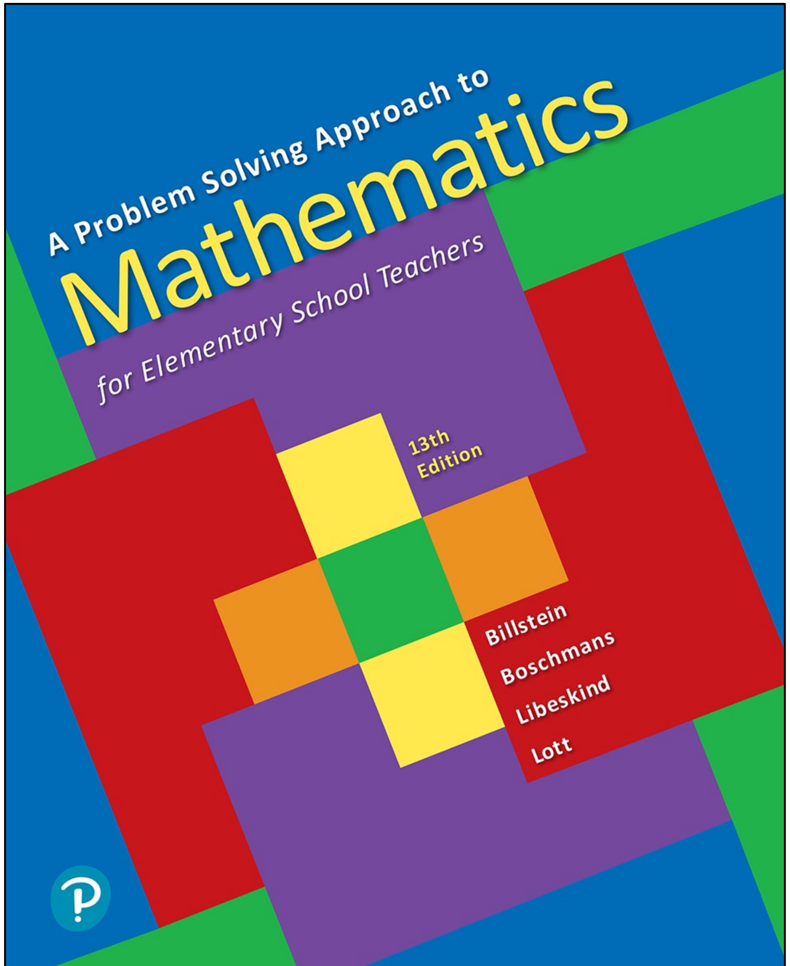
A Problem Solving Approach to Mathematics for Elementary School Teachers

Thirteenth Edition



Chapter 3

Numeration Systems and Whole Number Operations

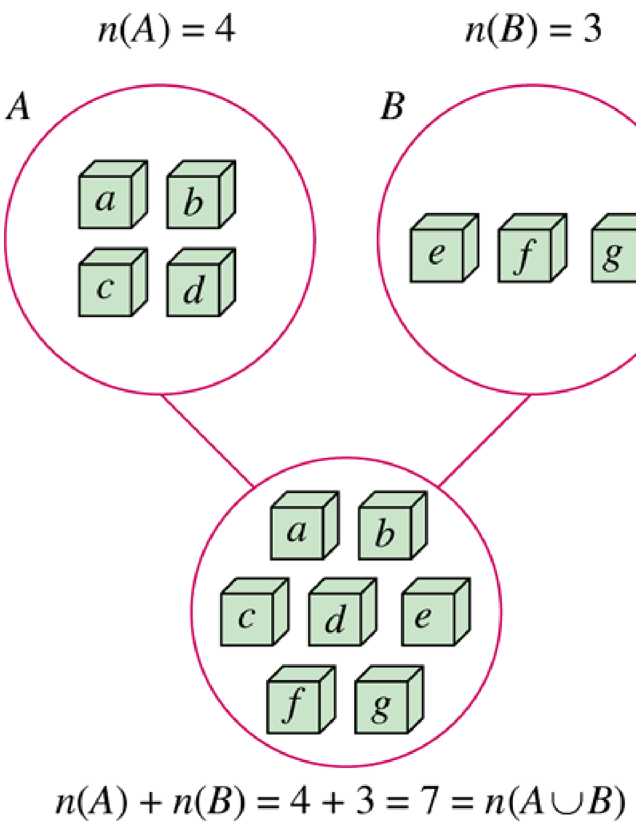
Section 3-2 Addition of Whole Numbers

- Number relationships including comparing and ordering.
- The meaning of addition by studying various models.
- Properties of addition and how to use them.
- Addition algorithms, including the standard algorithm, and how to use them to solve problems.
- Addition with number bases other than ten.
- Mental addition computational skills and estimation.

