $(-1,0)$, as well as the vertex at $(-2,-1)$.

Connect the dots with a smooth curve in a U shape to get our parabola.

Our Graph

If the leading coefficient a in $y = ax^2 + bx + c$ is *negative*, the parabola will end up having an upside-down U shape. The process to graph it is identical, we just need to be very careful of how our signs operate. Remember, if a is negative, then ax^2 will also be negative because we only square the x, not the a.

Example 3. Graph the parabola

$y = -3x^2 + 12x - 9$ Find the key points

$y = -9$ y-intercept is $y = c$, point $(0, -9)$

$0 = -3x^2 + 12x - 9$ To find the x-intercepts, this equation

$0 = -3(x^2 - 4x + 3)$ Factor out GCF first, then factor rest

$0 = -3(x-3)(x-1)$ Set each factor with a variable equal to zero

$x - 3 = 0$ or $x - 1 = 0$ Solve each equation

$\underline{+3 = +3}$ $\underline{+1 = +1}$

$x = 3$ or $x = 1$ Our x-intercepts, points $(3,0)$ and $(1,0)$

$x = \dfrac{-12}{2(-3)} = \dfrac{-12}{-6} = 2$ To find the vertex, first use $x = \dfrac{-b}{2a}$

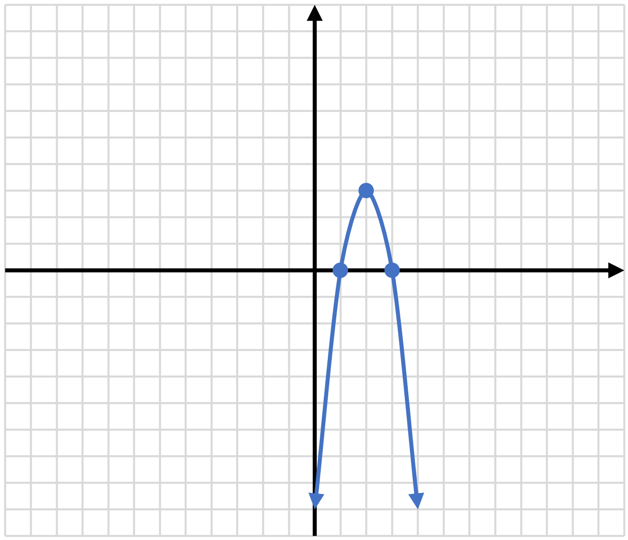
$y = -3(2)^2 + 12(2) - 9$ Plug this value into the equation to find the y-coordinate

$y = -3(4) + 24 - 9$ Evaluate

$y = -12 + 24 - 9$

$y = 3$ y-value of vertex

$(2,3)$ Vertex as a point



Graph the points $(0,-9)$, $(3,0)$, and $(1,0)$, as well as the vertex at $(2,3)$.

Connect the dots with a smooth curve in an upside-down U shape to get our parabola.

Our Graph

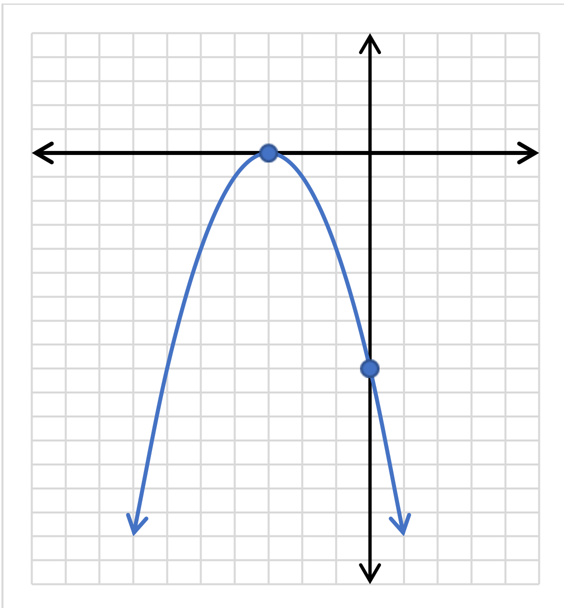
It is important to remember the graph of all quadratics is a parabola with the same U shape (either opening up or opening down). If you plot your points and they cannot be connected in the correct U shape, then at least one of your points must be wrong. Go back and check your work!

