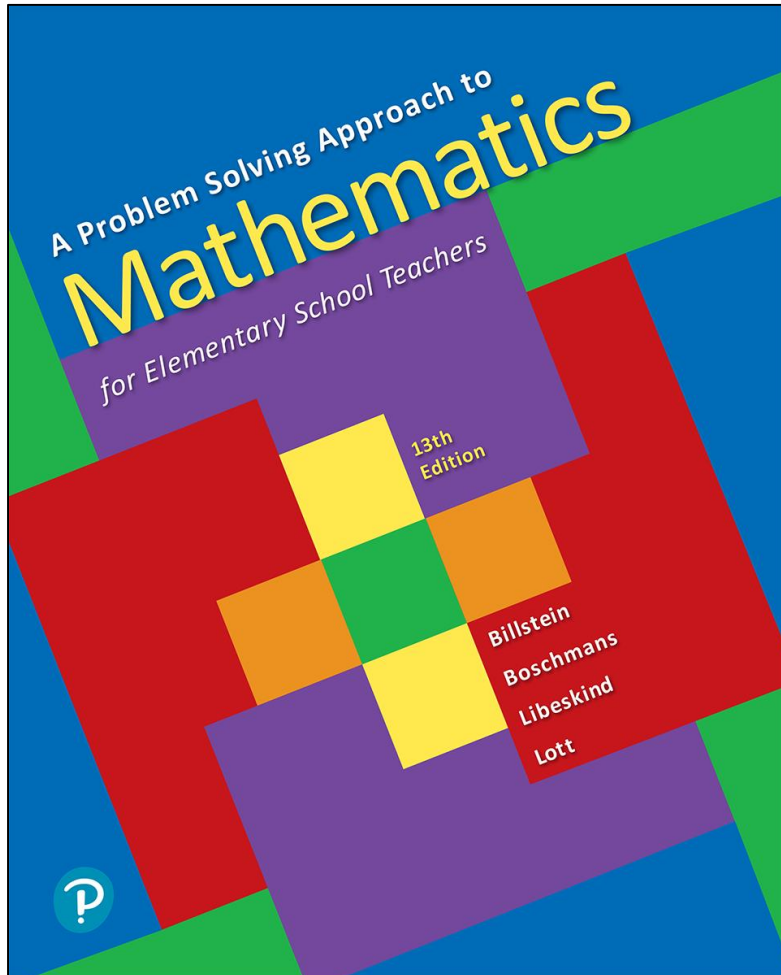


A Problem Solving Approach to Mathematics for Elementary School Teachers

Thirteenth Edition



Chapter 13

Area, Pythagorean Theorem, and Volume

Section 13-5 Volume, Mass, and Temperature

Students will be able to understand and explain

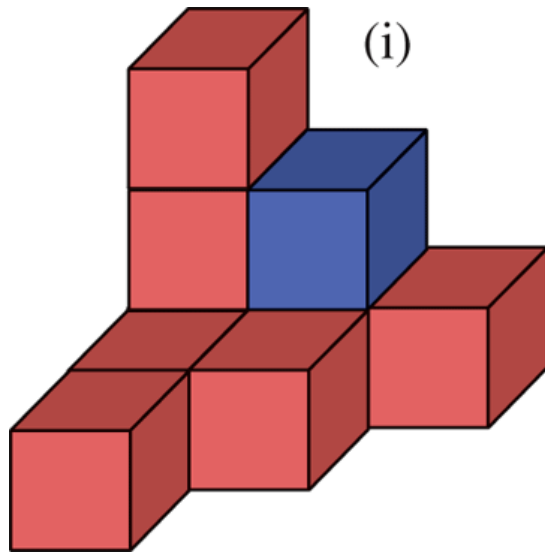
- Volumes of prisms, cylinders, pyramids, cones, and spheres.
- Converting metric measures of volume.
- Converting English measures of volume.
- Ratios of volume of similar figures.
- Measures of mass and capacity.

Volume and Surface Area

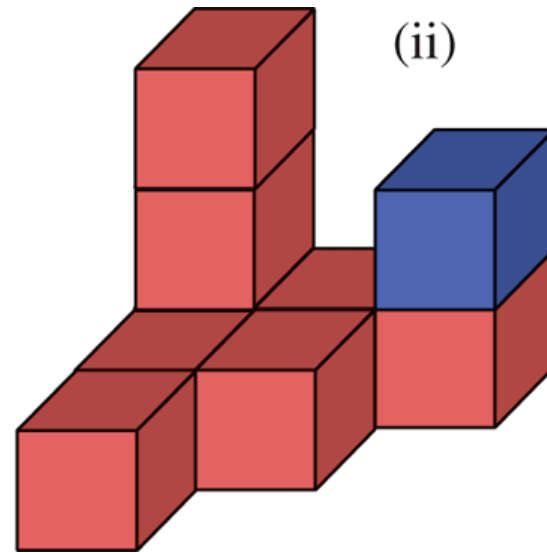
What is the volume of each figure?

9 cubic units

What is the surface area of each figure?