Set Model

A set model is one way to represent addition of whole numbers. Suppose Jane has 4 blocks in one pile and 3 in another. If she combines the two groups, how many objects are there in the combined group?

Note that the sets must be disjoint (have no elements in common) or an incorrect conclusion can be drawn.



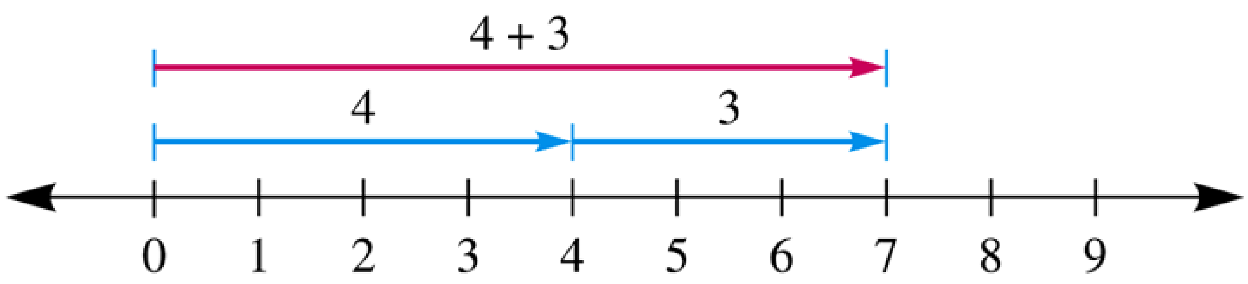
Definition of Addition of Whole Numbers

Let *A* and *B* be two disjoint finite sets.

If $n(A) = a$ and $n(B) = b$, then $a + b = n(A \cup$

Number-Line Model (1 of 2)

1. Kenzo has 4 feet of red ribbon and 3 feet of white ribbon. How many feet of ribbon does he have altogether?
2. One day, Lora drank 4 ounces of orange juice in the morning and 3 ounces at lunchtime. If she drank no other orange juice that day, how many ounces of orange juice did she drink for the entire day?



Number-Line Model (2 of 2)

Students need to understand that the sum represented by any two directed arrows can be found by placing the endpoint of the first directed arrow at 0 and then joining to it the directed arrow for the second number with no gaps or overlaps.

The sum of the numbers can then be read.

Definition of Less Than

For any whole numbers a and b, a is less than b, written $a < b$, if, and only if, there exists a natural number k such that $a + k = b$.

$a \leq b$ means $a < b$ or $a = b$.

$a > b$ is the same as $b < a$.

Mastering Basic Addition Facts (1 of 3)

Counting On

The strategy of *counting on* is an addition strategy where addition is performed by counting on from one of the numbers, for example, 5 + 3 can be computed by starting at 5 and counting 6, 7, 8. The addition 7 + 4 can be performed by starting at 7 and counting 8, 9, 10, 11.

Mastering Basic Addition Facts (2 of 3)

Doubles. Use of *doubles*, such as 3 + 3, receives special attention. After students master doubles, *doubles* + 1 and *doubles* + 2 can be learned easily. For example, if a student knows 6 + 6 = 12, then using the associative property
6 + 7 = 6 + (6 + 1) = (6 + 6) + 1, or 1 more than the double of 6, or 13.

Mastering Basic Addition Facts (3 of 3)

Making 10: Another strategy is that of *making* 10 and then adding any remaining.

One way is to use ten frames to make a ten.

Whole Number Addition Properties (1 of 4)

Closure Property of Addition of Whole Numbers

If *a* and *b* are whole numbers, then *a* + *b* is a whole number.

