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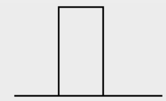
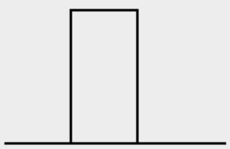
Use the completed handout to complete the notes.

Volume of a Solid of Revolution: Using the Disk Method

Another application of the definite integral is the computation of the volume of a particular type of three-dimensional solid, called a *solid of revolution*. A solid of revolution is obtained by revolving a region in the plane about a line. This line is called the *axis of revolution*.



Revolving this rectangle, we get a disk—a hockey puck. Compute the volume of the disk:



- cross-sectional area is the area of a =
- width = Therefore, volume =

Revolving a More General Region

- The volume of a solid of revolution is approximated by the of the
- The radius of each disc is R_i = for some
- The width of each disk is

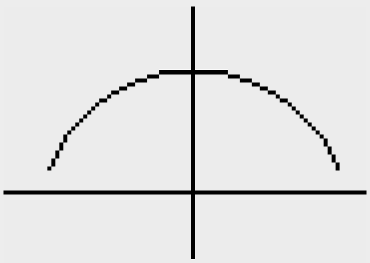
The *approximate* volume of the solid of revolution is

The *exact* volume of the solid of revolution is

Note: In the case that the **x-axis is the axis of revolution**, then $R(x)$ =

Watch this video for an example of determining the volume of a solid of revolution.

Exercise 1: Determine the volume of the solid of revolution formed by revolving the region bounded by $f(x) = \sqrt{\cos(x)}$, the x-axis, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, about the x-axis.

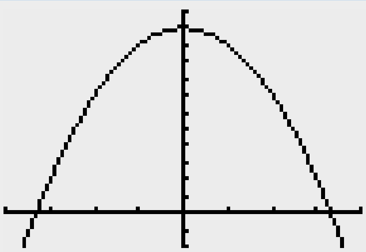


Note: In this exercise, $R(x)$ =

since the is the axis of revolution.

Volume =

Exercise 2: Determine the volume of the solid of revolution formed by revolving the region bounded by $f(x) = 11 - x^2$ and y = 2 about the line y = 2.



What is the axis of revolution?

What are a and b? That is, for what x-values does f intersect y = 2? To answer this, we solve

Note that, in this exercise $R(x) \neq f(x)$ since the is ***not*** the

In fact, the radius is the height of the graph of $f(x)$. Thus $R(x)$ =

Vol. = $\pi \int_a^b [R(x)]^2\, dx = \pi \int_{-3}^{3} [\qquad]^2\, dx = \pi \int_{-3}^{3} [\qquad]^2\, dx = \pi \int_{-3}^{3} [\qquad]^2\, dx$

When you determine the volume of a solid of revolution using the *disk method*, your diagram will resemble one of the two following diagrams. **Note that with the *disk method*, the representative rectangle that you draw will be *perpendicular* to the axis of revolution**.

Horizontal Axis of Revolution

volume =

Vertical Axis of Revolution

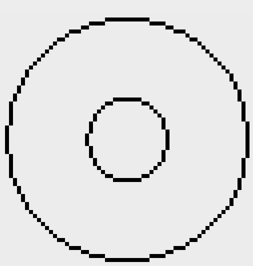
volume =

Annulus

The disk method can be generalized to a method called the *washer method*. Before we discuss the washer method, we must review the definition of an annulus.

Definition: An annulus is a circular region in the plane that has had a smaller, inner, centered, circular region removed.

Watch this video explanation of an annulus.



What is the area of an annulus?

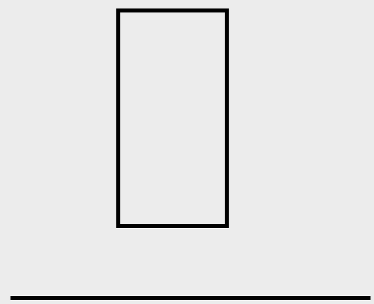
Area =

=

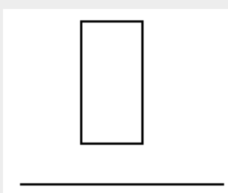
=

Volume of a Solid of Revolution: Using the Washer Method

The disc method can be generalized, or extended, to cover solids of revolution with holes, by replacing the representative disc with a representative washer.