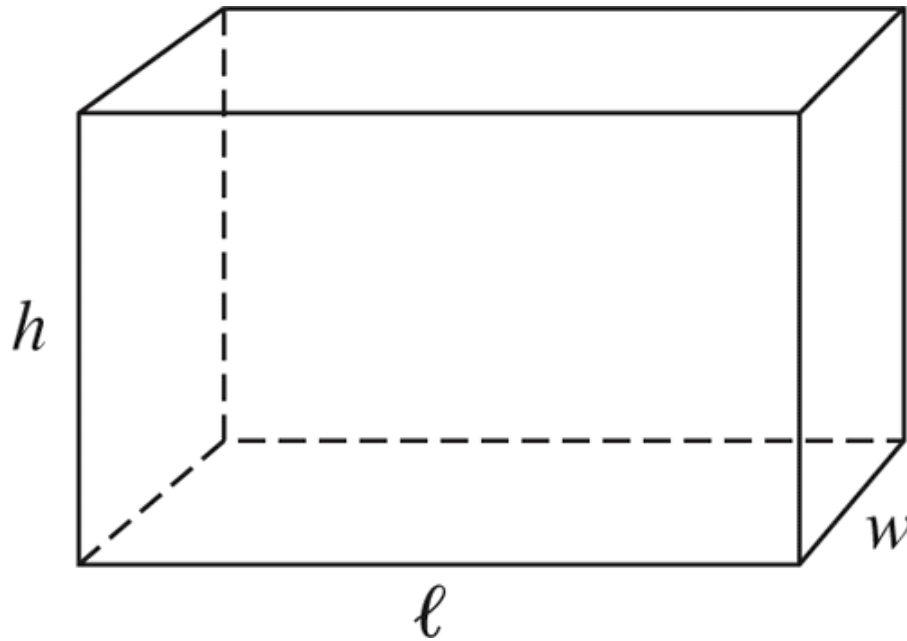


34 square units



36 square units

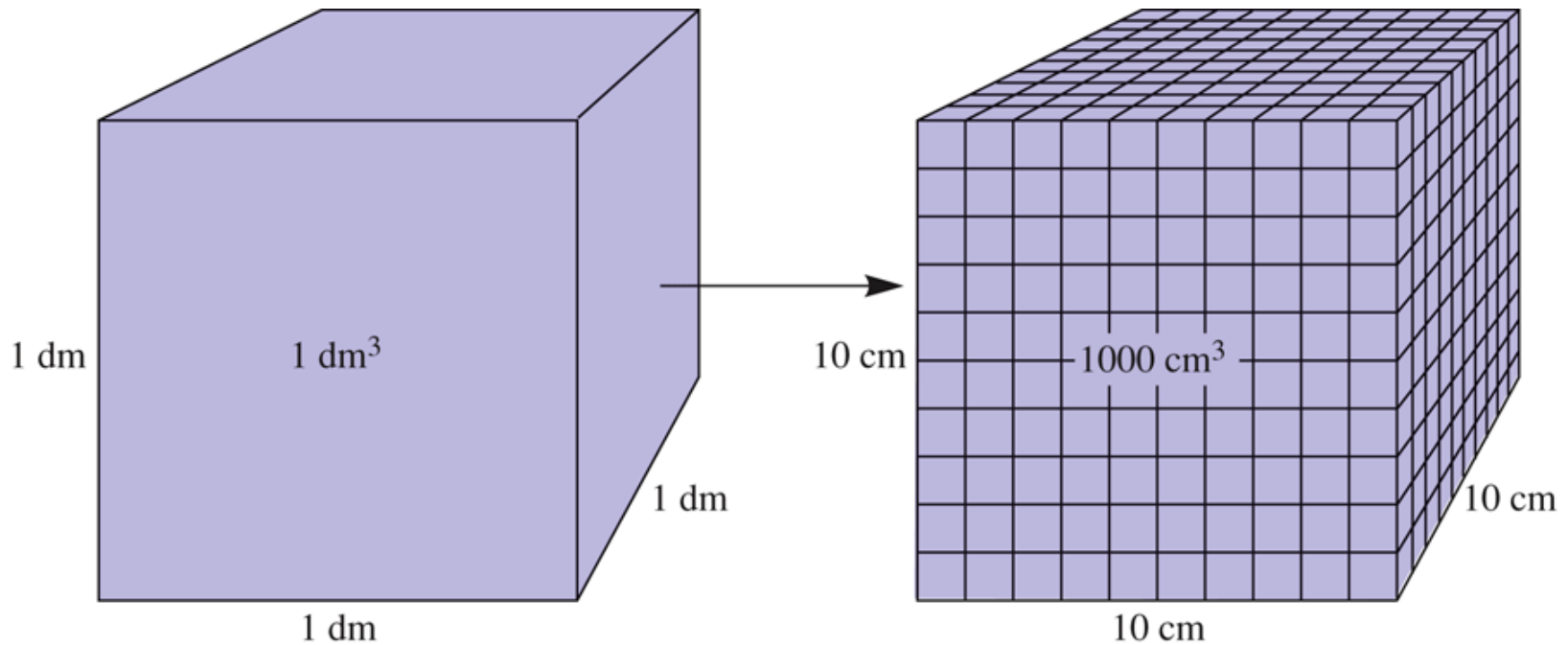
Volume of Right Rectangular Prisms



Right rectangular prism

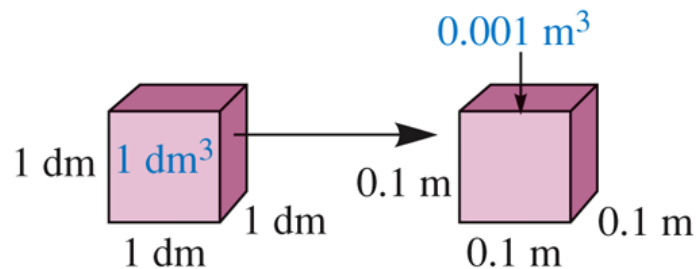
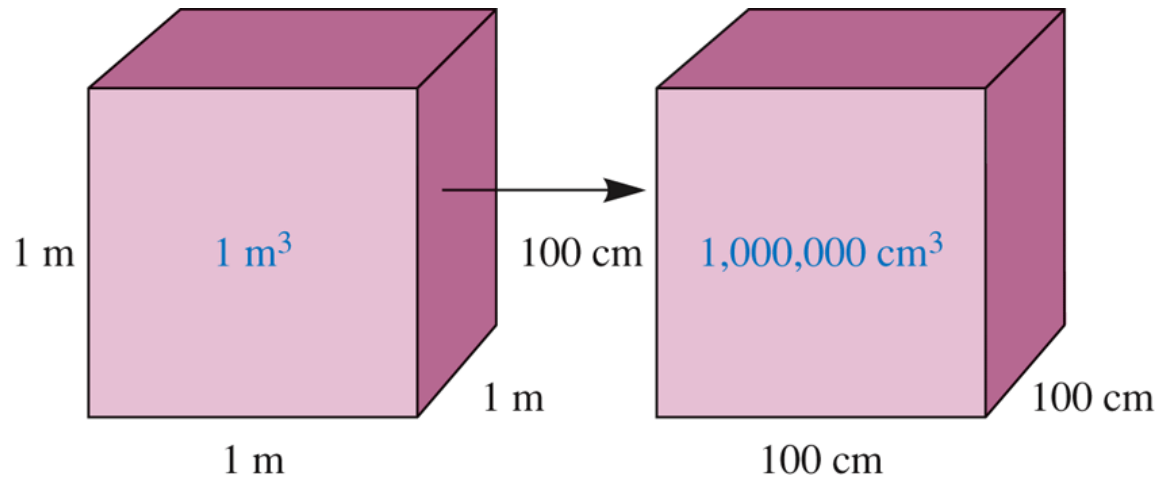
$$\text{Volume} = lwh$$

