Analyzing the Graph of a Function

We will use all of the techniques developed in this chapter and in earlier chapters in order to make a comprehensive analysis of different functions *using algebra only*, *if possible*.

Exercise 1: Analyze the graph of $f(x) = -2x^4 + 24x^3 - 96x^2 + 128x$.

Domain:

Vertical Asymptote(s):

First Derivative and Critical Point(s):

$f'(x) = -8x^3 + 72x^2 - 192x + 128 = -8\left(x^3 - 9x^2 + 24x - 16\right)$

Since the coefficients add up to 0, $x = 1$ is a root of f'. Thus, is a factor of f'.

Therefore, the critical *points* of *f* are ***(1 , f(1)) = (1 , 54)*** and ***(4 , f(4)) = (4 , 0)***.

Sign Chart for First Derivative, Interval Descriptions, and Classification of Critical Numbers:

Second Derivative and *Candidates* for Inflection Point(s):

$f''(x) =$ $= -24\left(x^2 - 6x + 8\right) =$

The solutions to $f''(x) = 0$ are *candidates* for the x-coordinates of inflection points. Thus, *(2 , f(2)) = (2 , 32)* and *(4 , f(4)) = (4 , 0)* are *candidates* for inflection points.

Sign Chart for Second Derivative, Interval Descriptions, and Determination of Candidates:

Intercepts:

To determine the *x*-intercept(s), set *y = f(x)* equal to *0* and solve for *x*. In this case, solving $0 = -2x^4 + 24x^3 - 96x^2 + 128x$ might be too hard algebraically. However, we notice that $x = 0$ solves this polynomial equation because there is no nonzero constant term. Also, in the work on page 1 for the critical points, after we found $x = 4$ to be a critical number, we calculated that its y-coordinate is 0, which indicates that $x = 4$ is a solution to the polynomial equation above. Thus, two of the x-intercepts of the graph of f are *(0 , 0)* and *(4 , 0)*. It is left to you to confirm, using synthetic division to factor *f(x)*, that $f(x) = x(x-4)^3$, which means that the graph of f has only two x-intercepts: *(0 , 0)* and *(4 , 0)*.

To determine the *y*-intercept, set *x* equal to *0*. We get $y = 0$, so the y-intercept is *(0 , 0)*.

End Behavior and Horizontal Asymptote(s):

As $x \to -\infty$, $f(x) \to$ and as $x \to \infty$, $f(x) \to$.

We know these two facts from College Algebra, since f is an - degree

with .

Therefore, because the end behavior is unbounded, there are

Exercise 2: Analyze the graph of $g(x)=\dfrac{3x^2-12}{2x^2-18}$.

Domain:

Vertical Asymptote(s):

First Derivative and Critical Point(s):

x-intercept(s):

y-intercept:

End Behavior and Horizontal Asymptote(s):

Second Derivative and *Candidates* for Inflection Point(s):

$$g''(x) \quad = \quad \frac{d}{dx}\left(\frac{-60x}{\left(2x^2-18\right)^2}\right) \quad = \quad \frac{(-60x)'\left(2x^2-18\right)^2-(-60x)\left(\left(2x^2-18\right)^2\right)'}{\left[\left(2x^2-18\right)^2\right]^2}$$

$$= \quad \frac{-60\left(2x^2-18\right)^2-(-60x)\cdot 2\left(2x^2-18\right)^1\cdot 4x}{\left(2x^2-18\right)^4} \quad =$$

Second Derivative Test to Classify the Critical Point:

Exercise 3: Analyze the graph of $h(x) = \dfrac{\sin(x)}{1+\cos(x)}$.

Domain:

Vertical Asymptote(s):

First Derivative and Critical Point(s):

$$h'(x) \quad = \quad \frac{(\sin(x))'(1+\cos(x)) - \sin(x)(1+\cos(x))'}{[1+\cos(x)]^2} \quad =$$

$$= \qquad\qquad = \qquad\qquad =$$

$h'(x) = \dfrac{1}{1+\cos(x)}$ = *0* for

Therefore,

Note: $h'(x) > 0$ on each interval of its domain; h is increasing on each of these intervals.

<u>Second Derivative and *Candidates* for Inflection Point(s)</u>:

$$h'(x) = \frac{1}{1+\cos(x)} = (1+\cos(x))^{-1} \Rightarrow h''(x) =$$

$$h''(x) = \frac{\sin(x)}{(1+\cos(x))^2}$$ = *0* for *x*-values such that --that is, for *x* =

<u>Sign Chart to Test Candidates for Inflection Points</u>:

<u>*x*-intercept(s)</u>:

<u>*y*-intercept</u>:

<u>End Behavior and Horizontal Asymptote(s)</u>:
From a graph, we see that the limit of *h(x)* does not exist as $x \to -\infty$ or as $x \to \infty$. Therefore, there are no horizontal asymptotes.