Revisiting Indeterminate Forms of Limits

Suppose that, upon evaluating the limit $\lim_{x \to x_0} \frac{f(x)}{g(x)}$ by direct substitution, we get $\frac{0}{0}$. In earlier chapters, we were occasionally able to genetically alter (factor or apply some other algebraically invasive procedure) the molecular structure of the functions f and g. Sometimes we were not able to do so, and we had to rely therefore on a table to evaluate the limit.

We will now study a powerful tool, called *L'Hôpital's Rule*. The reason why ***L'Hôpital's Rule can only be used when direct substitution has rendered an indeterminate form*** will be discussed more fully in Calculus II.

L'Hôpital's Rule

Theorem: Let f' and g' be continuous at $x = x_0$.

Then $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0}$ ________ .

Exercise 1: Example with indeterminate form $\frac{0}{0}$. Evaluate $\lim_{x \to 2} \frac{3x^2 - 12}{x - 2}$.

You may recall this example from pages 2 and 3 of Handout 2.2, Exercises 2a and 2b.

Direct Substitution:

L'Hôpital's Rule:

Exercise 2: Another example with indeterminate form $\frac{0}{0}$. Evaluate $\lim_{x \to 0} \frac{e^{2x} - 1}{x}$.

Direct Substitution:

L'Hôpital's Rule:

Exercise 3: Example with indeterminate form $\frac{\infty}{\infty}$. Evaluate $\lim_{x\to\infty} \frac{\ln(x)}{x}$.

$$\frac{\lim_{x\to\infty} \ln(x)}{\lim_{x\to\infty} x} =$$

L'Hôpital's Rule:

Exercise 4: Example applying L'Hôpital's Rule more than once. Evaluate $\lim_{x\to-\infty} \frac{x^2}{e^{-x}}$.

$$\frac{\lim_{x\to-\infty} x^2}{\lim_{x\to-\infty} e^{-x}} =$$

L'Hôpital's Rule:

Exercise 5: Example with indeterminate form $0 \cdot \infty$. Evaluate $\lim_{x\to\infty} e^{-x}\sqrt{x}$.

Exercise 6: Example with indeterminate form 1^{∞}. Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$.

Exercise 7: Find the ***error*** in the following sequence of steps.

$$\lim_{x\to 0}\frac{e^{2x}-1}{e^{x}} = \lim_{x\to 0}\frac{\left(e^{2x}-1\right)'}{\left(e^{x}\right)'} = \lim_{x\to 0}\frac{2e^{2x}}{e^{x}} = \lim_{x\to 0}\frac{2\left(e^{x}\right)^{2}}{e^{x}} = \lim_{x\to 0}2e^{x} = 2$$ (**incorrect**)

Note: See the textbook for examples involving other indeterminate forms: 0^{0} and $\infty-\infty$.