Converting Metric Measures of Volume (1 of 2)



$$1 \text{ dm}^3 = 1000 \text{ cm}^3$$

Converting Metric Measures of Volume (2 of 2)



$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

$$1 \text{ dm}^3 = 0.001 \text{ m}^3$$

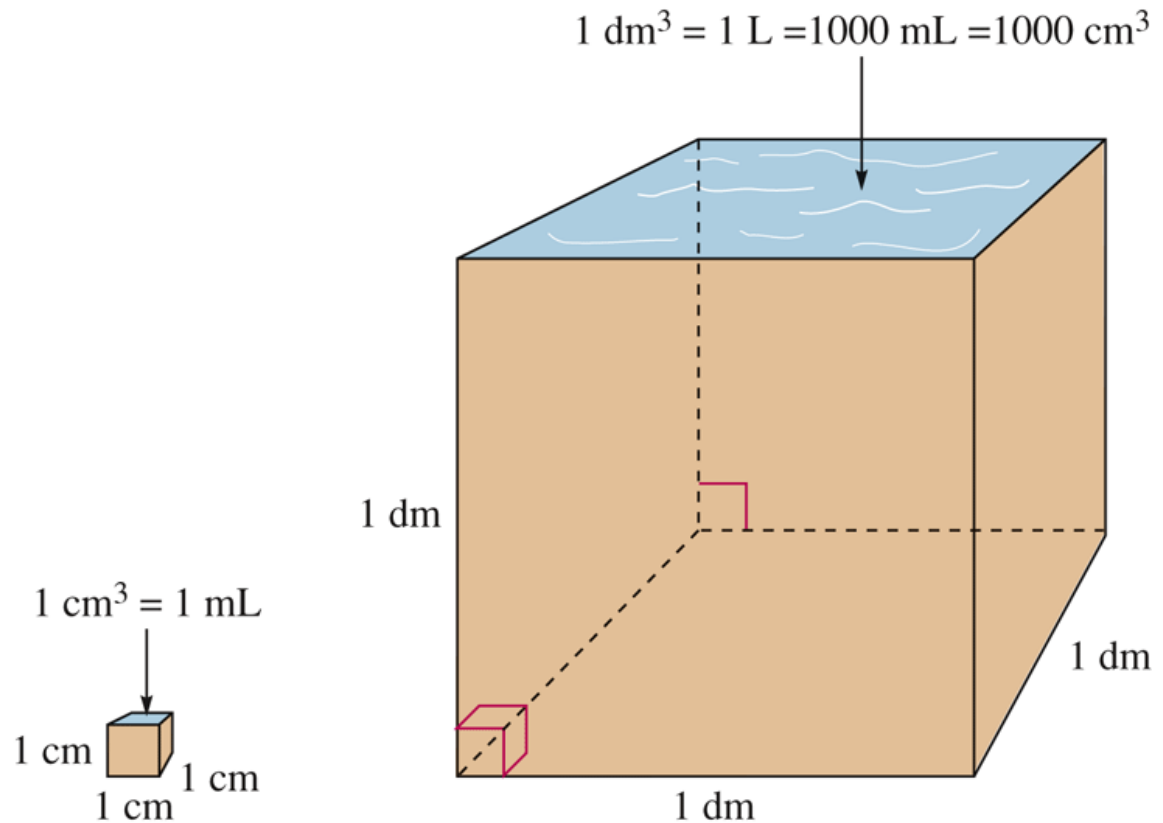
Example 27

Convert each of the following:

