

## Section 5.2: Operations on Functions

**Objective: Combine functions using sum, difference, product, and quotient of functions.**

We can combine functions using four common operations. The four basic operations on functions are addition, subtraction, multiplication, and division. The notation for these functions is as follows.

<b>Addition</b>	$(f + g)(x) = f(x) + g(x)$
<b>Subtraction</b>	$(f - g)(x) = f(x) - g(x)$
<b>Multiplication</b>	$(f \cdot g)(x) = f(x) \cdot g(x)$
<b>Division</b>	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ for $g(x) \neq 0$

### EVALUATING FUNCTIONS

When we evaluate the sum/difference/product/quotient of two functions, we evaluate each function independently and then perform the given operation with both results.

**Example 1.** Perform the indicated operation.

Let  $f(x) = x^2 - x - 2$  and  $g(x) = x + 1$ .

$$\begin{aligned} &\text{Find } (f + g)(-3) \\ &= f(-3) + g(-3) \end{aligned}$$

$$\begin{aligned} &= 10 + (-2) \\ &= 8 \end{aligned}$$

Evaluate  $(f + g)$  at  $x = -3$

Evaluate  $f$  at  $x = -3$ ;

$$f(-3) = (-3)^2 - (-3) - 2$$

$$f(-3) = 9 + 3 - 2$$

$$f(-3) = 10$$

Evaluate  $g$  at  $x = -3$ :

$$g(-3) = (-3) + 1$$

$$g(-3) = -2$$

Add the two results together

Our Answer

The process is the same regardless of the operation being performed.

**Example 2.** Perform the indicated operation.

Let  $h(x) = 2x - 4$  and  $k(x) = -3x + 1$ .

$$\begin{aligned} \text{Find } (h \cdot k)(5) \\ = h(5) \cdot k(5) \end{aligned}$$

$$\begin{aligned} &= (6)(-14) \\ &= -84 \end{aligned}$$

Evaluate  $(h \cdot k)$  at  $x = 5$

Evaluate  $h$  at  $x = 5$ ;

$$h(5) = 2(5) - 4$$

$$h(5) = 10 - 4$$

$$h(5) = 6$$

Evaluate  $k$  at  $x = 5$ :

$$k(5) = -3(5) + 1$$

$$k(5) = -15 + 1$$

$$k(5) = -14$$

Multiply the two results together

Our Answer

## COMBINING FUNCTIONS

In the examples above, we evaluated the sum and quotient of two functions at a given value of the variable. Often, instead of evaluating, we are asked to create a new function by performing an operation on the two given functions. We combine the two functions using the indicated operation, writing each in parentheses, and then simplifying the expression.

**Example 3.** Perform the indicated operation.

Let  $f(x) = 2x - 4$  and  $g(x) = x^2 - x + 5$ .

$$\begin{aligned} \text{Find } (f - g)(x) \\ = f(x) - g(x) \end{aligned}$$

$$= (2x - 4) - (x^2 - x + 5)$$

$$= 2x - 4 - x^2 + x - 5$$

$$= -x^2 + 3x - 9$$

Find the difference of two functions

Replace  $f(x)$  with  $(2x - 4)$  and  $g(x)$  with  $(x^2 - x + 5)$

Subtract by distributing the negative

Combine like terms

Our Answer

The parentheses are very important when we are combining  $f(x)$  and  $g(x)$  using a given operation. In the previous example, we needed the parentheses to know to distribute the negative.

**Example 4.** Perform the indicated operation.

Let  $f(x) = x^2 - 4x - 5$  and  $g(x) = x - 5$ .

Find  $\left(\frac{f}{g}\right)(x)$ .

$$\begin{aligned} &= \frac{f(x)}{g(x)} \\ &= \frac{(x^2 - 4x - 5)}{(x - 5)} \\ &= \frac{(x - 5)(x + 1)}{(x - 5)} \\ &= x + 1 \end{aligned}$$

Find the quotient of two functions

Replace  $f(x)$  with  $(x^2 - 4x - 5)$  and  $g(x)$  with  $(x - 5)$

Simplify the fraction; we must first factor

Divide out common factor of  $x - 5$

Our Answer

In the examples below, we will combine and evaluate functions. Notice the input value in these examples is a variable expression.