Whenever you have a perfect square trinomial quadratic equation, you will have only one unique *x*-intercept, and that *x*-intercept will also be the vertex of the parabola.

Example 4. Graph the parabola

$y=-x^2-6x-9$	Find the key points
$y=-9$	*y*-intercept is $y=c$, point $(0,-9)$
$0=-x^2-6x-9$	To find the *x*-intercept in this equation,
$0=-1(x^2+6x+9)$	Factor out GCF first, then factor the trinomial
$0=-1(x+3)(x+3)$	Set each factor with a variable equal to zero
$x+3=0$ or $x+3=0$	Solve each equation
$-3=-3$ $\quad$ $-3=-3$	
$x=-3$ or $x=-3$	Since they are the same value, the *x*-intercept is $(-3,0)$
$x=-\frac{-6}{2(-1)}=-\frac{-6}{-2}=-3$	To find the vertex, first use $x=\frac{-b}{2a}$
$y=-(-3)^2-6(-3)-9$	Plug this value into the equation to find the *y*-coordinate
$y=-(9)+18-9$	Evaluate
$y=-9+18-9$	
$y=0$	*y*-value of vertex
$(-3,0)$	Vertex as a point

Notice that the *x*-intercept and the vertex are the same point $(-3,0)$. This occurs whenever you have a perfect square trinomial as your quadratic equation. This is because whenever you factor a perfect square trinomial, both factors are identical. By setting each factor equal to zero there is only one unique solution.



Graph the *y*-intercept $(0,-9)$ and the vertex $(-3,0)$.

Connect the dots with a smooth curve in an upside-down U shape to get our parabola.

Our Graph

It is important to remember the graphs of all quadratics are parabolas with the same basic U shape. The differences come from the vertex being shifted to a different location, the curve opening up or down, and how quickly the curve opens.

Practice Exercises
Section 4.4: Parabolas

Find the vertex and intercepts. Use this information to graph each parabola.

1) $y = x^2 - 2x - 8$

2) $y = x^2 - 2x - 3$

3) $y = 2x^2 - 12x + 10$

4) $y = 2x^2 - 12x + 16$

5) $y = -2x^2 + 12x - 18$

6) $y = -2x^2 + 12x - 10$

7) $y = -3x^2 + 24x - 45$

8) $y = -3x^2 + 12x - 9$

9) $y = -x^2 + 4x + 5$

10) $y = -x^2 + 4x - 3$

11) $y = -x^2 + 6x - 5$

12) $y = -2x^2 + 16x - 30$

13) $y = -2x^2 + 16x - 24$

14) $y = 2x^2 + 4x - 6$

15) $y = 3x^2 + 12x + 9$

16) $y = 5x^2 + 30x + 45$

17) $y = 5x^2 - 40x + 75$

18) $y = 5x^2 + 20x + 15$

19) $y = -5x^2 - 60x - 175$

20) $y = -5x^2 + 20x - 15$

21) $y = 3x^2 - 6x + 1$

22) $y = 9x^2 - 18x + 4$

23) $y = -6x^2 - 18x - 11$

24) $y = x^2 - 4x + 5$

25) $y = -3x^2 + 6x - 5$

26) $y = x^2 + 6x + 10$

27) $y = x^2 + 8x + 16$

28) $y = -x^2 + 10x - 25$

ANSWERS to Practice Exercises
Section 4.4: Parabolas

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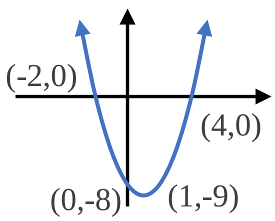