a. $5 \text{ m}^3 = \underline{5,000,000} \text{ cm}^3$

b. $12,300 \text{ mm}^3 = \underline{12.3} \text{ cm}^3$

Converting Metric Measures of Volume



$$1 \text{ liter} = 1 \text{ dm}^3 = 1000 \text{ cm}^3$$

Metric Volumes

Unit	Symbol	Relation to Liter
kiloliter	kL	1000 L
hectoliter	hL	100 L
dekaliter	daL	10 L
liter	L	1 L
deciliter	dL	0.1 L
centiliter	cL	0.01 L
milliliter	mL	0.001 L

Example 28

Convert each of the following:

a. $27 \text{ L} = \underline{27,000} \text{ mL}$

b. $362 \text{ mL} = \underline{0.362} \text{ L}$

c. $3 \text{ mL} = \underline{3} \text{ cm}^3$

d. $3 \text{ m}^3 = \underline{3000} \text{ L}$

Converting English Measures of Volume

Basic Units of Volume:

cubic inch (in.³)

cubic foot (ft³)

cubic yard (yd³)

Volume equivalents:

$$1 \text{ gal} = 231 \text{ in.}^3$$

$$1 \text{ qt} = \frac{1}{4} \text{ gal}$$

Example 29 (1 of 2)

Convert each of the following:

a. $45 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$

$$45 \text{ yd}^3 = 45 \text{ yd}^3 \cdot \frac{(3 \text{ ft})^3}{1 \text{ yd}^3} = 45 \text{ yd}^3 \cdot \frac{27 \text{ ft}^3}{1 \text{ yd}^3} = 1215 \text{ ft}^3$$

b. $4320 \text{ in.}^3 = \underline{\hspace{2cm}} \text{ yd}^3$

$$4320 \text{ in.}^3 = 4320 \text{ in.}^3 \cdot \frac{\left(\frac{1}{36} \text{ yd}\right)^3}{1 \text{ in.}^3} \approx 0.0926 \text{ yd}^3 \approx 0.1 \text{ yd}^3$$

Example 29 (2 of 2)

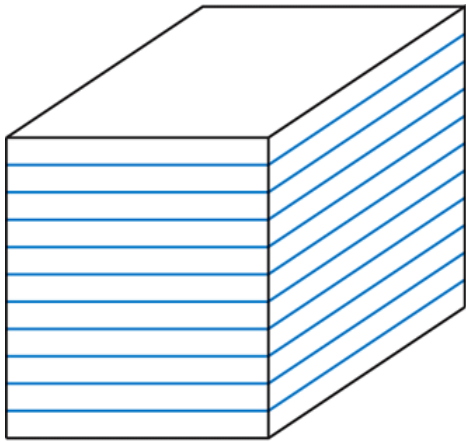
c. $10 \text{ gal} = \underline{\hspace{2cm}} \text{ ft}^3$

$$10 \text{ gal} = 10 \text{ gal} \cdot \frac{231 \text{ in.}^3}{1 \text{ gal}} \cdot \frac{\left(\frac{1}{12} \text{ ft}\right)^3}{1 \text{ in.}^3} \approx 1.337 \text{ ft}^3$$

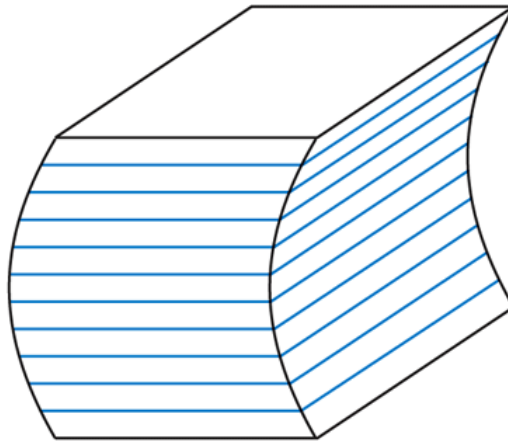
d. $3 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ yd}^3$

$$3 \text{ ft}^3 = 3 \text{ ft}^3 \cdot \frac{\frac{1}{27} \text{ yd}^3}{1 \text{ ft}^3} = \frac{1}{9} \text{ yd}^3 \approx 0.1 \text{ yd}^3$$