**Example 5.** Perform the indicated operation.

Let  $f(x) = 2x - 1$  and  $g(x) = x + 4$ .

Find  $(f + g)(x^2)$ .

$$\begin{aligned} &= f(x^2) + g(x^2) \\ &= [2(x^2) - 1] + [(x^2) + 4] \\ &= 2x^2 - 1 + x^2 + 4 \\ &= 3x^2 + 3 \end{aligned}$$

Find the sum of two functions

Evaluate  $f(x)$  at  $x^2$  and evaluate  $g(x)$  at  $x^2$

Distributing the  $+$  does not change the problem

Combine like terms

Our Answer

**Example 6.** Perform the indicated operation.

Let  $f(x) = 2x - 1$  and  $g(x) = x + 4$ .

Find  $(f \cdot g)(3x)$ .

$$\begin{aligned} &= f(3x) \cdot g(3x) \\ &= [2(3x) - 1] \cdot [(3x) + 4] \\ &= (6x - 1)(3x + 4) \\ &= 18x^2 + 24x - 3x - 4 \\ &= 18x^2 + 21x - 4 \end{aligned}$$

Find the product of two functions

Evaluate  $f(x)$  at  $3x$  and evaluate  $g(x)$  at  $3x$

Multiply  $2(3x)$

FOIL

Combine like terms

Our Answer

**APPLICATIONS OF OPERATIONS ON FUNCTIONS**

**Example 7.** A college has two campuses that opened at the same time. The function  $A(x) = 200x + 500$  gives the enrollment at campus A  $x$  years after opening. The function  $B(x) = 100x + 1000$  gives the enrollment at campus B  $x$  years after opening.

Find  $(A + B)(x)$ .

$$= A(x) + B(x)$$

$$= (200x + 500) + (100x + 1000)$$

$$= 300x + 1500$$

Find the sum of two functions

Replace  $A(x)$  with  $(200x + 500)$  and  $B(x)$  with  $(100x + 1000)$

Combine like terms

Our Answer

This combined function  $(A + B)(x) = 300x + 1500$  gives the total enrollment at both campuses of the college  $x$  years after opening.

**Example 8.** Use the functions from Example 7 to answer the question.

Find  $(A + B)(10)$ .

$$= 300(10) + 1500$$

$$= 3000 + 1500$$

$$= 4500$$

Since we already combined  $A(x) + B(x)$  to get  $(A + B)(x) = 300x + 1500$  in Example 7, evaluate  $(A + B)(x)$  when  $x = 10$ .

Multiply

Add

Our Answer

This answer tells us that the total enrollment at both campuses 10 years after opening is 4500 students.

## Practice Exercises

### Section 5.2: Operations on Functions

Perform the indicated operations.

- 1) Let  $f(x) = -4x + 1$  and  $g(x) = -2x - 1$ . Find  $(f + g)(5)$ .
- 2) Let  $g(x) = 3x + 3$  and  $f(x) = 2x - 2$ . Find  $(g + f)(9)$ .
- 3) Let  $f(x) = x^3 + 5x^2$  and  $g(x) = 2x + 4$ . Find  $(f + g)(3)$ .
- 4) Let  $g(x) = 3x + 1$  and  $f(x) = x^3 + 3x^2$ . Find  $(g \cdot f)(2)$ .
- 5) Let  $f(x) = -3x^2 + 3x$  and  $g(x) = 2x + 5$ . Find  $\left(\frac{f}{g}\right)(-4)$ .
- 6) Let  $g(x) = 4x + 3$  and  $h(x) = x^3 - 2x^2$ . Find  $(g - h)(-1)$ .
- 7) Let  $g(x) = x + 3$  and  $f(x) = -x + 4$ . Find  $(g - f)(3)$ .
- 8) Let  $g(x) = x^2 + 2$  and  $f(x) = 2x + 6$ . Find  $(g - f)(0)$ .
- 9) Let  $g(t) = t - 3$  and  $h(t) = -3t^3 + 6t$ . Find  $(g + h)(1)$ .
- 10) Let  $f(n) = n - 5$  and  $g(n) = 4n + 2$ . Find  $(f + g)(-8)$ .
- 11) Let  $h(t) = t + 5$  and  $g(t) = 3t - 5$ . Find  $(h \cdot g)(5)$ .
- 12) Let  $g(a) = 3a - 2$  and  $h(a) = 4a - 2$ . Find  $(g + h)(-10)$ .
- 13) Let  $h(n) = 2n - 1$  and  $g(n) = 3n - 5$ . Find  $\left(\frac{h}{g}\right)(0)$ .
- 14) Let  $g(x) = x^2 - 2$  and  $h(x) = 2x + 5$ . Find  $(g + h)(-6)$ .
- 15) Let  $f(a) = -2a - 4$  and  $g(a) = a^2 + 3$ . Find  $\left(\frac{f}{g}\right)(7)$ .