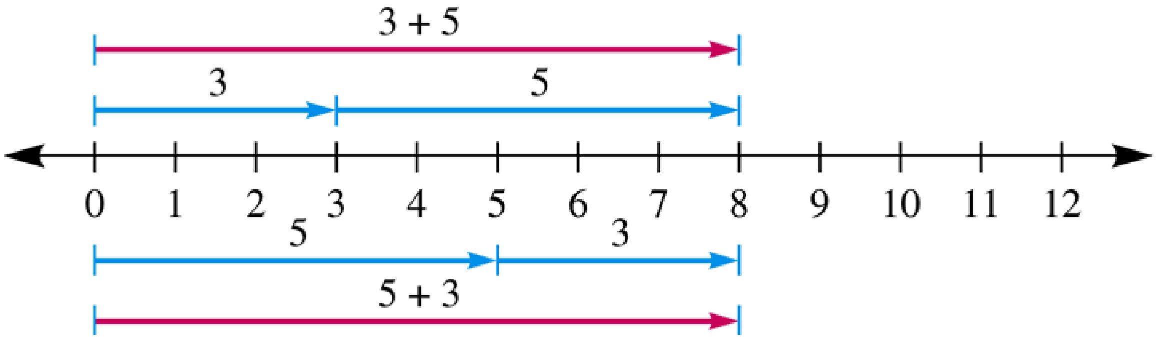
The closure property implies that the sum of two whole numbers **exists** and that the sum is a **unique** whole number.

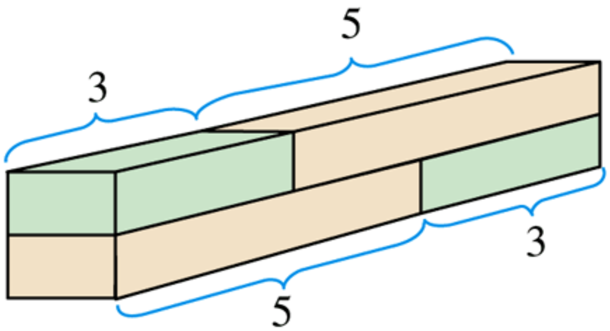
For example, 5 + 2 is a unique whole number, 7.

Whole Number Addition Properties (2 of 4)

Commutative Property of Addition of Whole Numbers

If *a* and *b* are any whole numbers, then $a + b = b + a$.





Whole Number Addition Properties (3 of 4)

Associative Property of Addition of Whole Numbers

If *a*, *b*, and *c* are any whole numbers, then

$$(a+b)+c=a+(b+c).$$

Whole Number Addition Properties (4 of 4)

Identity Property of Addition of Whole Numbers

There is a unique whole number, 0, the **additive identity**, such that for any whole number a,
$a + 0 = a = 0 + a$.

Example 7

Which properties of whole numbers are illustrated in each of the following statements?

a. 5 + 7 = 7 + 5 Commutative property of addition

b. 1001 + 733 is a unique whole number.