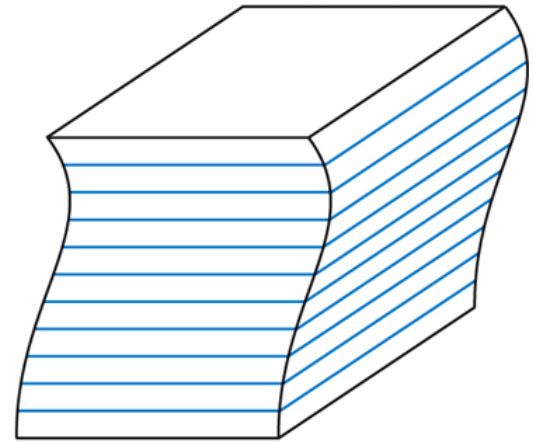
Volumes of Prisms and Cylinders



(a)



(b)



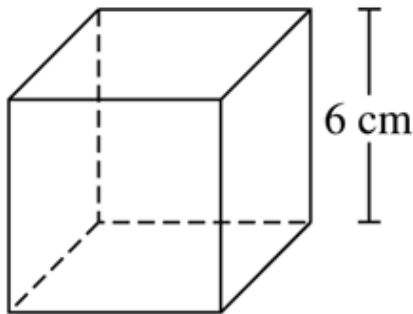
(c)

Cavalieri's Principle

Two solids each with a base in the same plane have equal volumes if every plane parallel to the bases intersects the solids in cross sections of equal area.

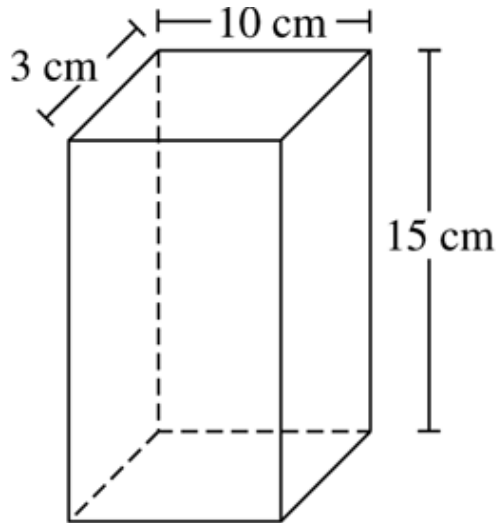
Example 30

Find the volume of each figure.