Revolving this rectangle, we get a washer. Compute the volume of the washer:



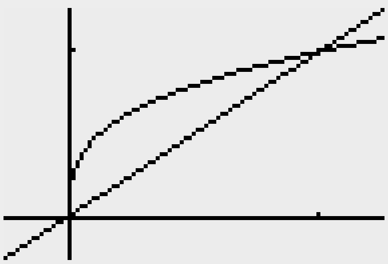
- cross-sectional area is the area of an =
- width = Therefore, volume =

Note that with the *washer method*, as with the disc method, the representative rectangle that you draw is *perpendicular* to the axis of revolution.

In general, by the washer method, the volume of the solid of revolution is

Watch this video example of the washer method.

Exercise 3: Determine the volume of the solid of revolution formed by revolving the region in the first quadrant bounded by the graphs of $y = x$ and $y = \sqrt[3]{x}$ about the x-axis.



First, determine a and b by solving

Hence, a = and b =

Note that $R(x)$ = and $r(x)$ =

Thus, the volume =

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Watch a video example with integration with respect to y.

Watch a video of Exercise 4.

Exercise 4: Determine the volume of the solid of revolution formed by revolving the region bounded by the graphs of $f(x) = 11 + x^2$, the x-axis, the y-axis, and $x = 3$ about the y-axis.

$R(y)$ =

$r(y)$ = distance from y-axis to the graph

= x-coordinate (of the function)

= x

Thus, solve $y = 11 + x^2$ for x.

More Example Videos

Volume of Revolution about a Vertical Line Washer Method video

Volume of Revolution Washer Method Example 1 video

Volume of Revolution Washer Method Example 2 video

Volume of Revolution Washer Method Example 3 video

Textbook Exercises: Section 7. 2

Problems: 5-37 odd, 47, 57, 61, 62, 65, 69a, 73

Textbook Exercises Videos: Section 7. 2

Problem 9

Problem 17

Problem 23

Problem 37

Textbook PowerPoints Slides: Section 7. 2

View a summary of the textbook reading in PowerPoint form.

Hyperlinks:

- Completed Handout: https://cwoer.ccbcmd.edu/math/math252/m252c7s2sol.pdf
- Solid of Revolution video: http://college.cengage.com/mathematics/blackboard/shared/content/video_explanations/v01023a.html
- Solid of Revolution Example video: https://www.youtube.com/embed/DRFyNHdVgUA?r=0
- Annulus video: https://college.cengage.com/mathematics/blackboard/shared/content/video_explanations/video_wrapper.html?filename=v01025a
- Washer Method video: https://college.cengage.com/mathematics/blackboard/shared/content/video_explanations/video_wrapper.html?filename=v01026a
- Integration with Respect to y video: https://www.youtube.com/embed/43AS7bPUORc?r=0
- Exercise 4 video: https://www.youtube.com/embed/iEv-LAVY3EM?r=0
- Volume of Revolution about a Vertical Line Washer Method : http://patrickjmt.com/volumes-of-revolution-volumes-about-vertical-lines-using-washers/
- Volume of Revolution Washer Method Example 1:http://patrickjmt.com/volumes-of-revolution-diskwashers-example-1/
- Volume of Revolution Washer Method Example 2: http://patrickjmt.com/volumes-of-revolution-diskwashers-example-2/

- Volume of Revolution Washer Method Example 3: http://patrickjmt.com/volumes-of-revolution-diskwashers-example-3/
- Textbook Exercises Video 7.2 Problem 9: youtu.be/6Udn9Cj4AdQ
- Textbook Exercises Video 7.2 Problem 17: youtu.be/burfVO5OnJ4
- Textbook Exercises Video 7.2 Problem 23: youtu.be/Rob9-qVo1ZM
- Textbook Exercises Video 7.2 Problem 37: youtu.be/NJuNkOqSyCo
- Textbook PowerPoint 7.2: https://cwoer.ccbcmd.edu/math/math252/Math252Section0702.pptx