Section 4.5: Quadratic Applications

Objective: Solve quadratic application problems.

The vertex of the parabola formed by the graph of a quadratic equation is either a maximum point or a minimum point, depending on the sign of *a*. If *a* is a *positive* number, then the vertex is the *minimum*; if *a* is a *negative* number, then the vertex is a *maximum*.

An example of a maximum would be the highest height of a ball that has been thrown into the air. An example of a minimum would be the minimum average cost to a company for a product that has been produced.

Example 1. Answer each of the following questions.

Terry is on the balcony of her apartment, which is 150 feet above the ground. She tosses a ball vertically upward. The ball's height above the ground as it travels is modeled by the quadratic equation $h = -16t^2 + 64t + 150$, where t is the amount of time (in seconds) the ball has been in flight and h is the height of the ball (in feet) at any particular time.

a. How many seconds will it take for the ball to reach its maximum height above the ground?

$h = -16t^2 + 64t + 150$ The time t is unknown; $a = -16$, $b = 64$, $c = 150$

$t = \frac{-(64)}{2(-16)} = \frac{-64}{-32} = 2$ Use $\frac{-b}{2a}$ to find the amount of time, t, that has passed when the ball reaches its maximum height

2 seconds The ball reaches its maximum height after 2 seconds

b. What is the ball's maximum height above the ground?

$h = -16t^2 + 64t + 150$ The time that has passed, t, is 2 seconds

$h = -16(2)^2 + 64(2) + 150$ Substitute the value 2 in for t everywhere in the equation

$h = -16(4) + 64(2) + 150$ Simplify

$h = -64 + 128 + 150 = 214$

214 feet So, the maximum height of the ball is 214 feet

c. How long does it take for the ball to hit the ground? Round to the nearest tenth of a second, if necessary.

$h = -16t^2 + 64t + 150$	The time that has passed, t, is unknown; When the ball hits the ground, its height h is zero
$0 = -16t^2 + 64t + 150$	Set the quadratic equation equal to zero and solve the equation; $a = -16$, $b = 64$, $c = 150$
$t = \dfrac{-(64) \pm \sqrt{(64)^2 - 4(-16)(150)}}{2(-16)}$	Use the quadratic formula to determine the time $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$t = \dfrac{-64 \pm \sqrt{4096 + 9600}}{-32}$	Simplify
$t = \dfrac{-64 \pm \sqrt{13696}}{-32}$	Use a calculator to evaluate each of solutions
$t \approx -1.7$ or $t \approx 5.7$	Time cannot be negative; so $t \approx -1.7$ is extraneous
5.7 seconds	So, the time lapsed when the ball hits the ground is approximately 5.7 seconds

Example 2. Solve.

The price to charge for a product is a key business decision. If the price is low, then the business may sell many items but will not make much profit per sale. If the price is high, the business will make a large profit per sale but they will have fewer sales. Some value in the middle is "just right" and will maximize profit.

A company has determined that if they charge a price x, in dollars, then their profit P, in thousands of dollars, is given by the equation $P = -x^2 + 120x - 2000$. To maximize profit, what price should the company choose? What is the maximum profit?

$P = -x^2 + 120x - 2000$	The price, x, is unknown; $a = -1$, $b = 120$, $c = -2000$
$x = \dfrac{-b}{2a} = \dfrac{-120}{2(-1)} = \dfrac{-120}{-2} = 60$	Use $\frac{-b}{2a}$ to find the price x that will maximize the profit.
$x = 60$ dollars	The company should choose \$60 as the price to maximize profit.

$P=-(60)^2+120(60)-2000$	Substitute the value 60 for x in the equation.
$P=-3600+7200-2000$	Simplify using the Order of Operations.
$P=1600$	The maximum profit is 1600 thousand dollars or \$1,600,000.

Example 3. Solve.

Arthur sells used cell phones. He has determined that his average cost to package and ship cell phones to customers is given by the equation $C=2x^2-60x+1700$, where x is the number of cell phones packaged and shipped every two weeks, and C is the average cost. How many cell phones must Arthur package and ship during the two-week period in order to minimize the average cost? What is the minimum average cost?

$C=2x^2-60x+1700$	The number of cell phones, x, is unknown; $a=2$, $b=-60$, $c=1700$
$x=-\frac{b}{2a}=-\frac{(-60)}{2(2)}=\frac{60}{4}=15$	Use $\frac{-b}{2a}$ to find the number of cell phones that will minimize the cost.
$x=15$	15 cell phones must be shipped to minimize the average cost.
$C=2(15)^2-60(15)+1700$	Substitute the value 15 for x in the equation.
$C=2(15)^2-60(15)+1700$ $C=2(225)-60(15)+1700$ $C=450-900+1700$	Simplify using the Order of Operations.
$C=1250$	The minimum average cost is 1250 dollars.

Practice Exercises
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Use the following information to answer questions 1 and 2.

George is standing on the top of a 275 foot building. He throws a ball straight up into the air. The ball's initial velocity is given as 48 ft/sec. The height h, in feet, of the ball after t seconds is given by the equation $h=-16t^2+48t+275$.

1) How long will it take for the ball to reach its maximum height above the ground?
2) What is the maximum height that the ball reaches?

Use the following information to answer questions 3-5.

Shelly is standing on a platform 100 feet above the ground. She tosses a baseball straight up into the air. The equation $h=-16t^2+64t+100$ models the ball's height h, in feet, above the ground t seconds after it was thrown.

3) How long will it take for the ball to reach its maximum height above the ground?
4) What is the maximum height that the ball reaches?
5) How many seconds does it take for the ball to finally hit the ground (rounded to the nearest tenth of a second)?

Use the following information to answer questions 6-8.

Les is standing on the ground. He launches a model rocket straight up into the air. The equation $h=-16t^2+64t$ models the rocket's height *h*, in feet, above the ground t seconds after it was launched.

6) How long will it take for the rocket to reach its maximum height above the ground?
7) What is the maximum height that the rocket reaches?
8) How many seconds does it take for the ball to finally hit the ground (rounded to the nearest tenth of a second)?

Use the following information to answer questions 9-10.

The equation $P=-0.001x^2+2.45x-525$ models the profit P, in dollars, for x lasagne meals sold each week at Mama Anna's Restaurant

9) How many lasagne meals should the restaurant sell each week in order to maximize its profit?
10) What would be the maximum weekly profit if they sell the necessary number of meals (rounded to the nearest cent)?

Use the following information to answer questions 11-12.

The equation $P=0.001t^2-0.24t+59.90$ closely models common stock XYZ's closing price, P, in dollars, after t days of trading on the market for the calendar year of 2015.

11) After how many days was XYZ stock at its lowest value?
12) What was the stock's lowest price for 2015?

ANSWERS to Practice Exercises
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1) 1.5 seconds
2) 311 feet
3) 2 seconds
4) 164 feet
5) 5.2 seconds
6) 2 seconds
7) 64 feet
8) 4 seconds
9) 1225 meals
10) $975.63
11) 120 days
12) $45.50