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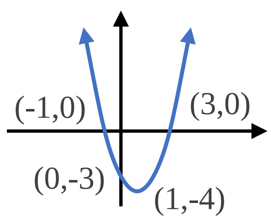

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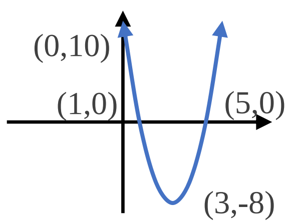

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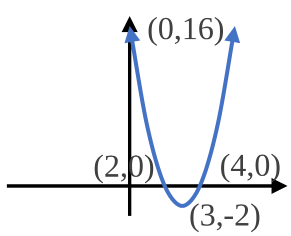

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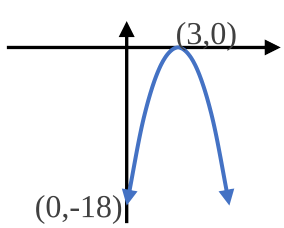

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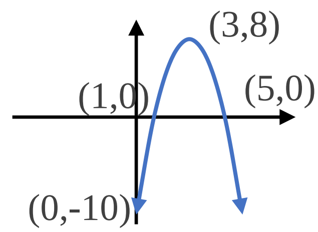

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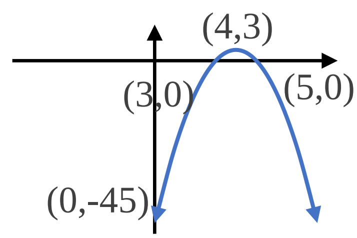

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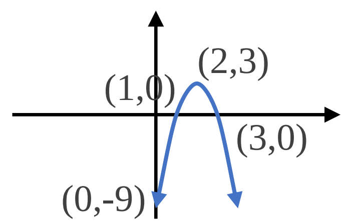

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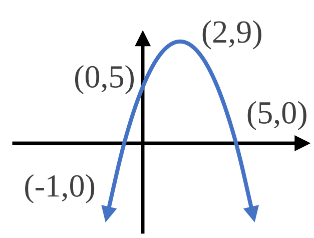

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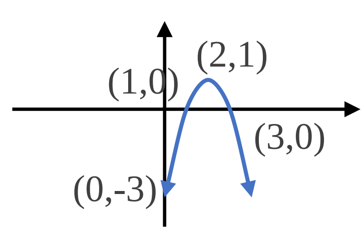

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 4.4 (continued)

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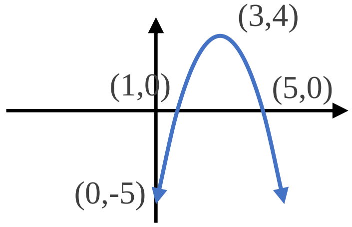

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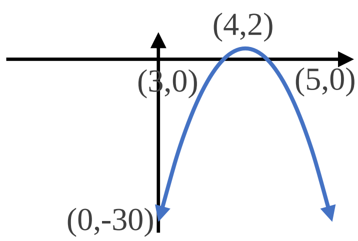

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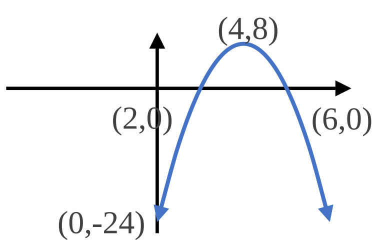

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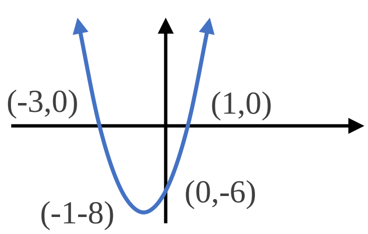

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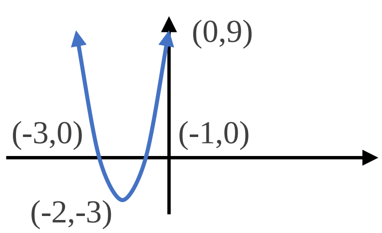