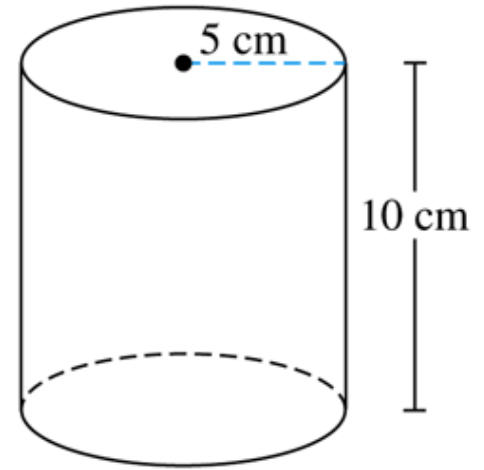
Cube

$$216 \text{ cm}^3$$



Right rectangular prism

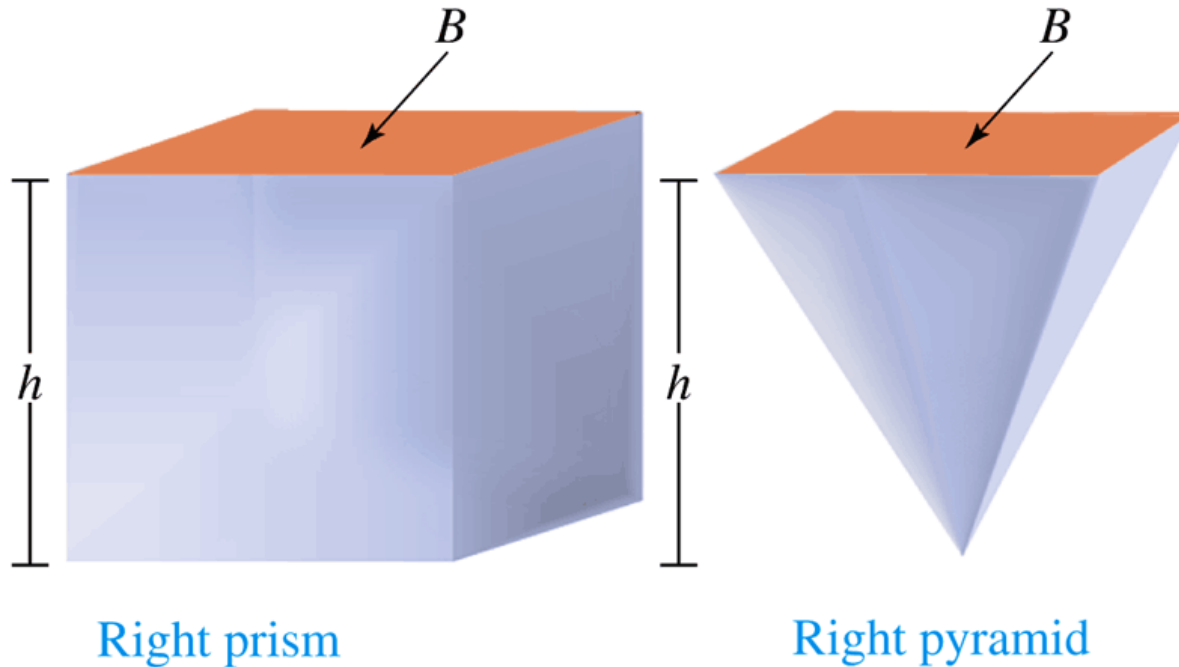
$$450 \text{ cm}^3$$



Right circular cylinder

$$250\pi \text{ cm}^3$$

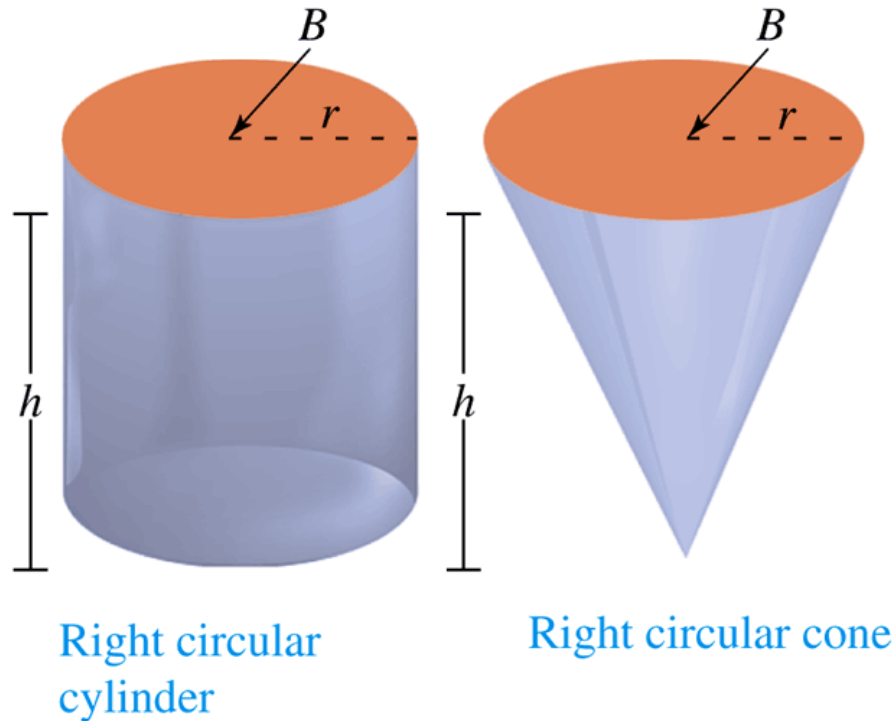
Volumes of Pyramids and Cones (1 of 2)



Right pyramid

Volume = $\frac{1}{3} Bh$ where B is the area of the base.

Volumes of Pyramids and Cones (2 of 2)

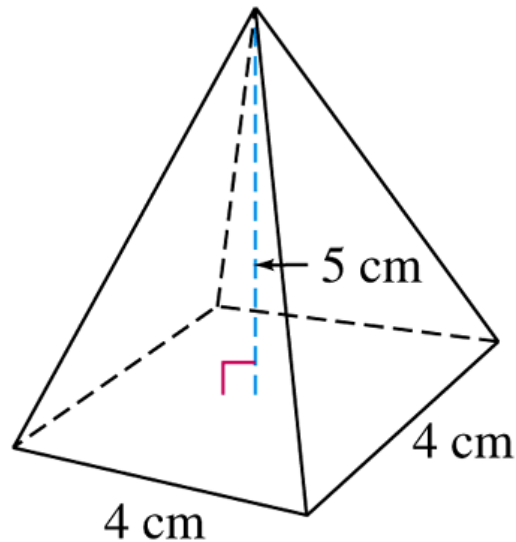


Right circular cone

$$\text{Volume} = \frac{1}{3} Bh = \frac{1}{3} \pi r^2 h$$

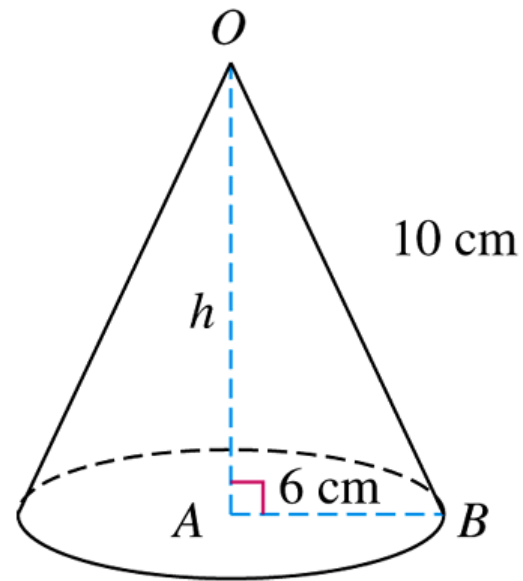
Example 31

Find the volume of each figure.