Closure property of addition

c. $(3+5)+7=(5+3)+7$

Commutative property of addition

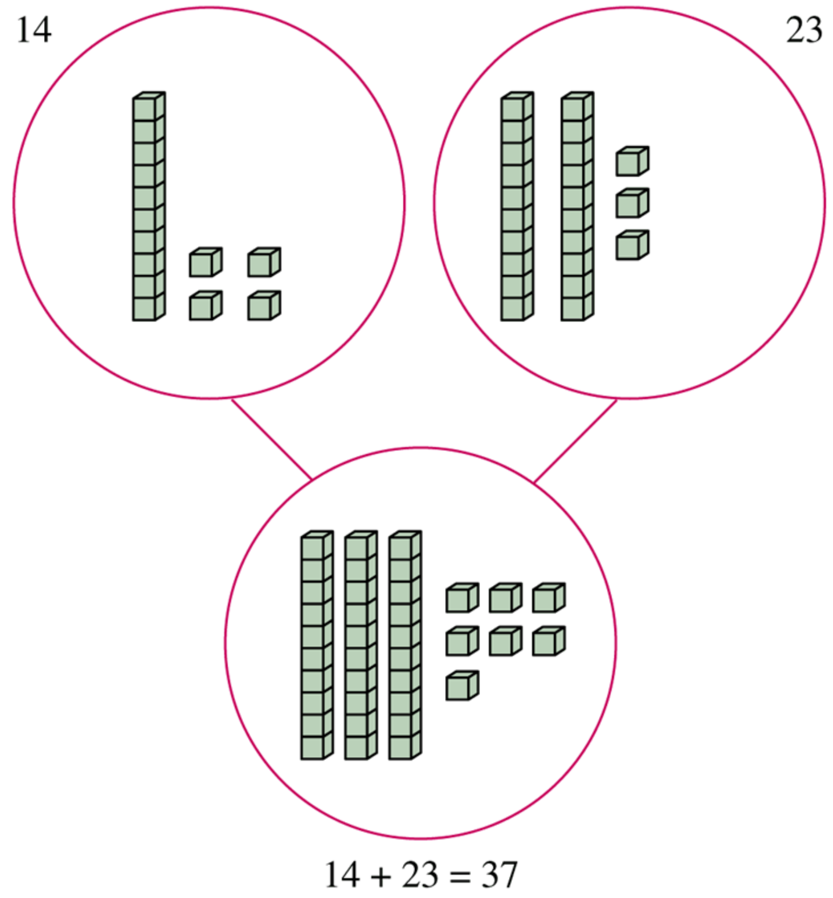
d. $(8+5)+2=2+(8+5)=(2+8)+5$

Commutative and associative properties of addition

Addition Algorithms (1 of 13)

Concrete model

$$\begin{array}{r} 14 \\ +23 \\ \hline 37 \end{array}$$



Addition Algorithms (2 of 13)

Expanded algorithm

$$\begin{array}{rl} 14 & \\ +23 & \\ \hline 7 & \text{(Add ones)} \\ +30 & \text{(Add tens)} \\ \hline 37 & \end{array}$$

Standard algorithm

$$\begin{array}{r} 14 \\ +23 \\ \hline 37 \end{array}$$

Addition Algorithms (3 of 13)

Expanded algorithm with regrouping

$$\begin{array}{rl} 37 & \\ +28 & \\ \hline 15 & \text{(Add ones)} \\ +50 & \text{(Add tens)} \\ \hline 65 & \end{array}$$

Addition Algorithms (4 of 13)

Standard algorithm with regrouping

$$\begin{array}{r} 1 \\ 37 \\ +28 \\ \hline 65 \end{array}$$

(Add ones, regroup, and add the tens)

Addition Algorithms (5 of 13)

Add two three-digit numbers with two regroupings.

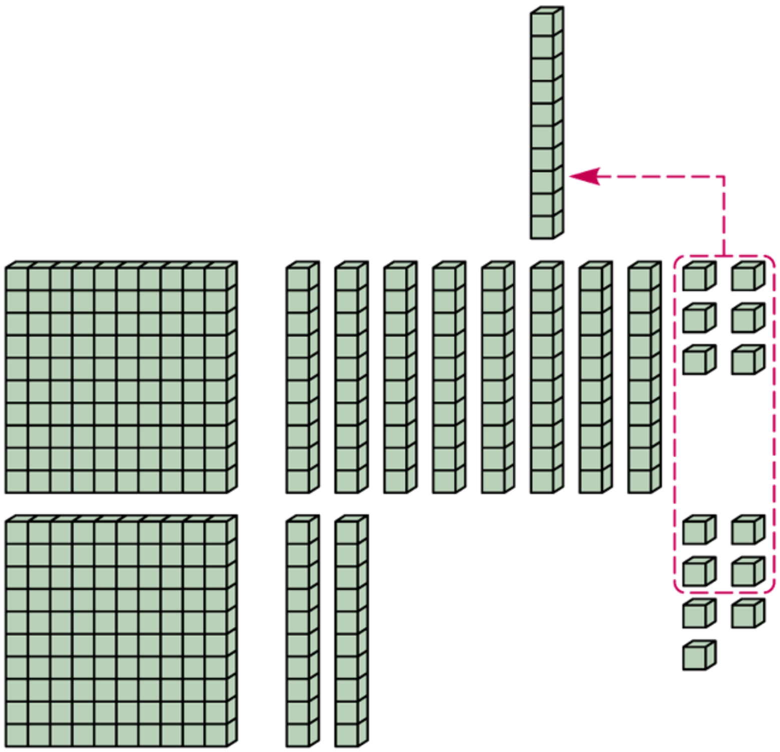
186 + 127

Add the ones and regroup.

$$6 \text{ ones} + 7 \text{ ones} = 13 \text{ ones}$$

$$13 \text{ ones} = 1 \text{ ten} + 3 \text{ ones}$$

$$\begin{array}{r} {}^{1} \\ 186 \\ +127 \\ \hline 3 \end{array}$$



Addition Algorithms (6 of 13)

Add two three-digit numbers with two regroupings. (continued)