*The Practice Exercises are continued on the next page.*

## Practice Exercises: Section 5.2 (continued)

**Perform the indicated operations.**

16) Let  $f(x) = x^2 - 5x$  and  $g(x) = x + 5$ . Find  $(f + g)(x)$ .

17) Let  $f(x) = 4x - 4$  and  $g(x) = 3x^2 - 5$ . Find  $(f + g)(x)$ .

18) Let  $f(x) = -3x + 2$  and  $g(x) = x^2 + 5x$ . Find  $(f - g)(x)$ .

19) Let  $g(n) = n^2 - 3$  and  $h(n) = 2n - 3$ . Find  $(g - h)(n)$ .

20) Let  $g(x) = 2x - 3$  and  $h(x) = x^3 - 2x^2 + 2x$ . Find  $(g - h)(x)$ .

21) Let  $g(t) = t - 4$  and  $h(t) = 2t$ . Find  $(g \cdot h)(t)$ .

22) Let  $g(x) = 4x + 5$  and  $h(x) = x^2 + 5x$ . Find  $(g \cdot h)(x)$ .

23) Let  $g(n) = n^2 + 5$  and  $f(n) = 3n + 5$ . Find  $\left(\frac{g}{f}\right)(n)$ .

24) Let  $f(x) = 2x + 4$  and  $g(x) = 4x - 5$ . Find  $(f - g)(x)$ .

25) Let  $g(a) = -2a + 5$  and  $f(a) = 3a + 5$ . Find  $\left(\frac{g}{f}\right)(a)$ .

26) Let  $g(t) = t^3 + 3t^2$  and  $h(t) = 3t - 5$ . Find  $(g - h)(t)$ .

27) Let  $h(n) = n^3 + 4n$  and  $g(n) = 4n + 5$ . Find  $(h + g)(n)$ .

28) Let  $f(x) = 4x + 2$  and  $g(x) = x^2 + 2x$ . Find  $\left(\frac{f}{g}\right)(x)$ .

**ANSWERS to Practice Exercises**  
**Section 5.2: Operations on Functions**

1)  $-30$

2)  $46$

3)  $82$

4)  $140$

5)  $20$

6)  $2$

7)  $5$

8)  $-4$

9)  $1$

10)  $-43$

11)  $100$

12)  $-74$

13)  $\frac{1}{5}$

14)  $27$

15)  $-\frac{9}{26}$

*The Answers to Practice Exercises are continued on the next page.*

## ANSWERS to Practice Exercises: Section 5.2 (continued)

16)  $x^2 - 4x + 5$

17)  $3x^2 + 4x - 9$

18)  $-x^2 - 8x + 2$

19)  $n^2 - 2n$

20)  $-x^3 + 2x^2 - 3$

21)  $2t^2 - 8t$

22)  $4x^3 + 25x^2 + 25x$

23)  $\frac{n^2 + 5}{3n + 5}$

24)  $-2x + 9$

25)  $\frac{-2a + 5}{3a + 5}$

26)  $t^3 + 3t^2 - 3t + 5$

27)  $n^3 + 8n + 5$

28)  $\frac{4x + 2}{x^2 + 2x}$