Right square pyramid

$$\frac{80}{3} \text{ cm}^3$$

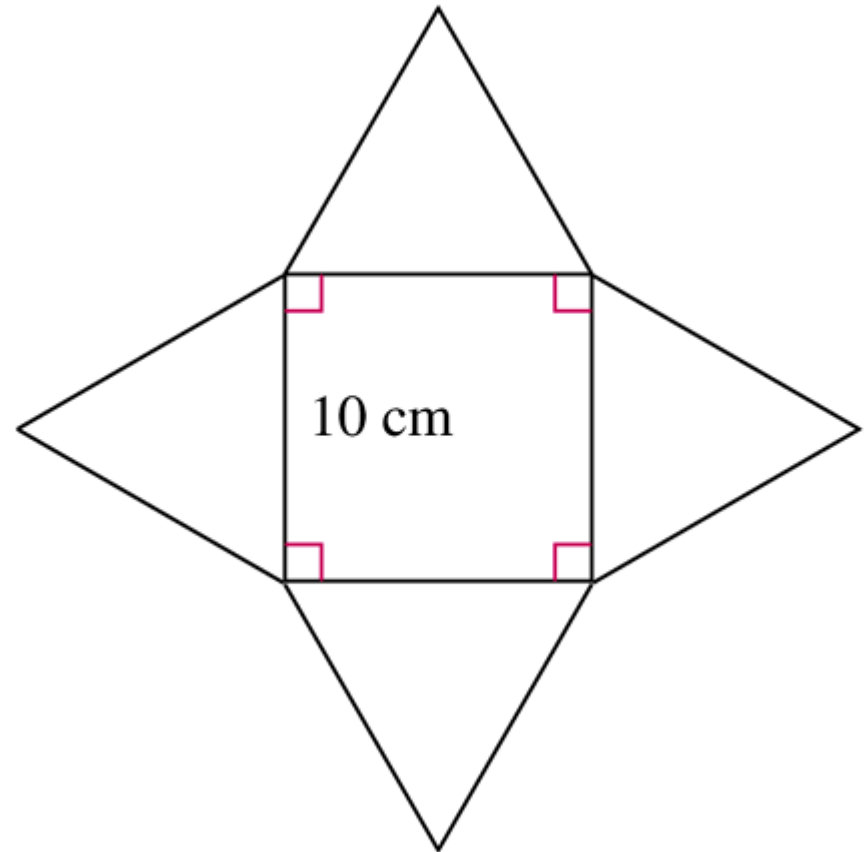


Right circular cone

$$96\pi \text{ cm}^3$$

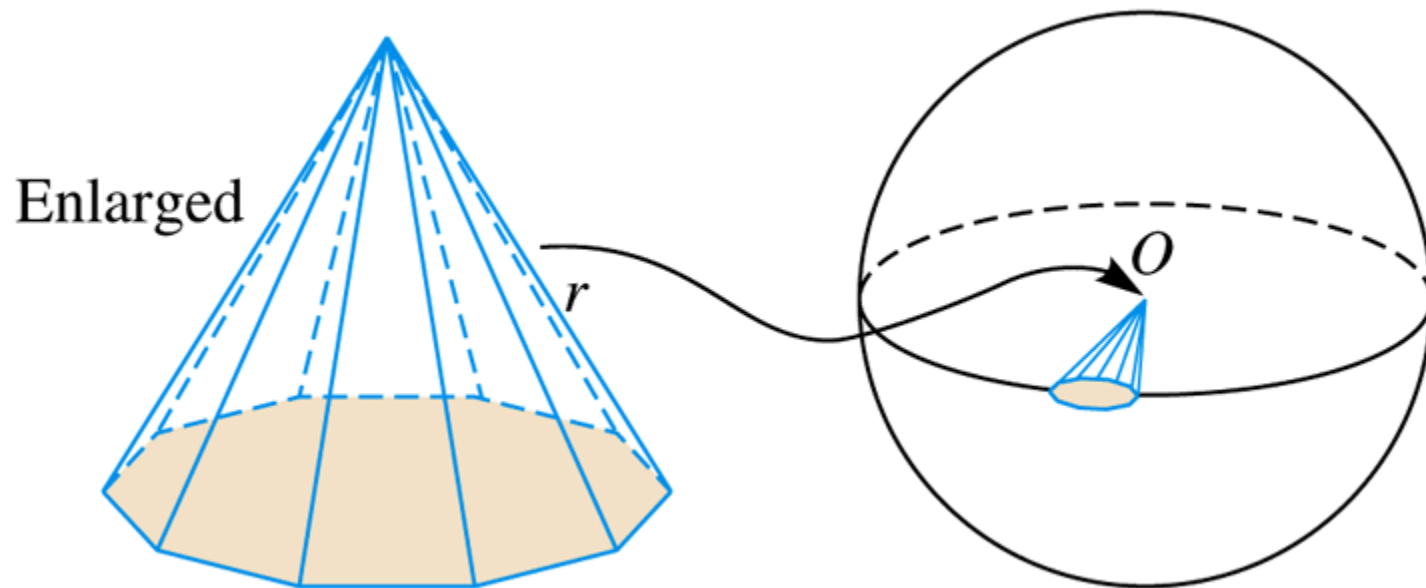
Example 5

Find the volume of the pyramid represented by the net shown. Each triangle is equilateral.



$$V = \frac{1}{3} \cdot 10^2 \cdot \sqrt{50} \approx 235.7 \text{ cm}^3$$

Volume of a Sphere



Sphere

$$\text{Volume} = \frac{4}{3} \pi r^2$$

Example 33

Find the volume of a sphere whose radius is 6 cm.

$$V = \frac{4}{3}\pi (6 \text{ cm})^3 = 288\pi \text{ cm}^3$$

Comparing Measurements of Similar Figures Theorems (1 of 2)

The ratio of any linear measurement of two similar figures, for example, length, width, height, perimeter, diagonal, diameter, slant height, have the same scale factor k .