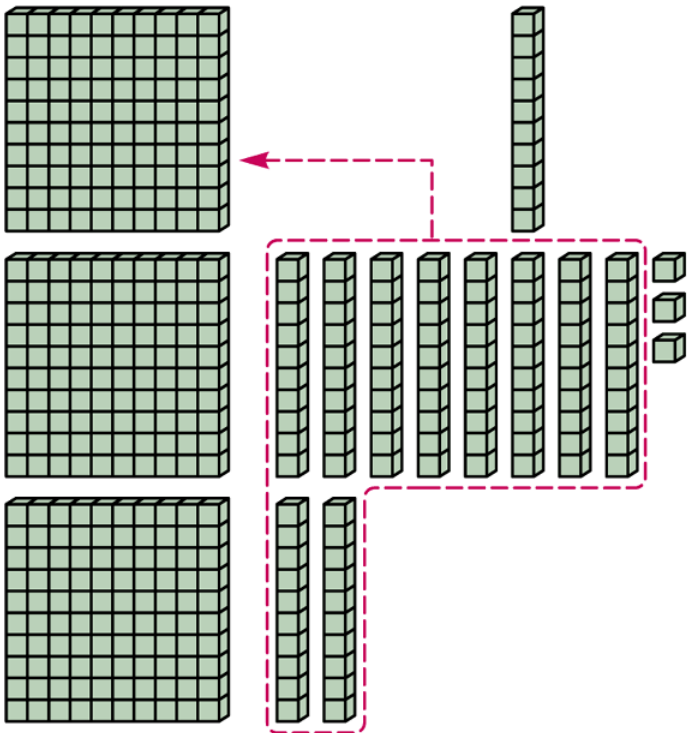
186 + 127

Add the tens and regroup.

$$1 \text{ ten} + 8 \text{ tens} + 2 \text{ tens} = 11 \text{ tens}$$

$$11 \text{ tens} = 1 \text{ hundred} + 1 \text{ ten}$$

$$\begin{array}{r} {}^{11} \\ 186 \\ \underline{+127} \\ 13 \end{array}$$



Addition Algorithms (7 of 13)

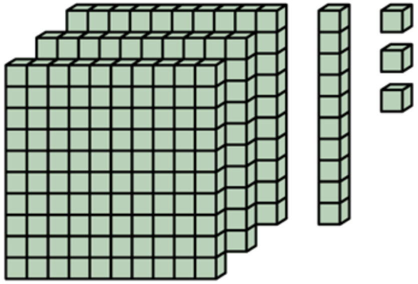
Add two three-digit numbers with two regroupings. (continued)

186 + 127

Add the hundreds.

$$1 \text{ hundred} + 1 \text{ hundred} + 1 \text{ hundred} = 3 \text{ hundreds}$$

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 186 \\ +127 \\ \hline 313 \end{array}$$



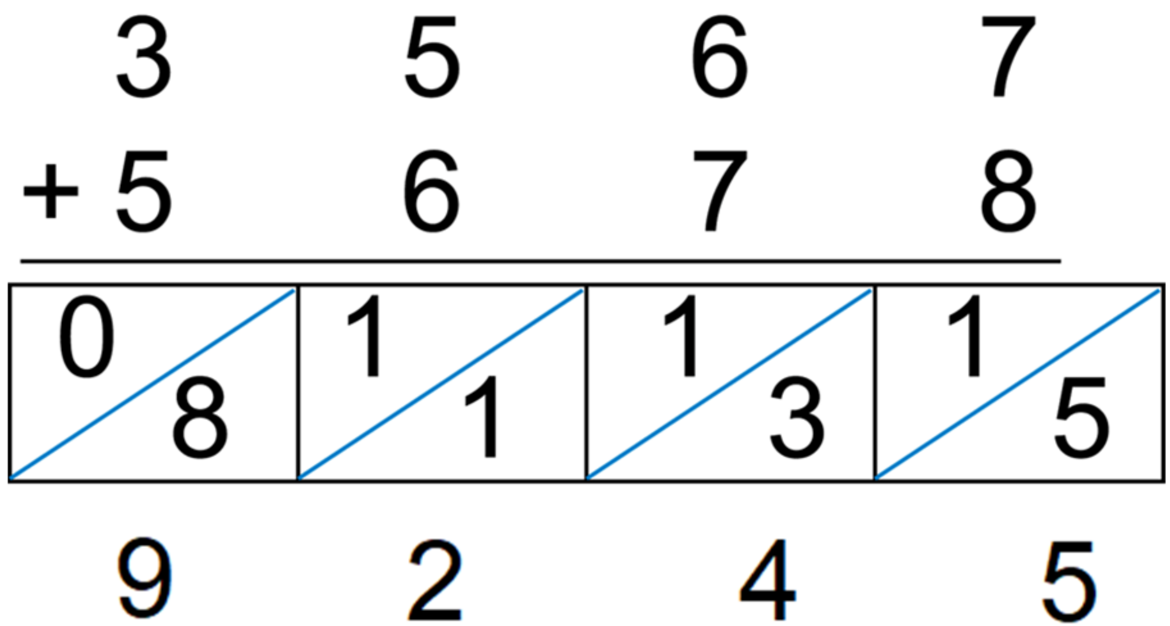
Addition Algorithms (8 of 13)