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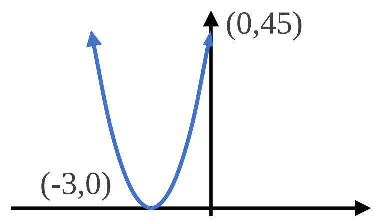

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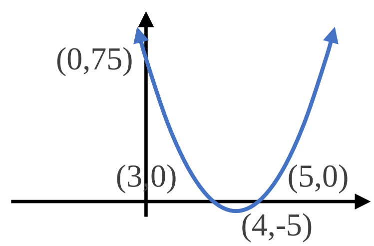

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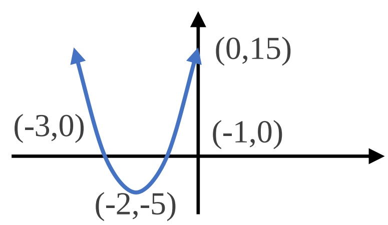

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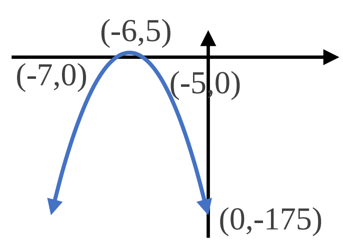

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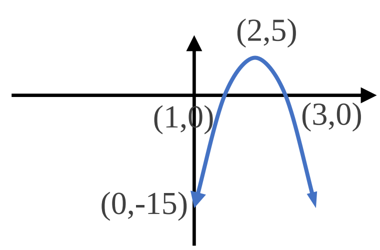

The Answers to Practice Exercises are continued on the next page.

ANSWERS to Practice Exercises: Section 4.4 (continued)

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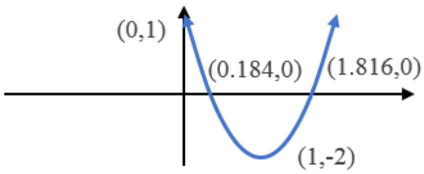

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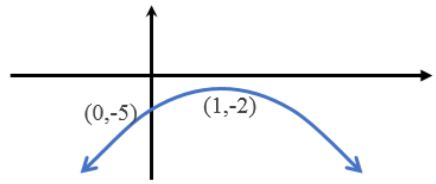

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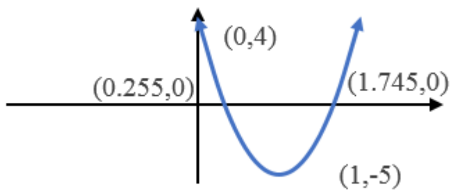

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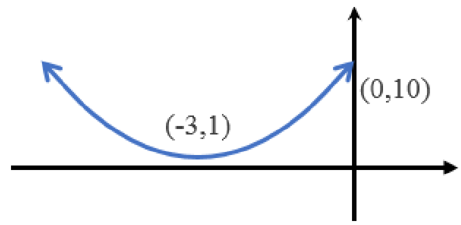

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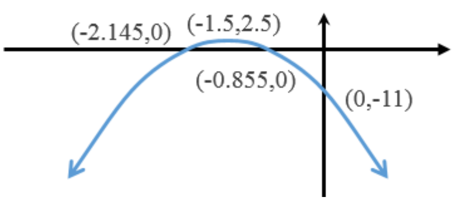

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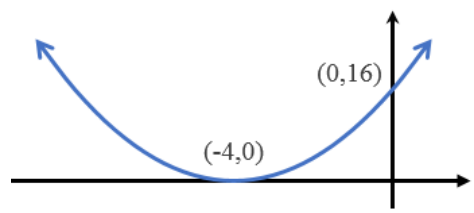

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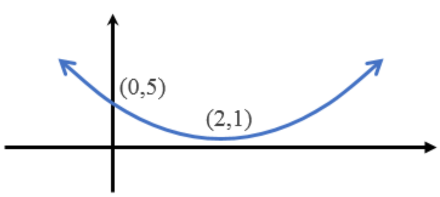

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