For similar triangles with scale factor k , the ratio of their areas is k^2 .

For similar polygons with scale factor k , the ratio of their areas is k^2 .

Comparing Measurements of Similar Figures Theorems (2 of 2)

If the scale factor of two similar figures is k , then the scale factor of the areas or surface areas is k^2 , and the ratio of volumes is k^3 .

Example 34a

How does the surface area of a sphere 10 in. in diameter compare with the surface area of a sphere 5 in. in diameter?

Any two spheres are similar.

The ratios of the diameters is 10:5 or 2.

Ratio of surface areas is 2^2 or 4.

The 10-in sphere has 4 times the surface area of the 5-inch sphere.

Example 34b

How does the volume of a sphere 10 in. in diameter compare with the volume of a sphere 5 in. in diameter?

The ratios of the volumes is 2^3 or 8.

The volume of a 10-in sphere has 8 times the volume of the 5-inch sphere.

Mass (1 of 2)

Mass: a quantity of matter

Weight: a force exerted by gravitational pull

Gram: fundamental unit of mass in the metric system

Mass (2 of 2)

Unit	Symbol	Relation to Gram
metric ton	T	1,000,000 g
kilogram	kg	1000 g
hectogram	hg	100 g
dekagram	dag	10 g
gram	g	1 g
decigram	dg	0.1 g
centigram	cg	0.01 g
milligram	mg	0.001 g

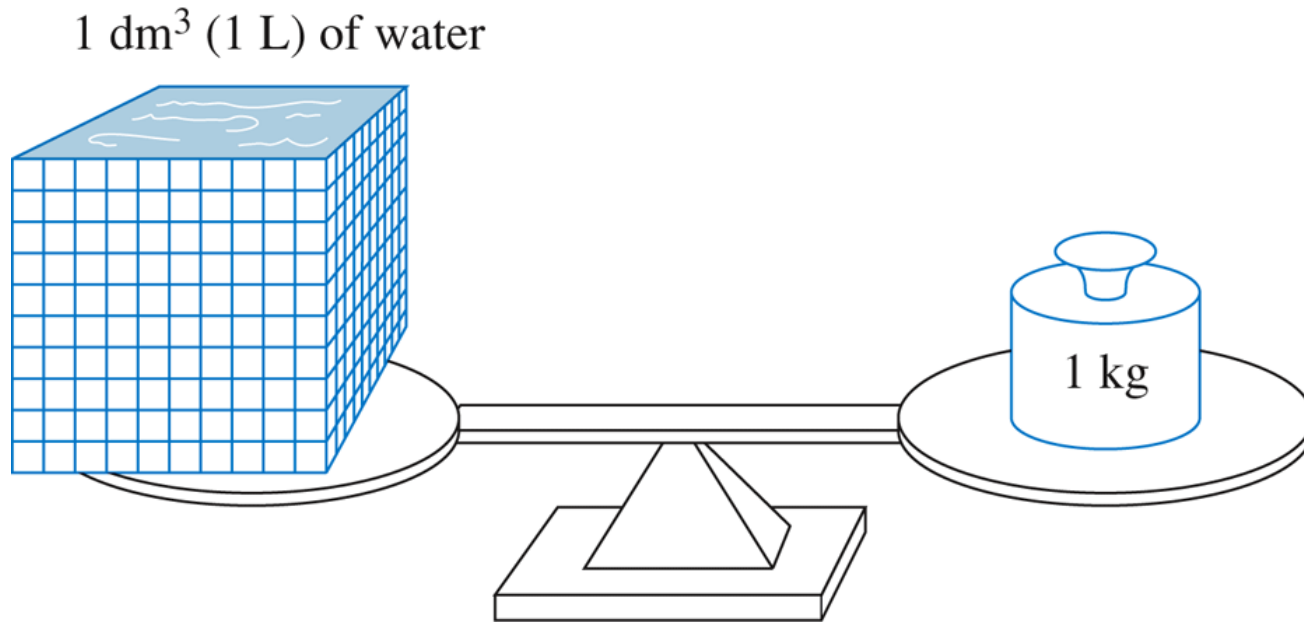
Example 35

Convert each of the following:

a. $34 \text{ g} = \underline{0.034} \text{ kg}$

b. $6836 \text{ kg} = \underline{6.836} \text{ t}$

Relationships Among Metric Units of Volume, Capacity, and Mass



1 cm³ (1 mL) of water = 1 g

1 dm³ (1 L) of water = 1 kg

Example 36

A waterbed measures $180 \text{ cm} \times 210 \text{ cm} \times 20 \text{ cm}$.

a. Approximately how many liters of water can it hold?

$$\begin{aligned} V &= 180 \text{ cm} \cdot 210 \text{ cm} \cdot 20 \text{ cm} = 756,000 \text{ cm}^3 \\ &= 756,000 \text{ mL} = 756 \text{ L} \end{aligned}$$

b. What is its mass in kilograms when it is full of water?

Because 1 L of water has a mass of 1 kg, 756 L of water has a mass of 756 kg, which is 0.756 t.