Left-to-Right Algorithm for Addition (Partial Sums)

$$
\begin{array}{rr}
 & 568 \\
 & \underline{+757} \\
(500+700)\rightarrow & 1200 \\
(60+50)\rightarrow & 110 \\
(8+7)\rightarrow & \underline{15} \\
 & 1325
\end{array}
$$

Addition Algorithms (9 of 13)

Lattice Algorithm for Addition



Column-Addition

In the column-addition algorithm, we add the numbers in each column, writing each sum directly beneath the column. Then we regroup each place one column at a time. Base-ten blocks are often used in the column-addition algorithm.

Opposite-Change Algorithm

In the opposite-change algorithm for addition, we decide which addend is closer to a nice number (typically 10 or a multiple of 10), then decide how to adjust that addend so that it ends in one or more zeros and adjust the other addend the opposite way. The goal is to make the addition problem a problem without the need to regroup.

Addition Algorithms (10 of 13)

Scratch Algorithm for Addition

$$\begin{array}{r} 87 \\ 6\not{5}\;\;2 \\ \underline{+49} \end{array}$$

Add the numbers in the units place starting at the top. When the sum is 10 or more, record this sum by scratching a line through the last digit added and writing the number of units next to the scratched digit.

Addition Algorithms (11 of 13)

Scratch Algorithm for Addition (continued)

$$\begin{array}{rl} 87 & \\ 6\not{5} & 2 \\ \underline{+4\not{9}} & 1 \end{array}$$

Continue adding the units, including any new digits written down. When the addition again results in a sum of 10 or more, repeat the process.

Addition Algorithms (12 of 13)

Scratch Algorithm for Addition (continued)

$$\begin{array}{r l} 2 & \\ 87 & \\ 6\not{5} & 2 \\ +4\not{9} & 1 \\ \hline 1 & \end{array}$$

When the first column of additions is completed, write the number of units, 1, below the addition line in the proper place value position. Count the number of scratches, 2, and add this number to the second column.

Addition Algorithms (13 of 13)

Scratch Algorithm for Addition (continued)

2			
	~~8~~ $_0$	7	
	6	~~5~~	2
+	~~4~~ $_0$	~~9~~	1
2	0	1	

Repeat the procedure for each successive column until the last column with non-zero values. At this stage, sum the scratches and place the number to the left of the current value.

Understanding Addition and Subtraction in Bases Other Than Ten (1 of 4)

Base-Five Addition Table

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

$$\begin{array}{rl} 144_{\text{five}} & \\ +\ 34_{\text{five}} & \\ \hline 13_{\text{five}} & (\text{Add } 4_{\text{five}} + 4_{\text{five}}) \\ 120_{\text{five}} & (\text{Add } 40_{\text{five}} + 30_{\text{five}}) \\ 100_{\text{five}} & (100_{\text{five}}) \\ 233_{\text{five}} & \end{array}$$

$$\begin{array}{r} 11\phantom{_{\text{five}}} \\ 144_{\text{five}} \\ +\ 34_{\text{five}} \\ \hline 233_{\text{five}} \end{array}$$

Mental Mathematics and Estimation for Whole-Number Operations

Mental mathematics

The process of producing an answer to a computation without using computational aids.

Computational estimation

The process of forming an approximate answer to a numerical problem.

Mental Mathematics: Addition (1 of 3)

1. Adding from the left

76	$70+20=90$	(Add the tens.)
$\underline{+25}$	$6+5=11$	(Add the ones.)
	$90+11=101$	(Add the two sums.)

2. Breaking up and bridging

76	$76+20=96$	Add the first number to the tens in the second number.
$\underline{+25}$	$96+5=101$	Add the sum to the units in the second number.

Mental Mathematics: Addition (2 of 3)

3. Trading off

76	$\rightarrow 76 + 4 = 80$	Add 4 to make a multiple of 10.
$\underline{+25}$	$\rightarrow 25 - 4 = 21$	Subtract 4 to compensate.
	$80 + 21 = 101$	Add the two numbers.

4. Using compatible numbers

Compatible numbers are numbers whose sums are easy to calculate mentally.

$$130 + 50 + 70 + 20 + 50 = 200 + 100 + 20 = 320$$

200 100

Mental Mathematics: Addition (3 of 3)

5. Making compatible numbers

$$\begin{array}{r} 76 \\ \underline{+25} \end{array} \quad \begin{array}{l} \rightarrow \; 75 + 25 = 100 \\ \rightarrow 100 + \;\; 1 = 101 \end{array}$$

75 + 25 adds to 100.

Add 1 more unit.

Computational Estimation (1 of 5)

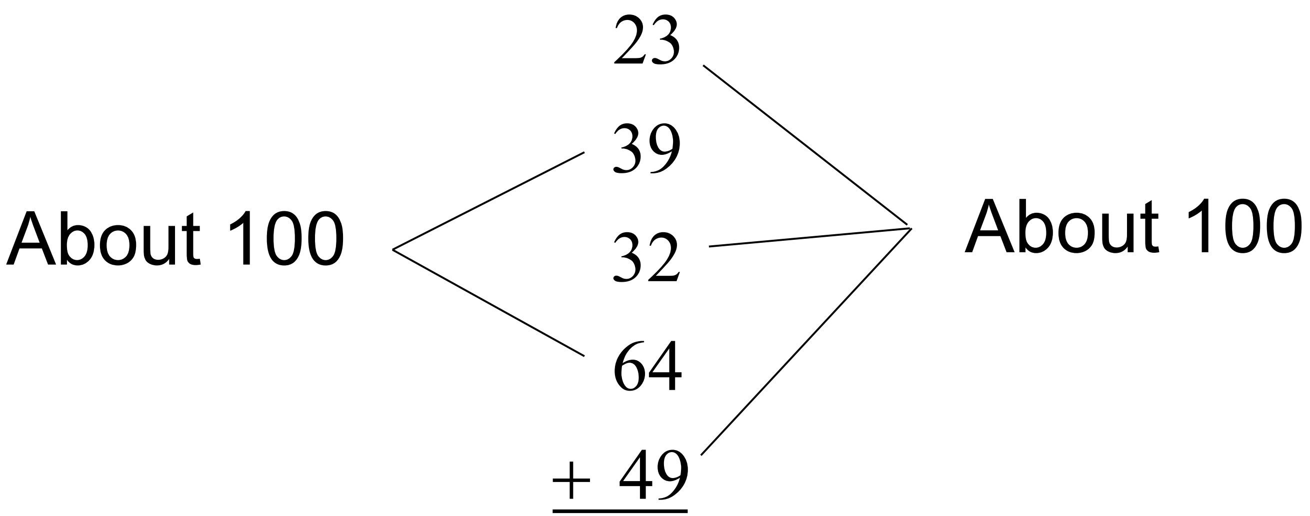
1. Front-end with adjustment

$$\begin{array}{r} 474 \\ 522 \\ +\ 231 \\ \hline \end{array}$$

- Add front-end digits: $4+5+2=11.$
- Place value = 1100.
- Adjust: $22+31\approx 50$ and $74\approx 70,$ so $50+70=120.$
- Adjusted estimate is $1100+120=1220.$

Computational Estimation (2 of 5)

1. Grouping to nice numbers



The sum is about 200.

Computational Estimation (3 of 5)

3. Clustering

Used when a group of numbers cluster around a common value

- Estimate the "average": about 5000
- Multiply the "average" by the number of values: $5 \times 5000 = 25,000$

$$\begin{array}{r} 4724 \\ 5262 \\ 5206 \\ 4992 \\ +\ 5331 \\ \hline \end{array}$$

Computational Estimation (4 of 5)

4. Rounding

$$\begin{array}{r} 7262 \\ \underline{-3806} \end{array} \rightarrow \begin{array}{r} 7000 \\ \underline{-4000} \\ 3000 \end{array}$$

Computational Estimation (5 of 5)

5. Using the range

Problem	Low Estimate	High Estimate
7262	7000	8000
+ 3806	+ 3000	+ 4000
	10,000	12,000